# Ultrasound Computed Tomography

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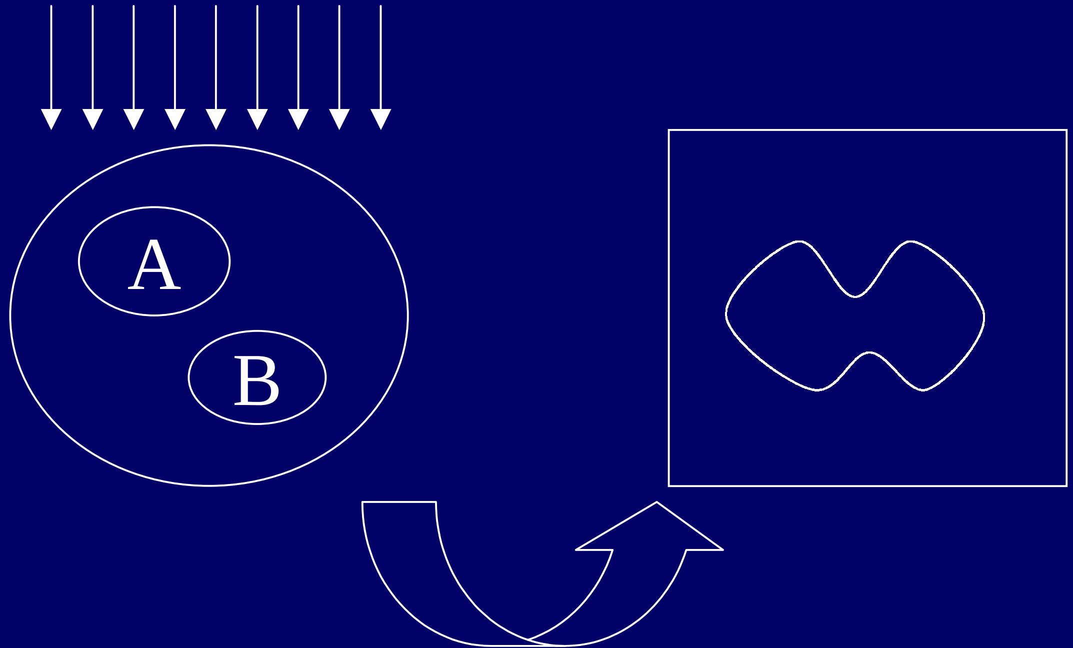
2002/05/15

# Introduction

- Conventional X-ray image is the superposition of all the planes normal to the direction of propagation.
- The tomography image is effectively an image of a slice taken through a 3-D volume.



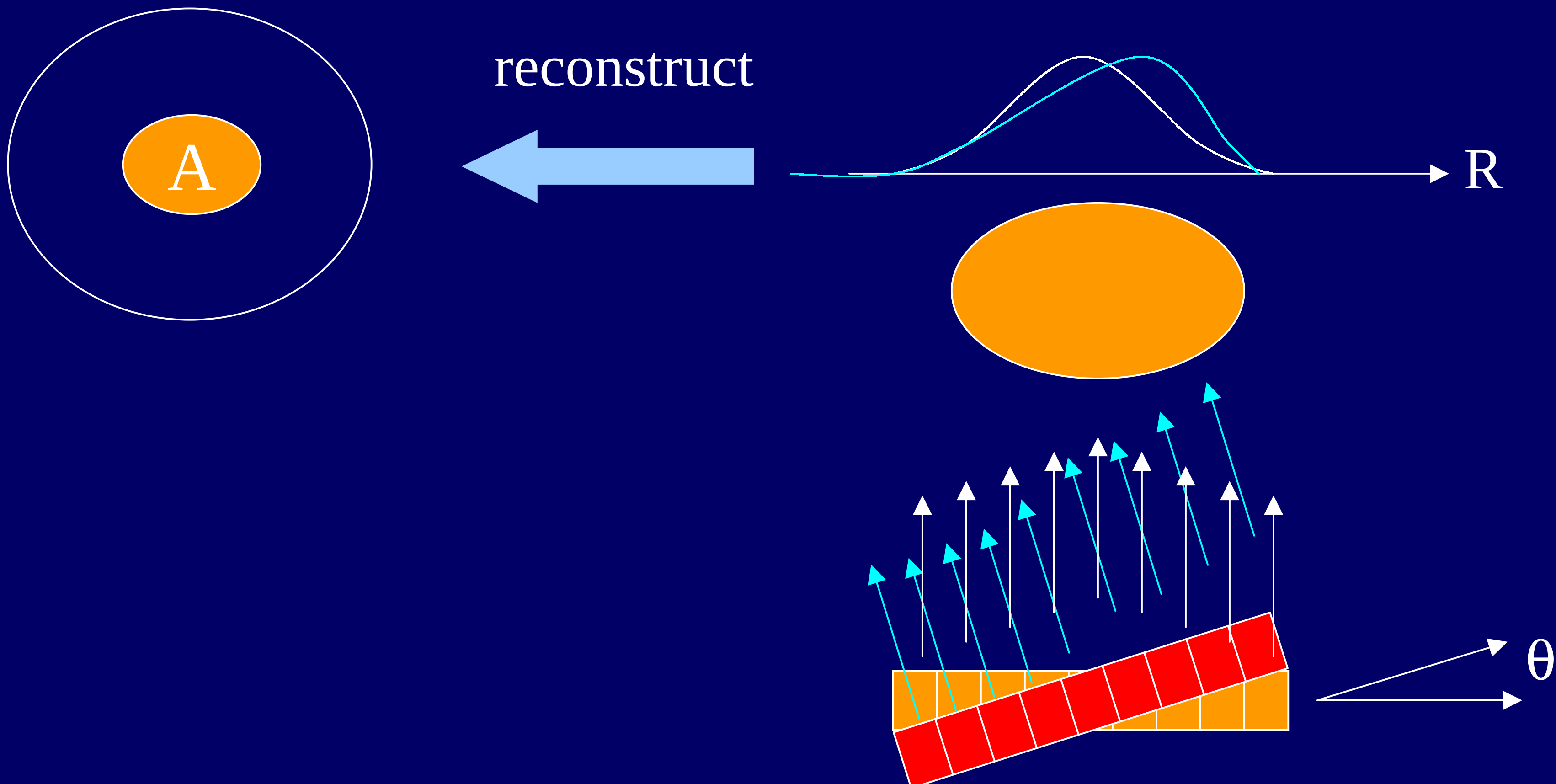
# Clinical application

- Problems of X-ray Mammography
- An estimated 10-15% of breast cancers evade detection by mammography
- Poor differentiation of malignant tumors from highly common cysts (while ultrasound can do so with accuracies of 90-100%)
- UCT can provide not only structural/density information, but also tissue compressibility and speed of sound maps

# Tomography

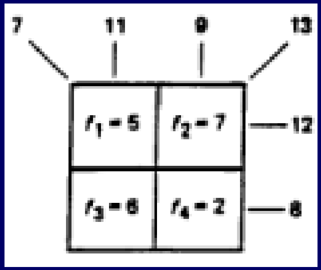
- Time of flight or intensity attenuation
- Array transducer : 1–D data (only $t_f$)
- Rotation : 2-D data ($t_f$ on range and angle $\theta$)
- Scan : 3-D data (y position, $t_f$ and angle $\theta$)

# Tomography

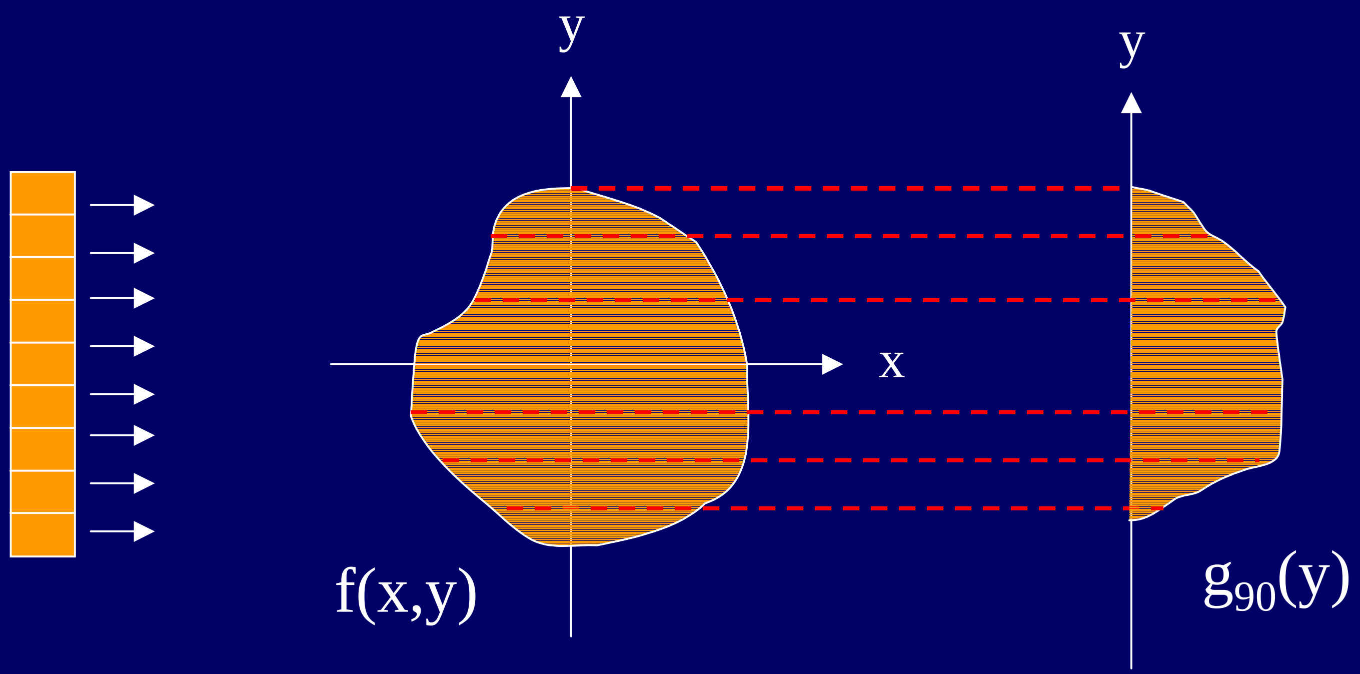


# Reconstruct method

- **Iterative method**
  - **Algebraic Reconstruction Technique (ART)**
- **Direct reconstruction**
  - **Fourier transform**
- **Alternative direct reconstruction**
  - **Back projection**



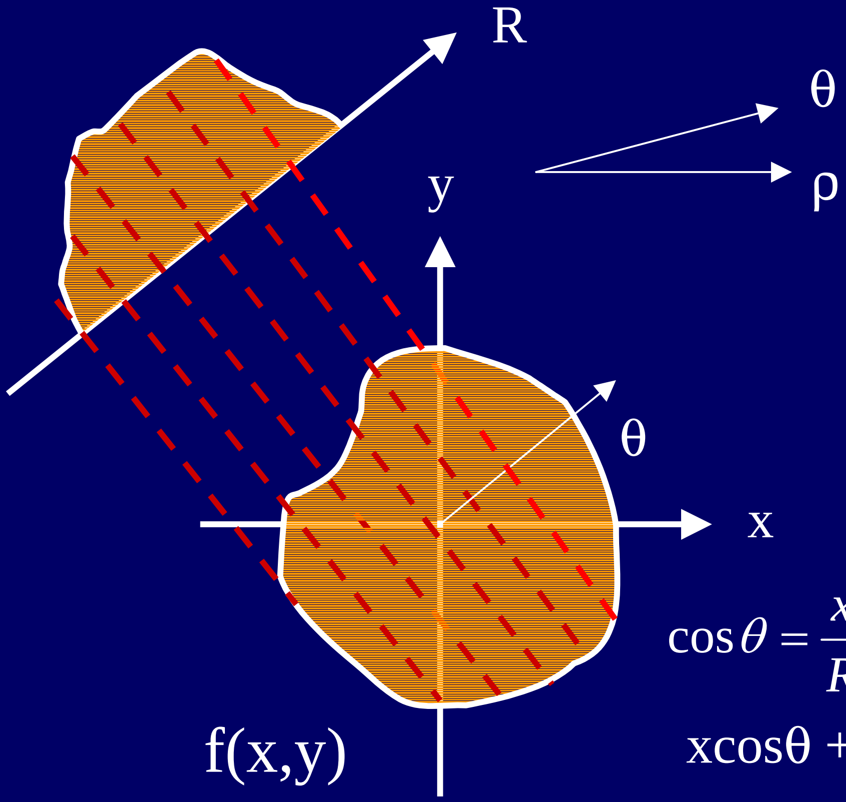
# Central Section Theorem




$$g(y) = \iint f(x,y)\delta(y - y')dxdy$$

$$= \int f(x,y)dx$$

# Ambiguity angle θ




$$\cos\theta = \frac{x}{R}, \sin\theta = \frac{y}{R}$$

$$x\cos\theta + y\sin\theta = R$$

$$g_\theta(R) = \iint f(x,y)\delta(x\cos\theta + y\sin\theta - R)dxdy$$

# Equations

- 1D FT of projection function

$$G_\theta(\rho) = (1D)\,F.T.\{g_\theta(R)\}$$

$$= \iiint f(x,y)\delta(x\cos\theta + y\sin\theta - R)\exp(-i2\pi\rho R)\,dxdydR$$

$$= \iint f(x,y)\exp(-i2\pi\rho(x\cos\theta + y\sin\theta))\,dxdy$$

$$= \iint f(x,y)\exp(-i2\pi(\rho\cos\theta x + \rho\sin\theta y))\,dxdy$$

$$= \iint f(x,y)\exp(-i2\pi(ux + vy))\,dxdy$$

# Cont'd

- 2D Fourier transform

$$f(u,v) = \iint f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

- $(u$,v) in polar coordinates is $(\rho cos\theta, \rho sin\theta)$

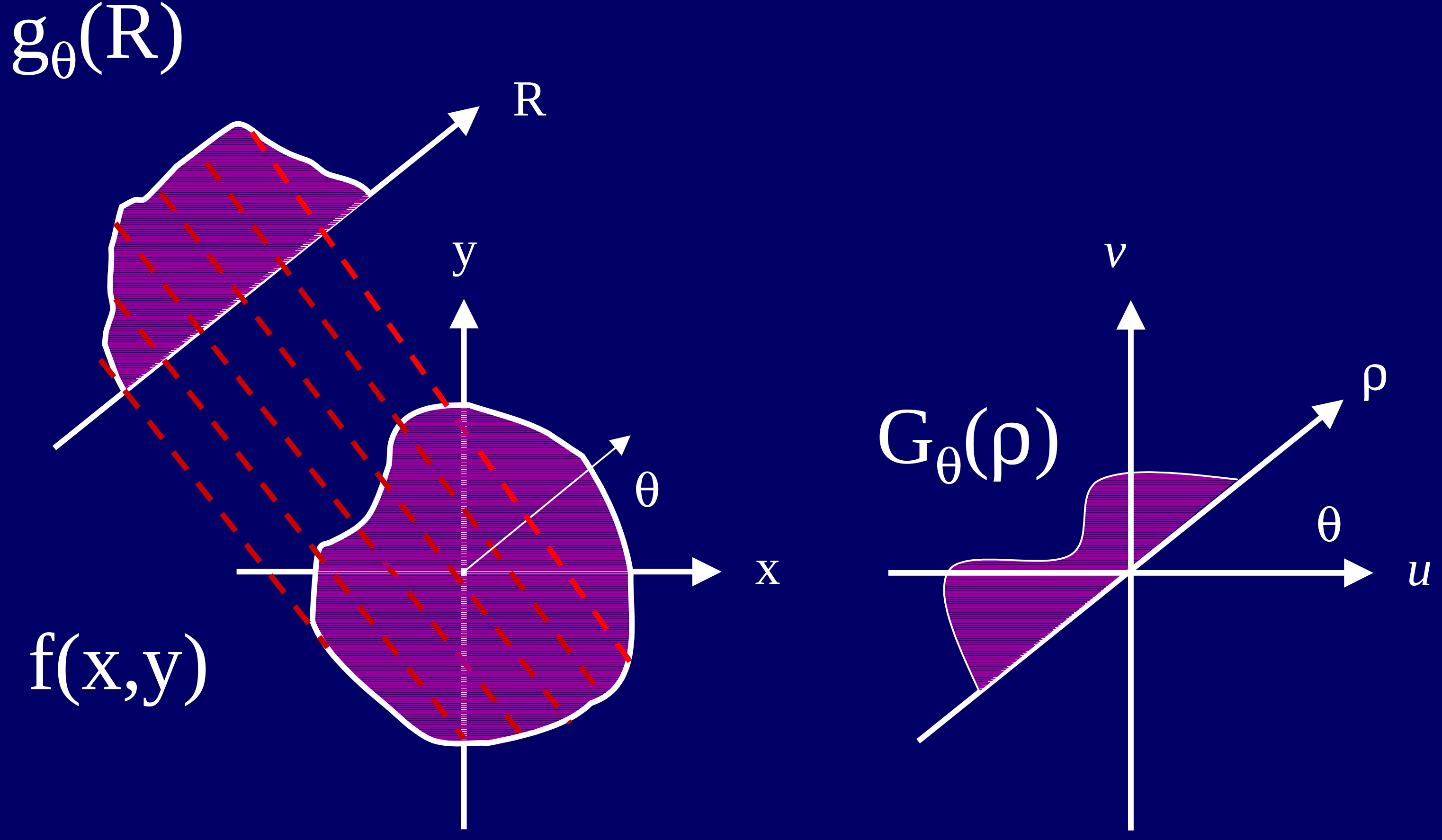
$$G_\theta(\rho) = f(u,v)\,|_{u=\rho\cos\theta,\, v=\rho\sin\theta} = f(\rho,\theta)$$

- 2 D inverse Fourier Transform

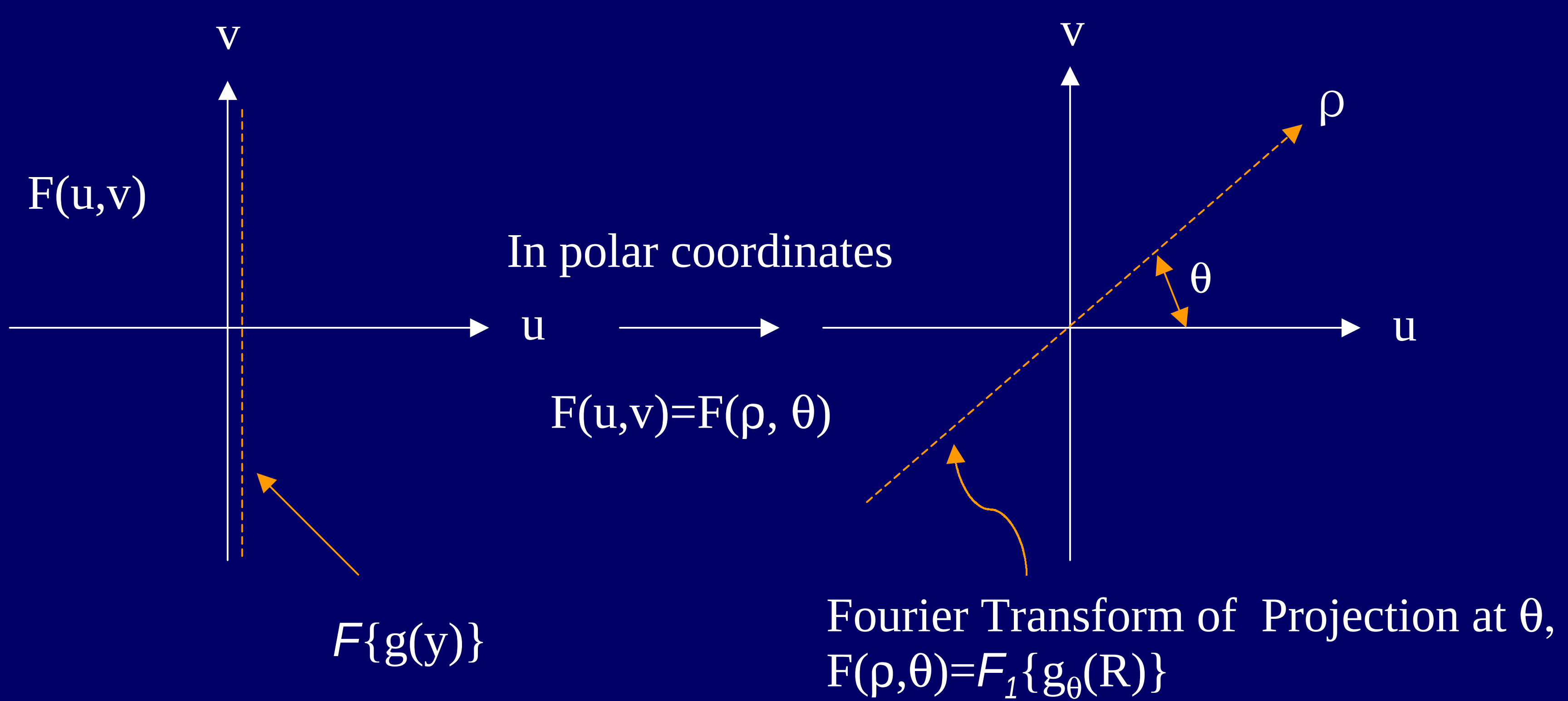
$$f(x,y) = \iint G_\theta(\rho)\exp[j2\pi(ux+vy)]dudv$$

$$= \int_0^{2\pi} d\theta \int_0^{\infty} G_\theta(\rho)\exp[j2\pi(\rho\cos x + \rho\sin y)]\rho d\rho$$

# figures



# Tomography


v
F(u,v)
u
$\mathcal{F}\{g(y)\}$
In polar coordinates
F(u,v)=F(ρ, θ)
v
ρ
θ
u
Fourier Transform of Projection at θ,
$F(\rho,\theta)=\mathcal{F}_1\{g_\theta(R)\}$

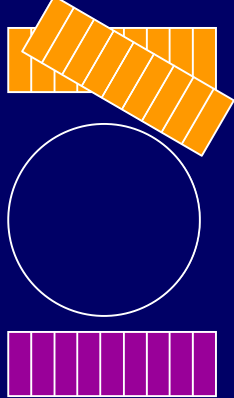
# Algorithm summation

- The Fourier transform of a projection at angle θ forms a line in the 2-D Fourier plane at this same angle.
- After filling the entire plane $F(\rho, \theta)$ with the transforms of the projections at all angles, the reconstructed density is provided by the two-dimensional inverse transform.
- 1. 1D FT each of the projections $g_\theta(R) \rightarrow G_\theta(\rho)$
- 2. Interpolate $F(r,q)$ to $F(u,v)$
  - solve coordinate problems (polar to rectangular coordinates)
- 3. Inverse 2D FT $G_\theta(\rho) \rightarrow F(x, y)$

# Simulation method

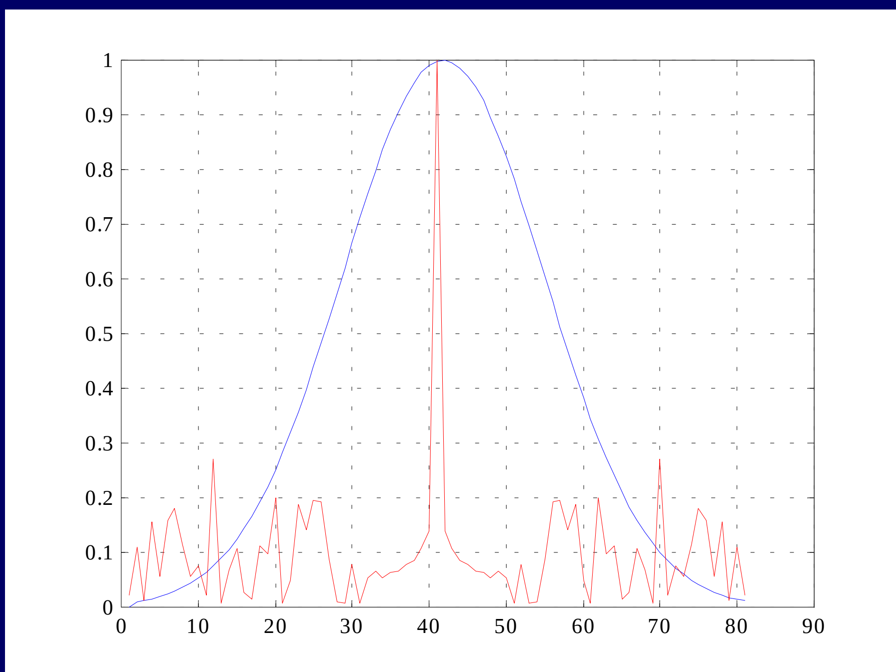
- Finite small point
- Gaussian envelope to time-of-flight $g_\theta(R)$
- FFT and phase compensation
- For loops for Integration

$$f(x,y)=\int_0^{2\pi}d\theta\int_0^{\infty}G_\theta(\rho)\exp[j2\pi(\rho\cos x+\rho\sin y)]\rho d\rho$$

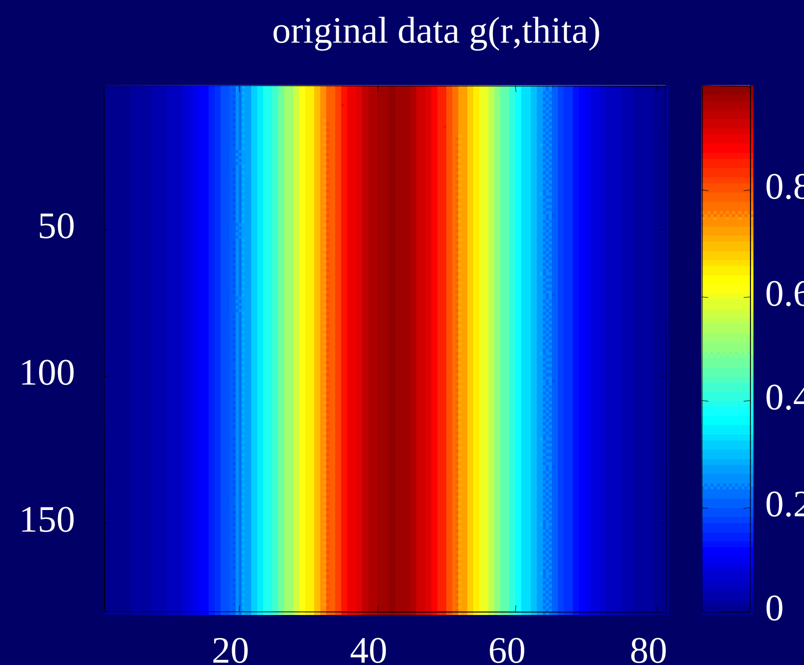


# Results I.

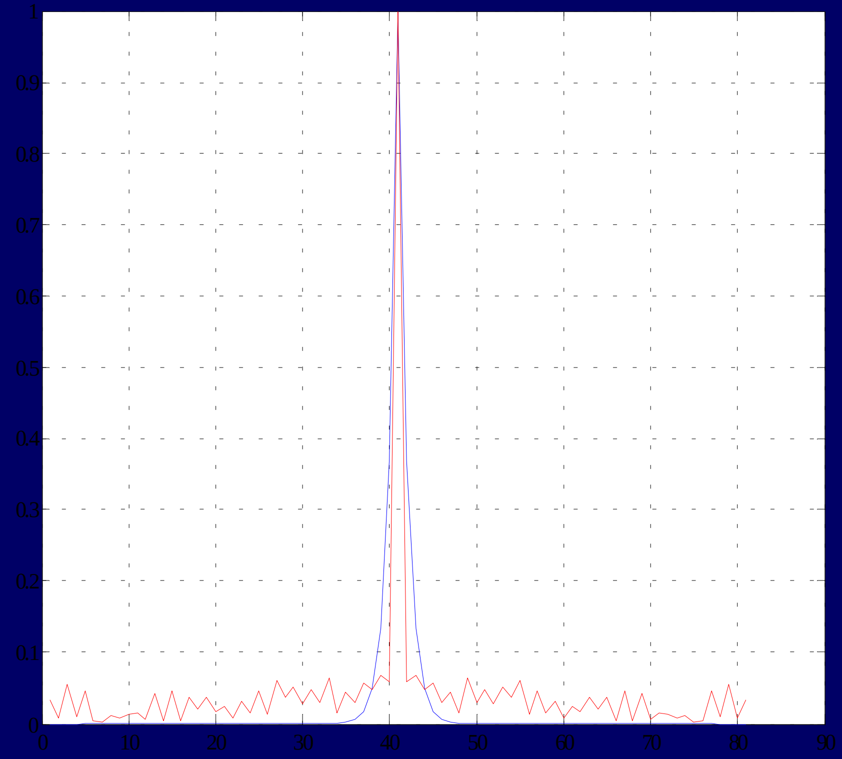
- Time of flight



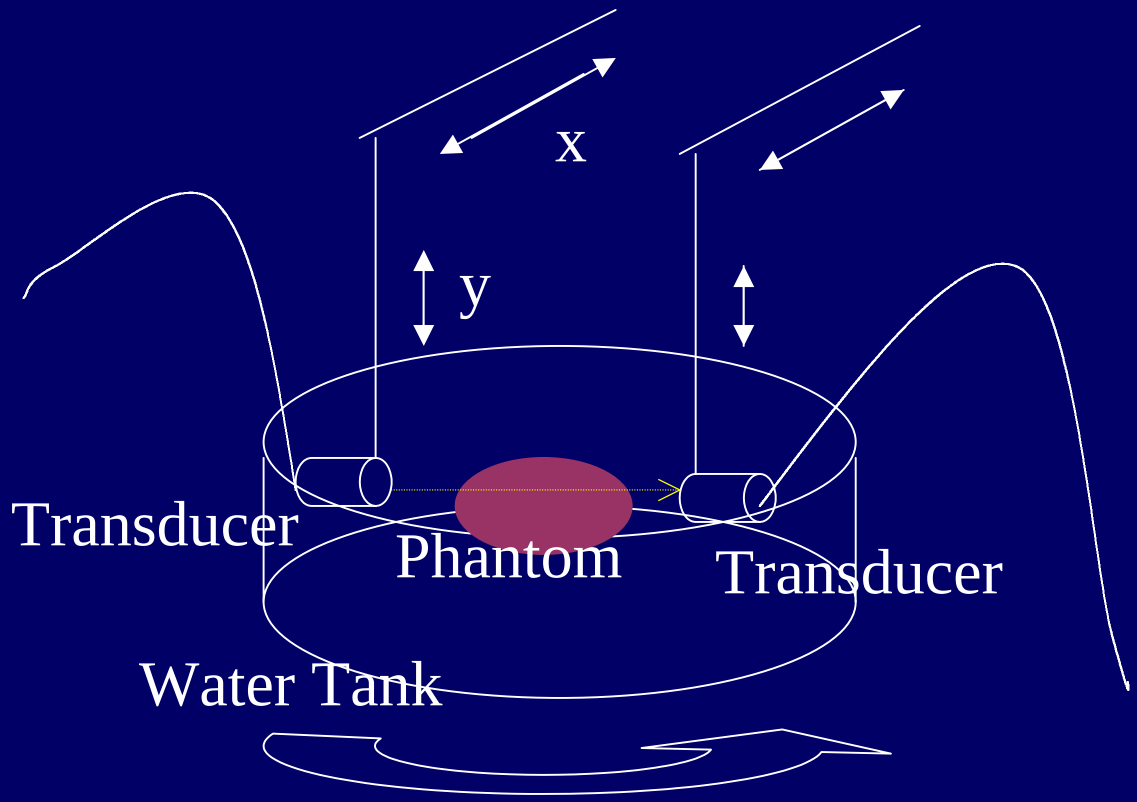

spectrum



# Results II.



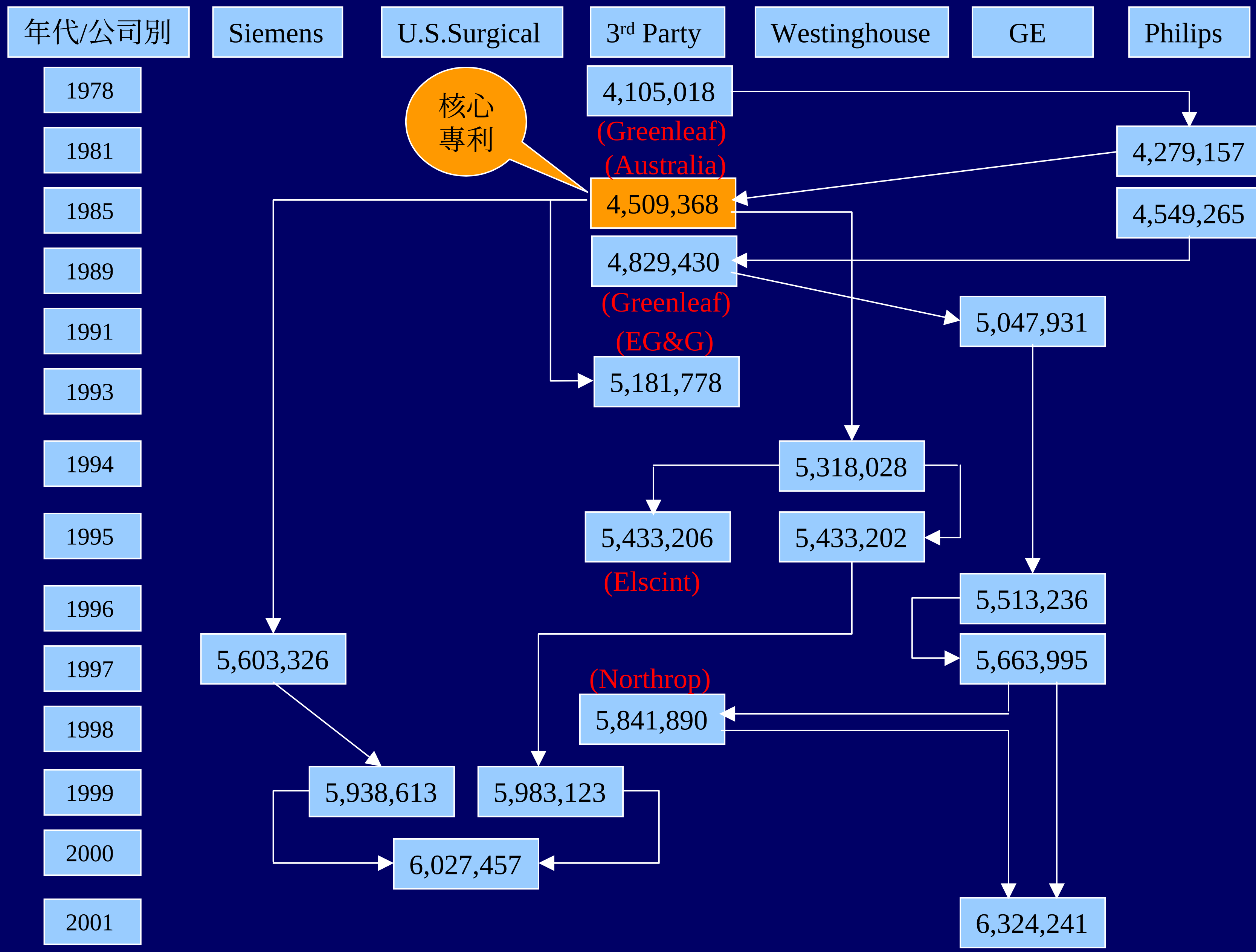
# Experiment architecture



# Future works

- Experimental data acquisition
- Other correct reconstruction algorithm
  - ART, Fourier Transform
  - Back propagation

# Patent Map



年代/公司別
Siemens
U.S.Surgical
3rd Party
Westinghouse
GE
Philips
1978
1981
1985
1989
1991
1993
1994
1995
1996
1997
1998
1999
2000
2001
核心
專利
4,105,018
(Greenleaf)
(Australia)
4,279,157
4,509,368
4,549,265
4,829,430
(Greenleaf)
5,047,931
(EG&G)
5,181,778
5,318,028
5,433,206
5,433,202
(Elscint)
5,513,236
5,603,326
5,663,995
(Northrop)
5,841,890
5,938,613
5,983,123
6,027,457
6,324,241

# Patent Map

| Method/ Process | Device/ Apparatus | System | 合計 |
| --- | --- | --- | --- |
| 17 | 11 | 7 | 35 |

# Patent Analysis

- James F. Greenleaf : measurement of the time-of-flight of acoustic signal ; reducing artifacts…
- Westinghouse : array scanner providing electronic scanning
- GE : helical or spiral scan, 3D reconstruction

# Patent Analysis

- Northrop Grumman : Multi-dimensional wavelet tomography (using wavelet decomposition upon the projection image)
- U.S.Surgical : combines mammography equipment (X-ray) with an ultrasonic transducer

# Core Patent

- 1985, The Commonwealth of Australia
- Ultrasound tomography, the apparatus comprising paired couples of transmission transducers and reflection transducers, the paired couples of transducer means being independently operable within a container of ultrasound transmission medium….

# Core Technique

- Pulses of acoustic energy are transmitted from a plurality of different directions through a plane of interest of a body to be examined. Time-of-flight of the pulses is measured for individual paths through the body, and from the data thus obtained the spatial distribution of the acoustic velocity through the plane or planes within the body is reconstructed using a mathematical reconstruction technique.

# Central Section Theorem

- 2D Fourier transform

$$F(u,v) = \iint f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

- Central Section Theorem

$$F(0,v) = \iint f(x,y)\exp(-j2\pi vy)dxdy$$

$$= \int \left[\int f(x,y)dx\right]\exp(-j2\pi vy)dy$$

$$= F_1\{g(y)\}$$