Ultrasound Computed Tomography

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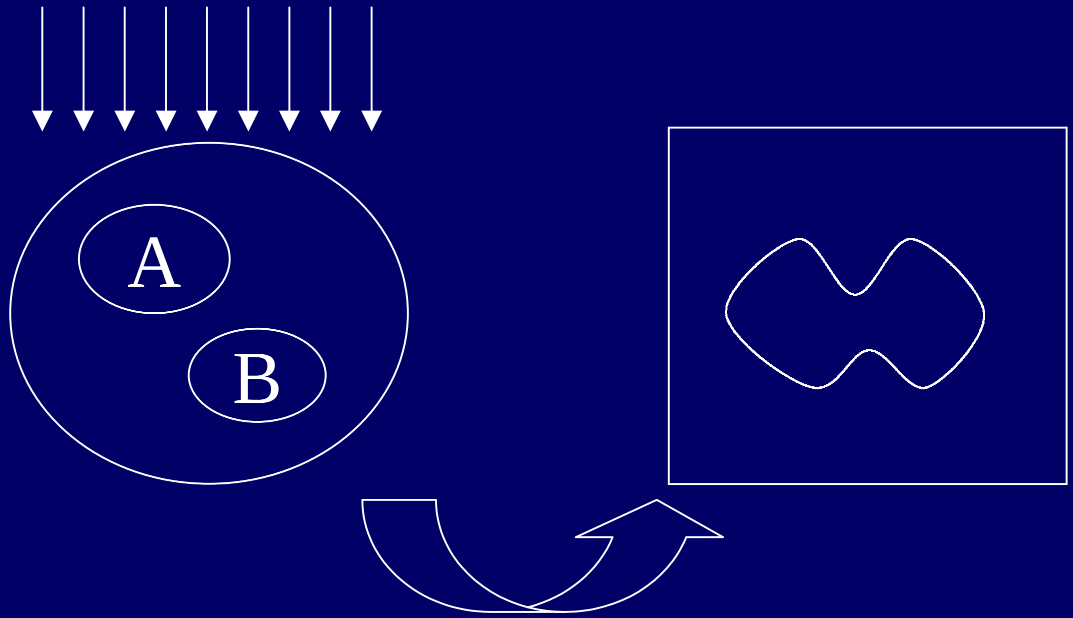
Introduction

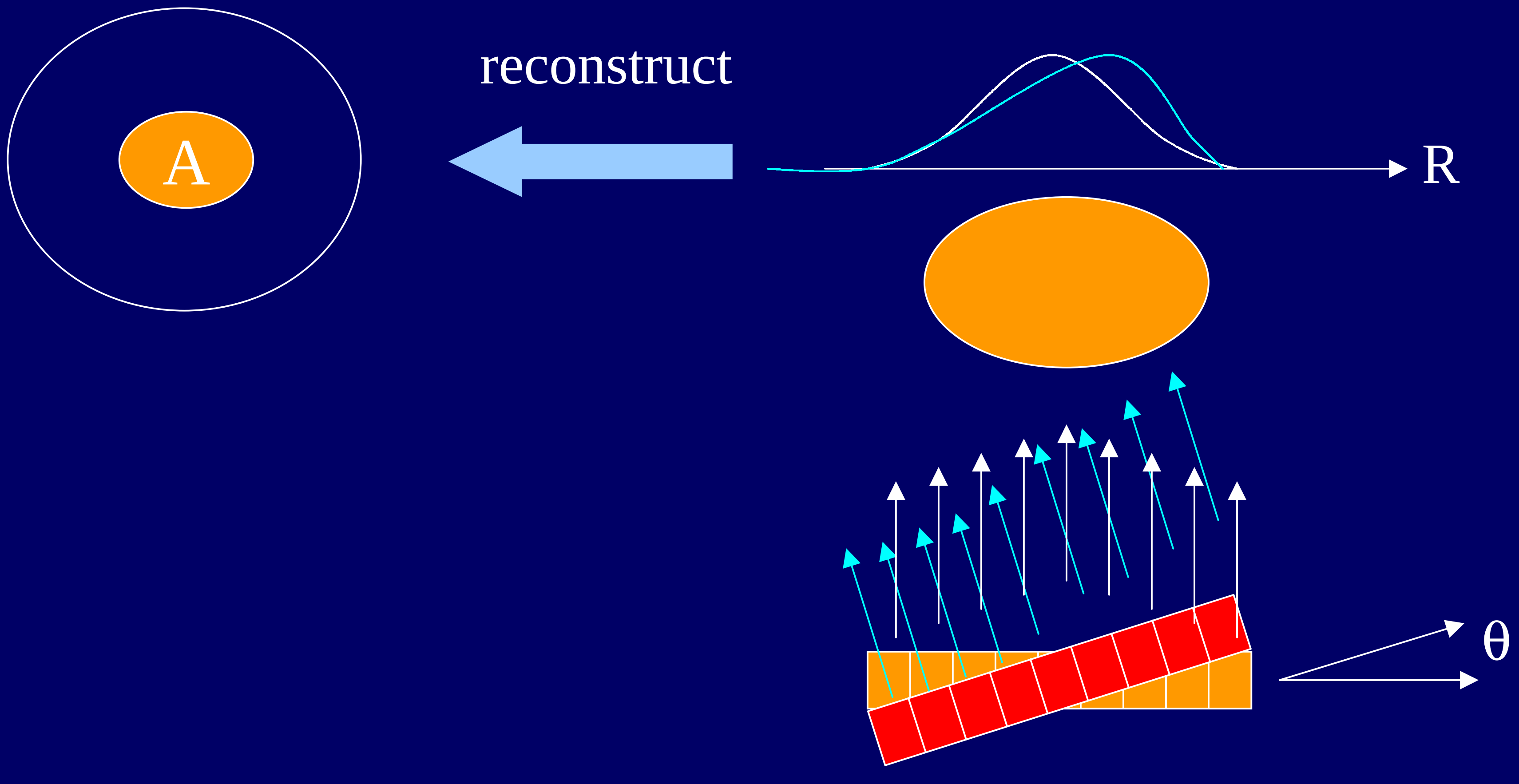
- Conventional X-ray image is the superposition of all the planes normal to the direction of propagation.
- The tomography image is effectively an image of a slice taken through a 3-D volume.
- An estimated 10-15% of breast cancers evade detection by mammography.
- Poor differentiation of malignant tumors from highly common cysts (while ultrasound can do so with accuracies of 90-100%).
- UCT can provide not only structural/density information, but also tissue compressibility and speed of sound maps

Tomography

- Time of flight or intensity attenuation
- Array transducer : 1–D data (only t_f)
- Rotation : 2-D data (t_f on range and angle θ)
- Scan : 3-D data (y position, t_f and angle θ)

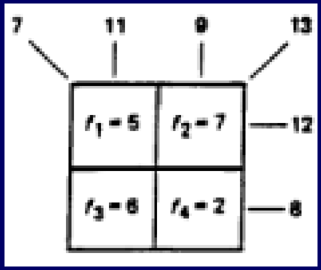


Tomography



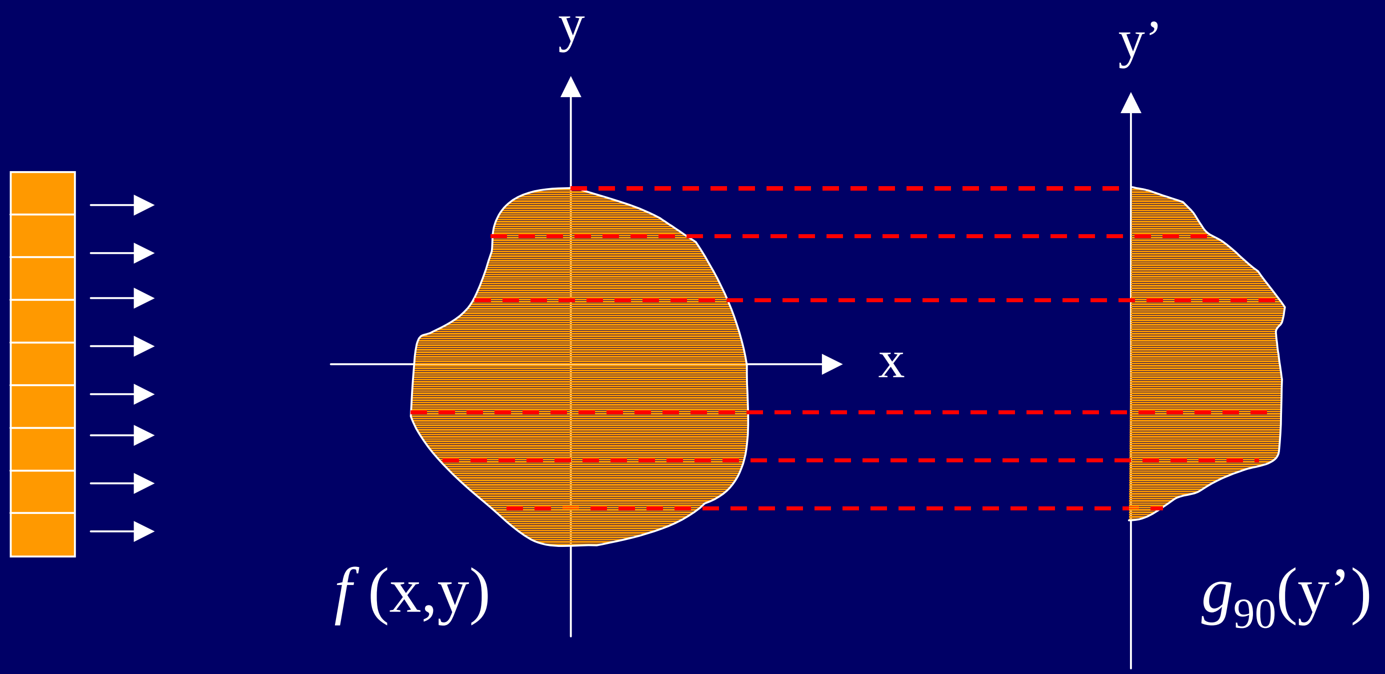
Reconstruction Method

- Attenuation method
- Iterative method
  - Algebraic Reconstruction Technique (ART)



- Direct reconstruction
  - Fourier transform
- Alternative direct reconstruction
  - Back projection
  - Filtered back projection

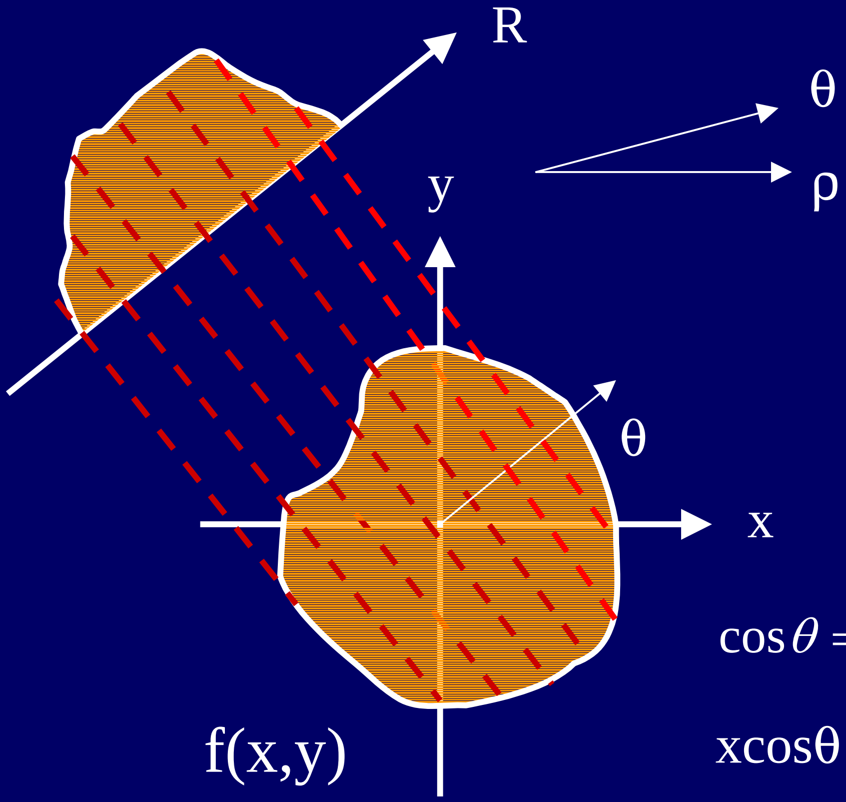
Central Section Theorem



$$g(y') = \iint f(x,y)\delta(y-y')dxdy$$

$$= \int f(x,y)dx$$

Ambiguity Angle θ



$$\cos\theta = \frac{x}{R}, \sin\theta = \frac{y}{R}$$

$$x\cos\theta + y\sin\theta = R$$

$$g_\theta(R) = \iint f(x,y)\delta(x\cos\theta + y\sin\theta - R)dxdy$$

Equations

- 1D FT of projection function

$$G_\theta(\rho) = (1D)\, F.T.\{g_\theta(R)\}$$

$$= \iiint \underline{f(x,y)\delta(x\cos\theta + y\sin\theta - R)}\exp(-i2\pi\rho \underline{R})dxdydR$$

$$= \iint f(x,y)\exp(-i2\pi\rho(x\cos\theta + y\sin\theta))dxdy$$

$$= \iint f(x,y)\exp(-i2\pi(\rho\cos\theta x + \rho\sin\theta y))dxdy$$

$$= \iint f(x,y)\exp(-i2\pi(ux + vy))dxdy$$ ⇐ 2D FT

Cont'd

- 2D Fourier transform

$$f(u,v) = \iint f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

- (*u*,v) in polar coordinates is (ρcos*s*θ, ρ*sin*θ)

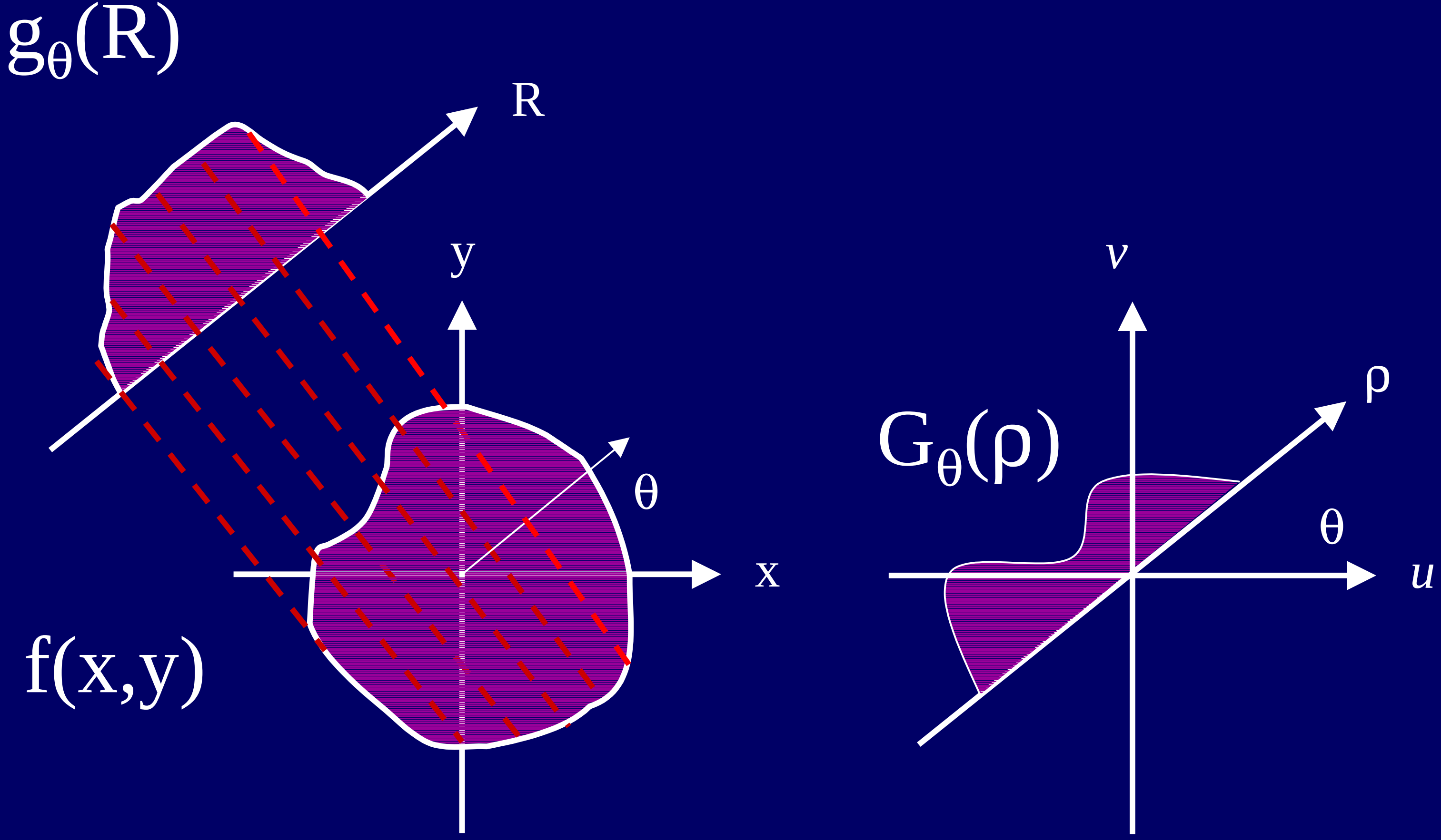
$$G_\theta(\rho) = f(u,v)\big|_{u=\rho\cos\theta,\, v=\rho\sin\theta} = f(\rho,\theta)$$

- 2 D inverse Fourier Transform

$$f(x,y) = \iint G_\theta(\rho)\exp[j2\pi(ux+vy)]dudv$$

$$= \int_0^{2\pi} d\theta \int_0^{\infty} G_\theta(\rho)\exp[j2\pi(\rho\cos x + \rho\sin y)]\rho d\rho$$

Figures



Algorithm

solve coordinate problems (polar to rectangular coordinates)

1. 1D FT each of the projections $g_\theta(R) \rightarrow G_\theta(\rho)$
2. Integrate $G_\theta(\rho)$ in θ and ρ

   -- avoid coordinate transformation problems

$$f(x,y) = \int_0^{2\pi} d\theta \int_0^{\infty} G_\theta(\rho) \exp[j2\pi(\rho\cos x + \rho\sin y)]\rho d\rho$$

$g_\theta(R)$ → $G_\theta(\rho) = F(u,v)$ → $f(x, y)$

1D FT

Back Projection Reconstruction

- Back projection (at one angle)

$$b_\theta(x,y)=\int_{-\infty}^{\infty} g_\theta(R)\delta(x\cos\theta+y\sin\theta-R)dR$$

- Integration of all ambiguity angle θ

$$f_b(x,y)=\int_0^{\pi}\int_{-\infty}^{\infty} g_\theta(R)\delta(x\cos\theta+y\sin\theta-R)dRd\theta$$

Cont'd

- Drawback of back projection method

$$f_b(x,y)=\int_{-\infty}^{\infty}\int_{0}^{\pi}\int_{-\infty}^{\infty}F(\rho,\theta)\delta(x\cos\theta+y\sin\theta-R)\exp(i2\pi\rho R)dRd\theta d\rho$$

$$=\int_{0}^{\pi}\int_{-\infty}^{\infty}F(\rho,\theta)\exp(i2\pi\rho(x\cos\theta+y\sin\theta-R)d\theta d\rho$$

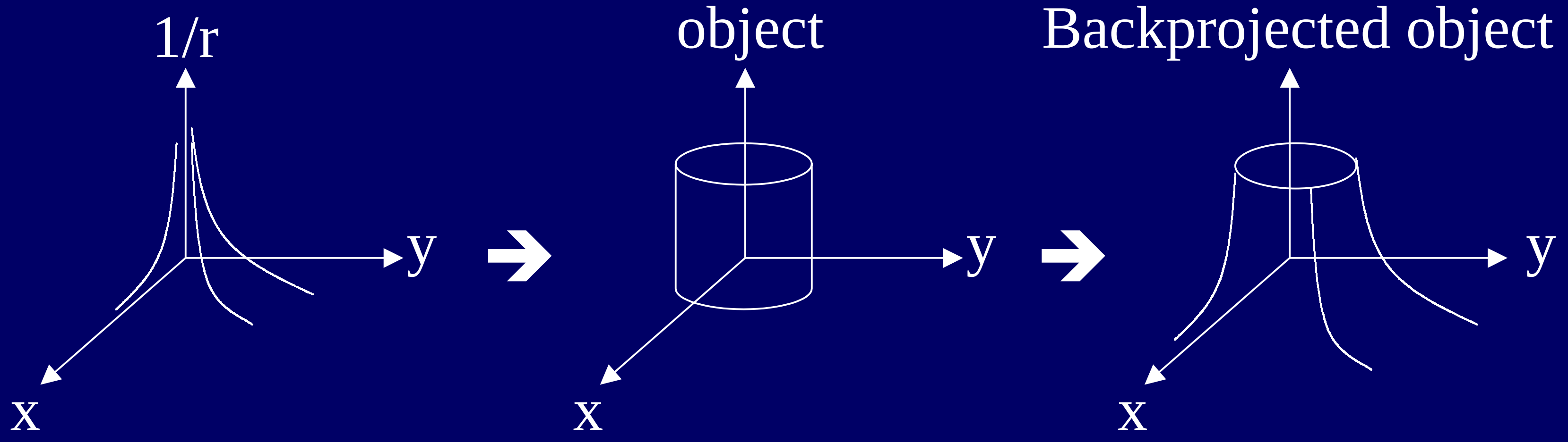
- So, we integrate this function after added 1/ρ

$$f_b(x,y)=\int_{0}^{2\pi}\int_{0}^{\infty}\frac{F(\rho,\theta)}{\rho}\exp(i2\pi\rho(x\cos\theta+y\sin\theta-R)\rho d\rho d\theta$$

$$=2D\ F^{-1}\left\{\frac{F(\rho,\theta)}{\rho}\right\}=f(x,y)\otimes 2D\ F^{-1}\left\{\frac{1}{\rho}\right\}=f(x,y)\otimes\frac{1}{r}$$

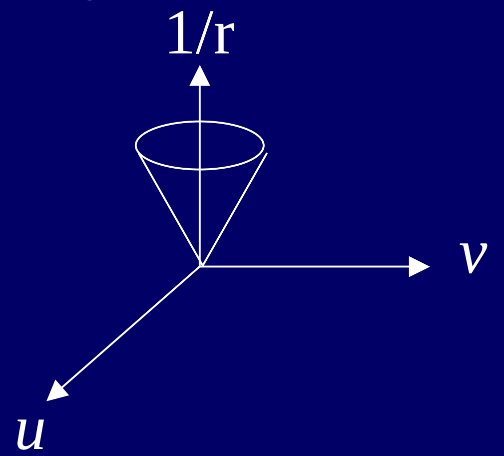
Disadvantage

- FT{1/ρ} ➔ 1/r : blurring function
- *f* (x,y) convolves 1/r
- Additional filter is needed



Filter Design

- Design filter at spatial frequency domain
- Cone filter

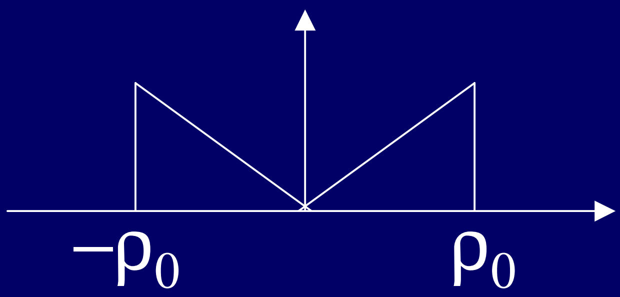


$$C(u,v) = \sqrt{u^2 + v^2}$$

- Spatial filter

$$C(\rho) = \rho_0 \left[rect\left(\frac{\rho}{2\rho_0}\right) - tri\left(\frac{\rho}{\rho_0}\right) \right]$$

$$C(R) = \rho_0^{\ 2} \left(2 \sin c(2\rho_0 R) - \sin c^2(\rho_0 R) \right)$$



Signal Processing Flow

- Backprojection all angles, $g_\theta(R) \rightarrow F_b(\theta)$
- Forward 2D FT to get: $\dfrac{F(\rho,\theta)}{\rho}$
- Apply filter on $\dfrac{F(\rho,\theta)}{\rho}$ to get F (u,v)
- Inverse 2D FT to f (x,y)

$$g_\theta(R) \rightarrow F_b(\theta) \rightarrow \frac{F(\rho,\theta)}{\rho} \rightarrow F(u,v) \rightarrow f(x,y)$$

2D FT   Filtering   2D IFT

Algebraic Reconstruction Technique (ART)

- The iterative process starts with all values set to a constant, such as the mean or 0.
- At each iteration, the difference between the measured projection data for a given projection and the sum of all reconstructed elements along the line defining the projection is computed.
- This difference is then evenly divided among the N reconstruction elements.

Algebraic Reconstruction Technique (ART)

1. Projection Step
2. Comparison Step
3. Backprojection Step
4. Update Step

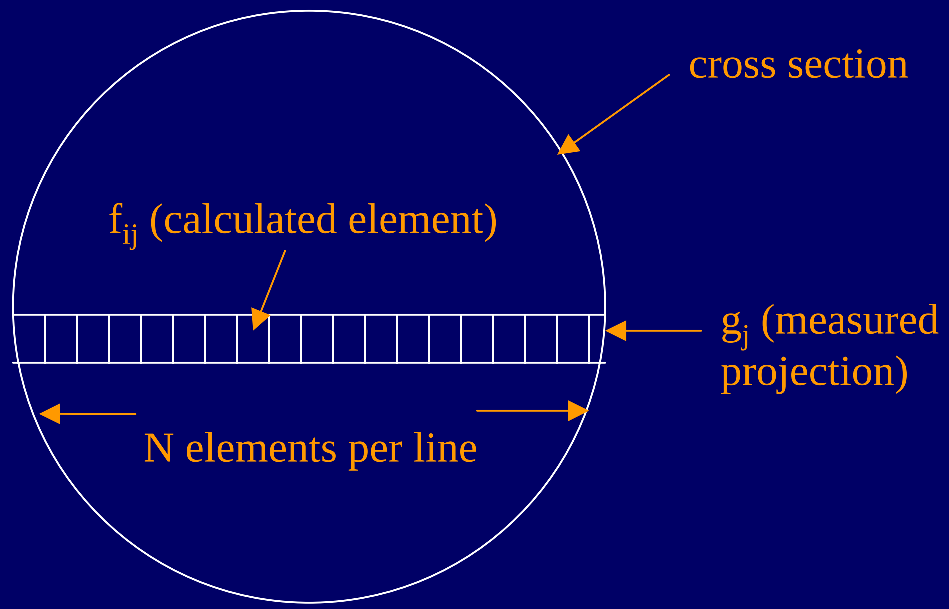
Additive ART :

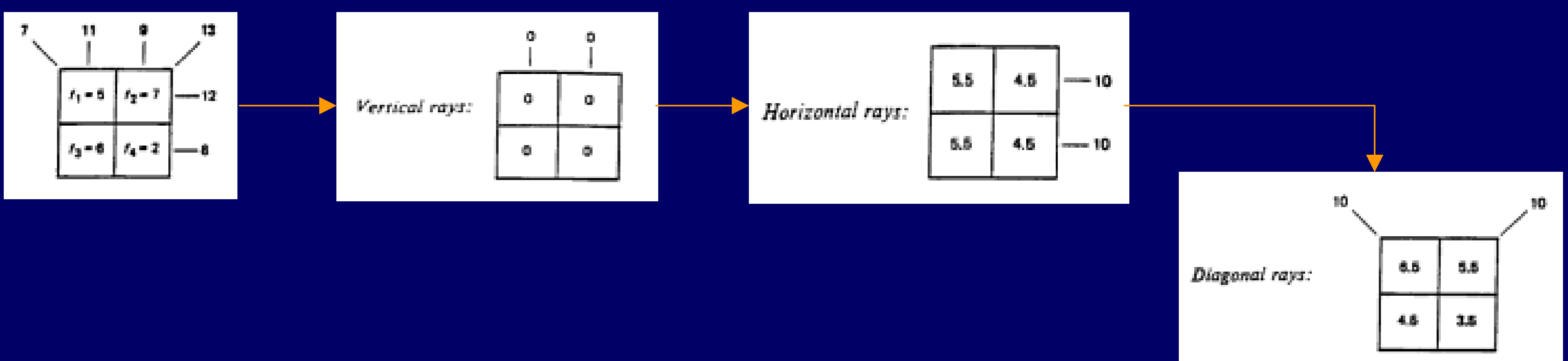
$$f_{ij}^{q+1} = f_{ij}^{q} + \frac{g_j - \sum_{i=1}^{N} f_{ij}^{q}}{N}$$

Multiplicative ART :

$$f_{ij}^{q+1} = \frac{g_j}{\sum_{i=1}^{N} f_{ij}^{q}} f_{ij}^{q}$$

Algebraic Reconstruction Technique (ART)

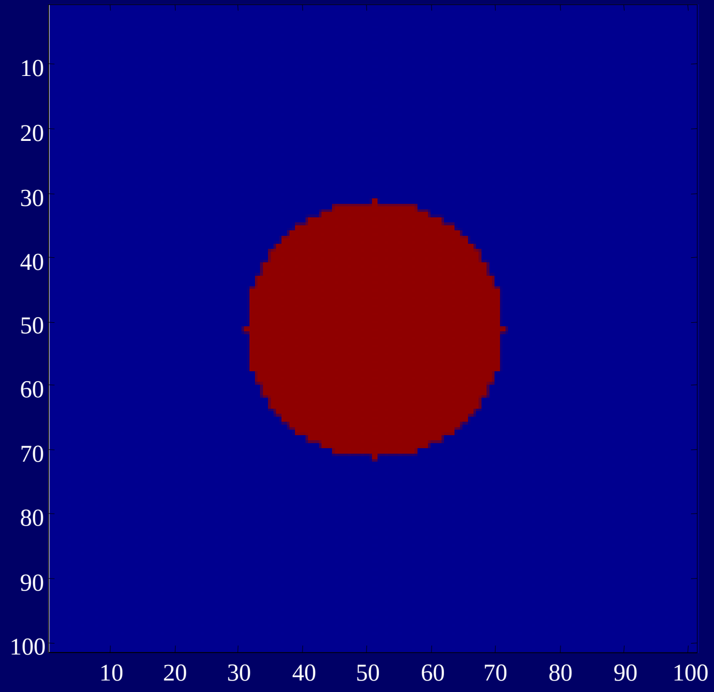




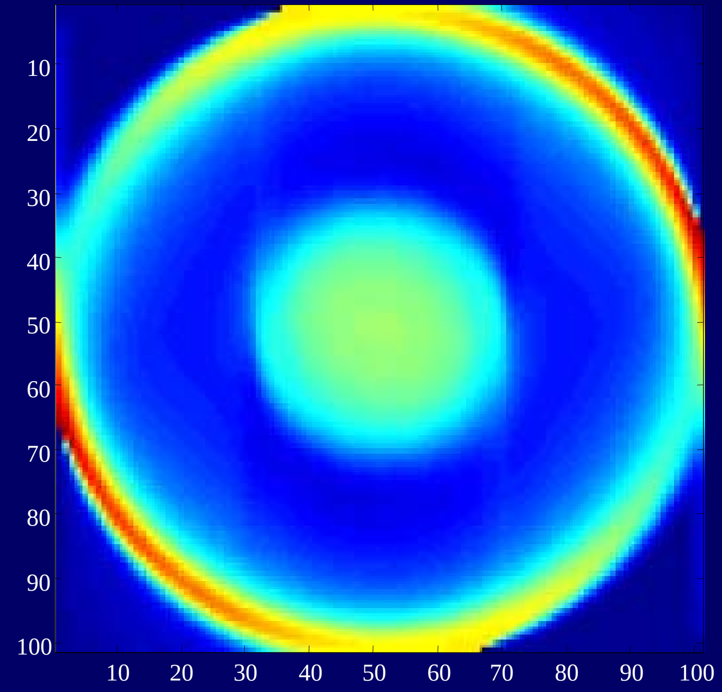
Simulation Results I.

•Using ART to reconstruct :

Simulated Data



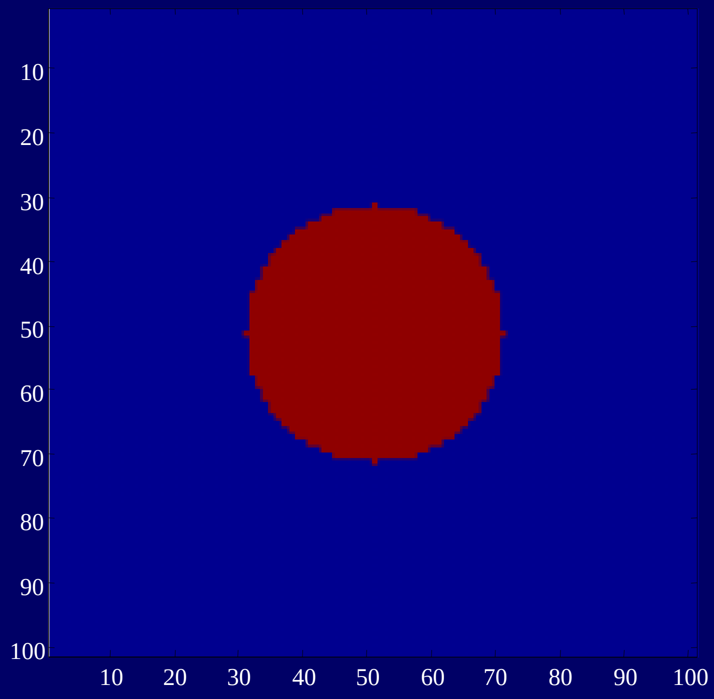
Reconstruction Result



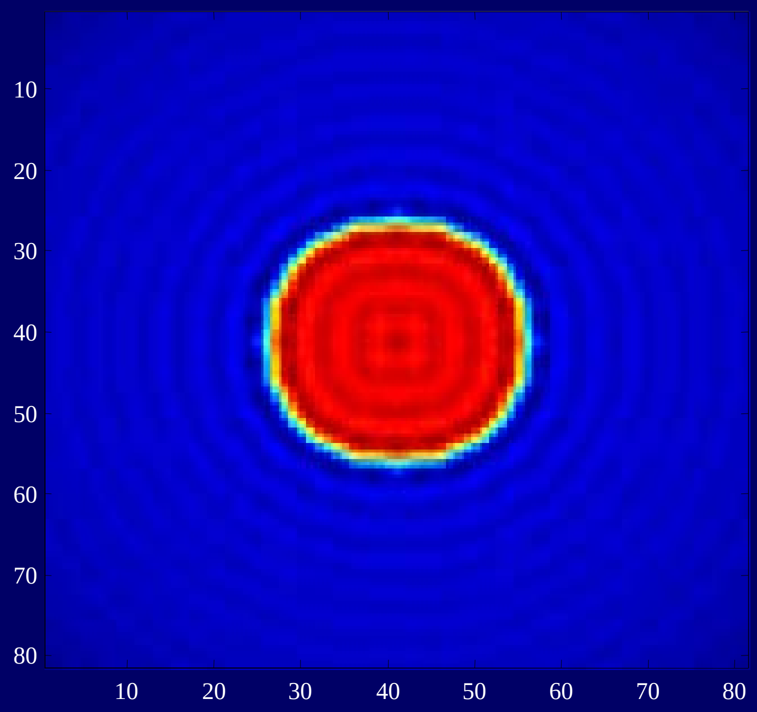
Simulation Results I.

- Using direct Fourier transform to reconstruct :

Simulated Data



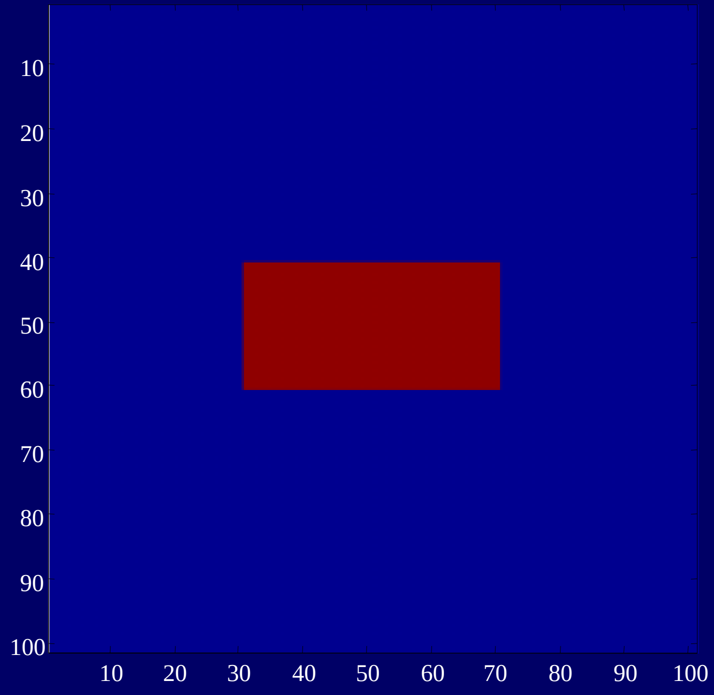
Reconstruction Result



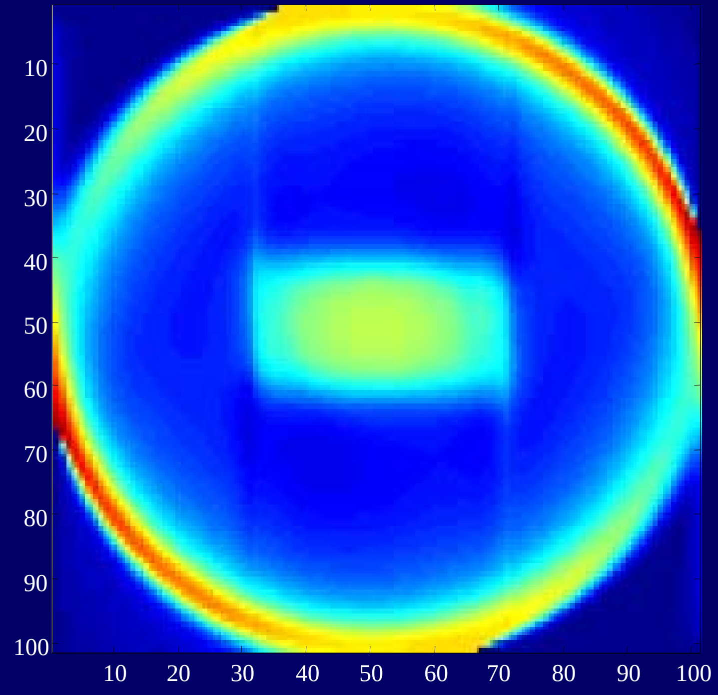
Simulation Results II.

- Using ART to reconstruct :

Simulated Data



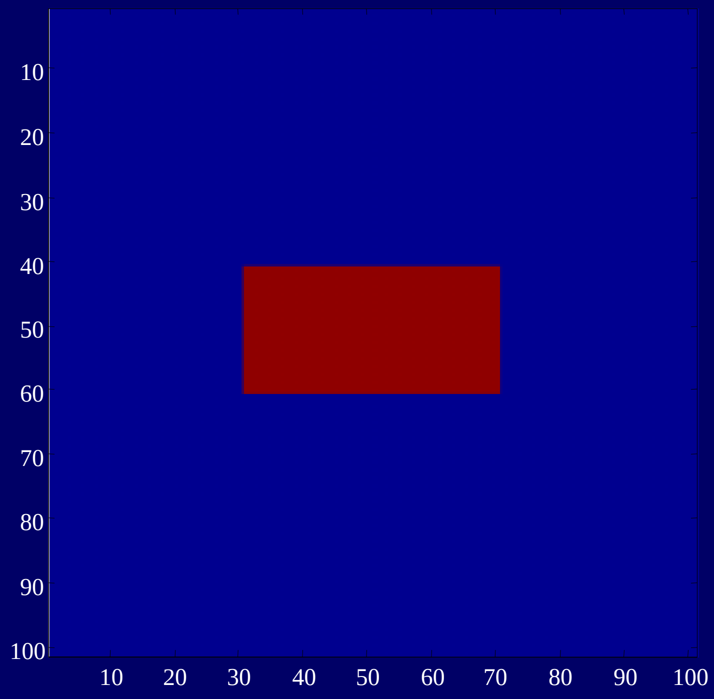
Reconstruction Result



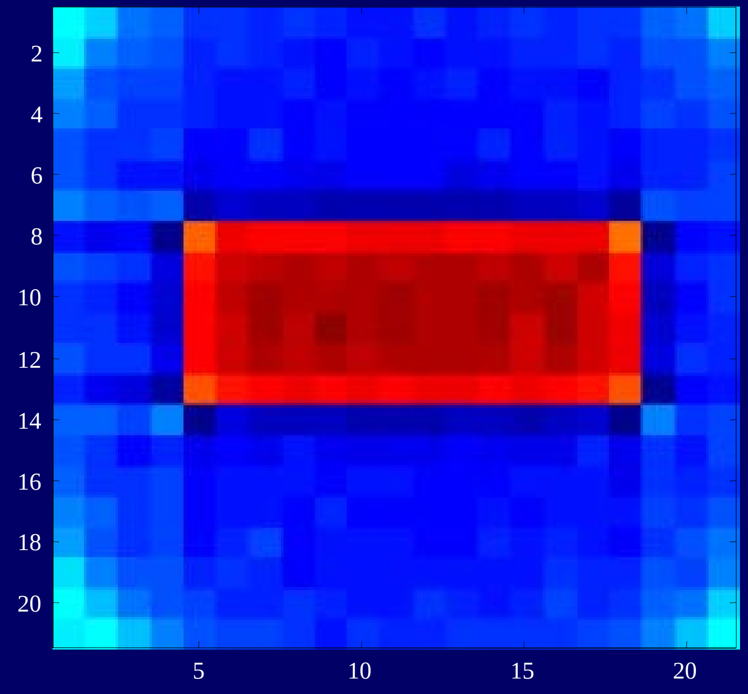
Simulation Results II.

•Using direct Fourier transform to reconstruct :

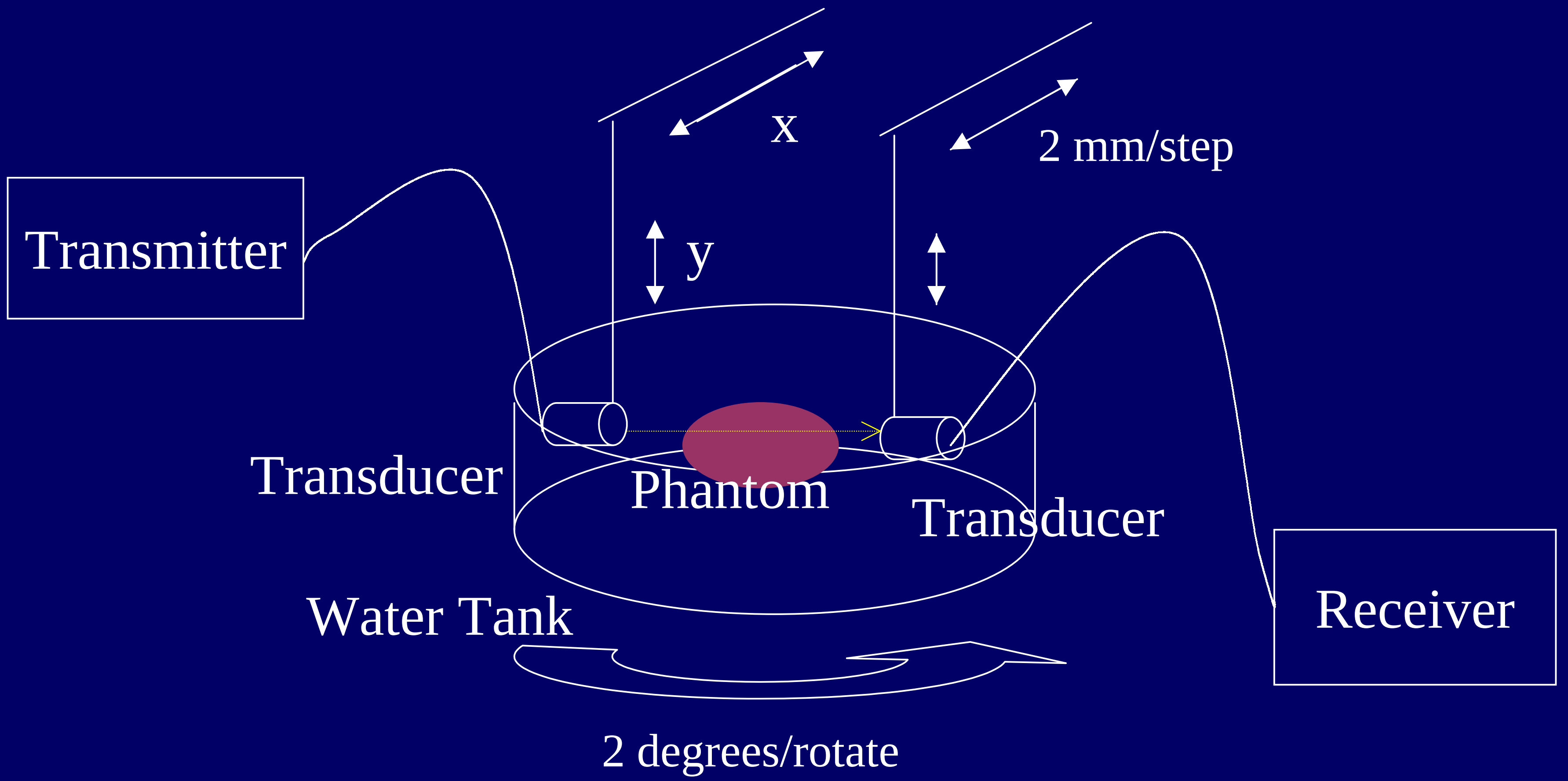
Simulated Data



Reconstruction Result

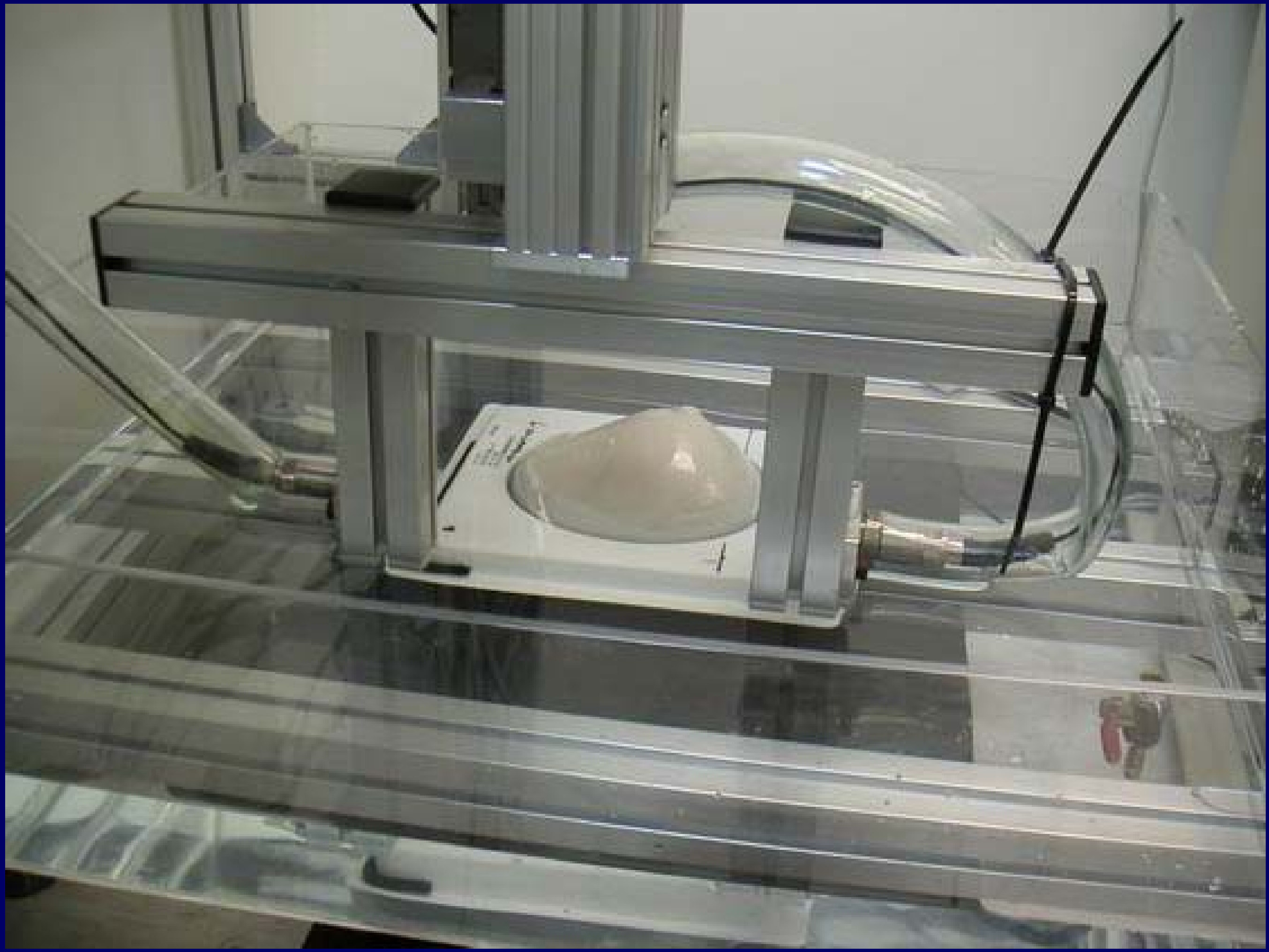


Experiment Architecture

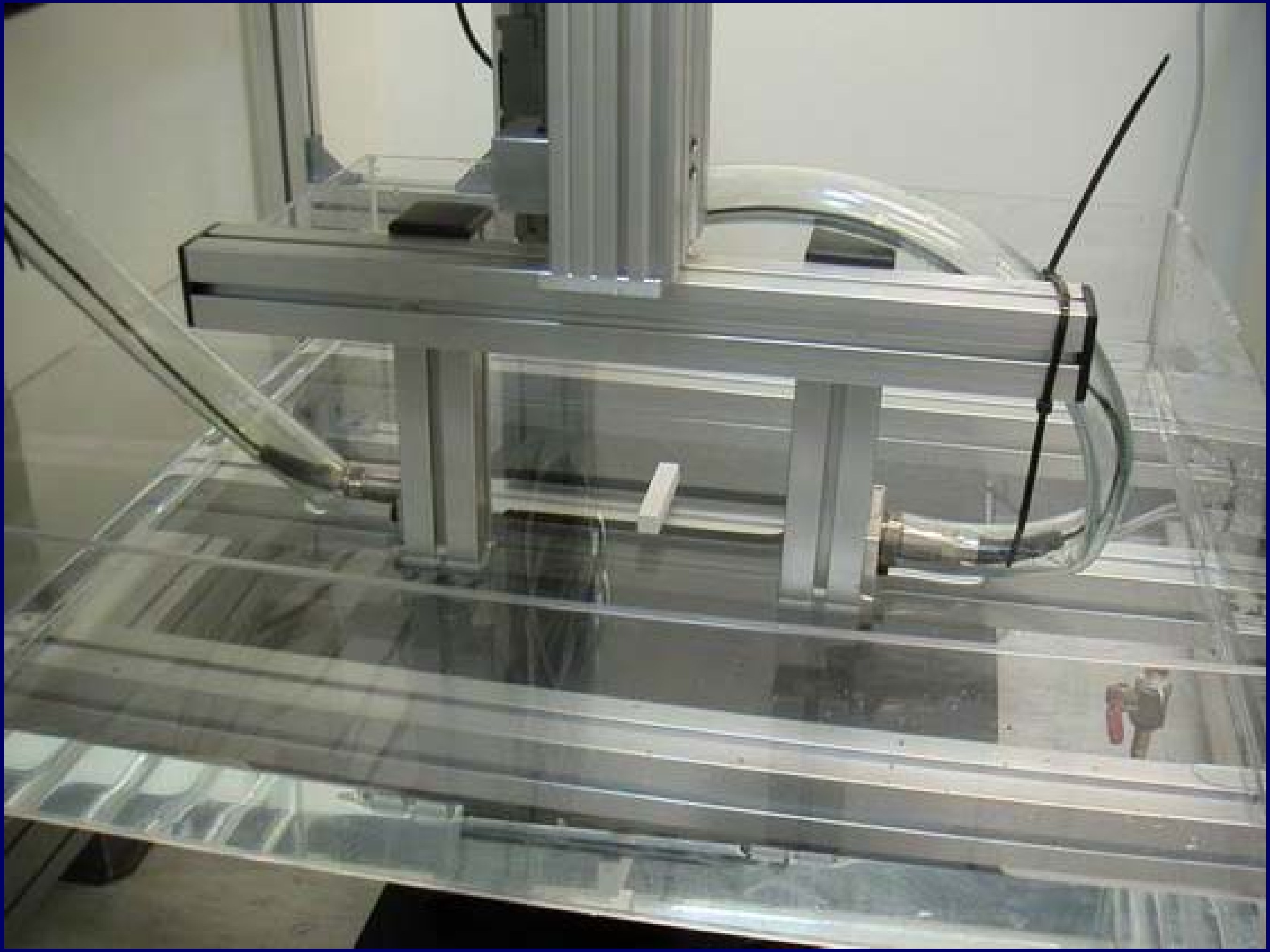


Experiment Architecture

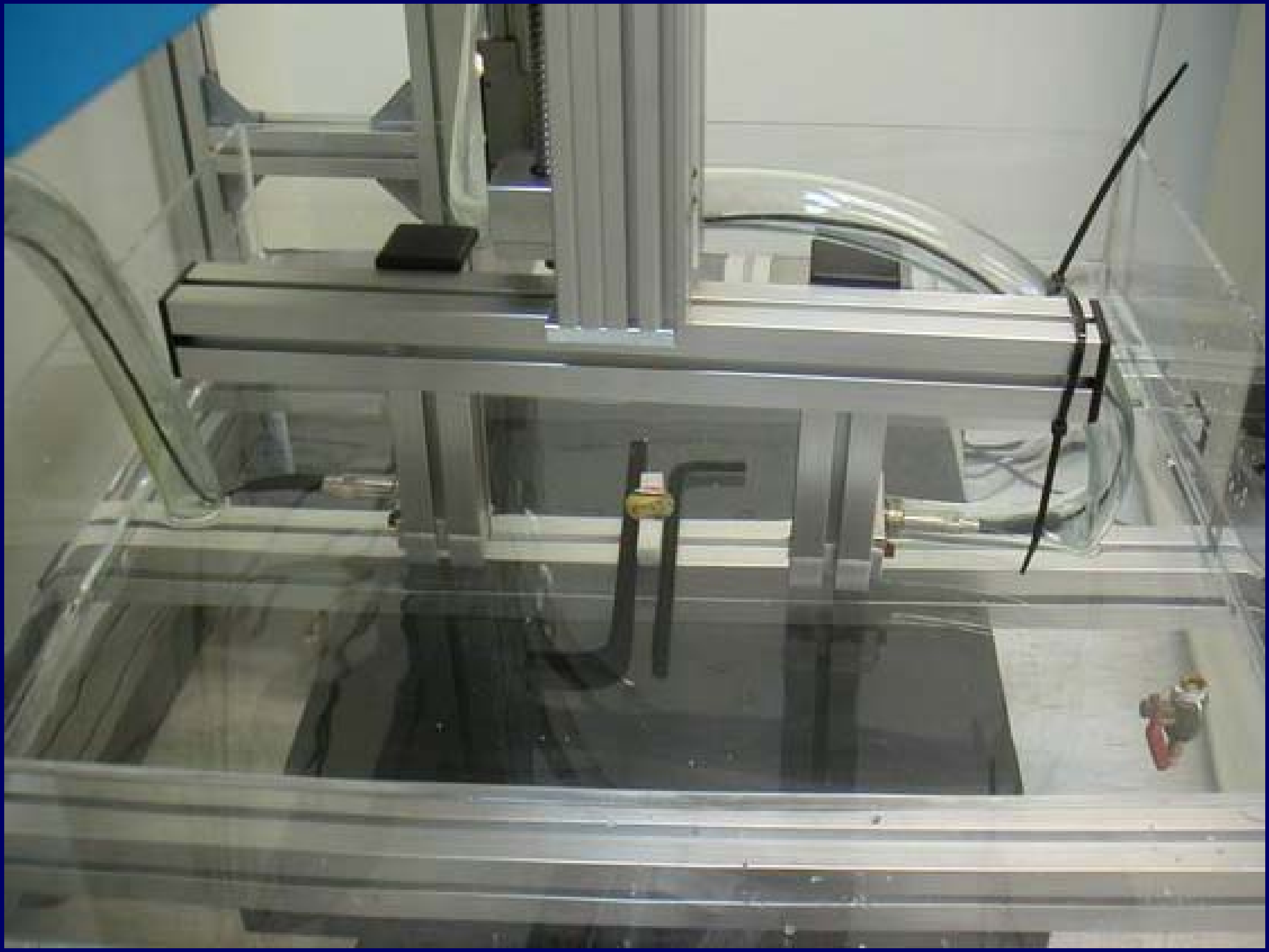
Breast Phantom



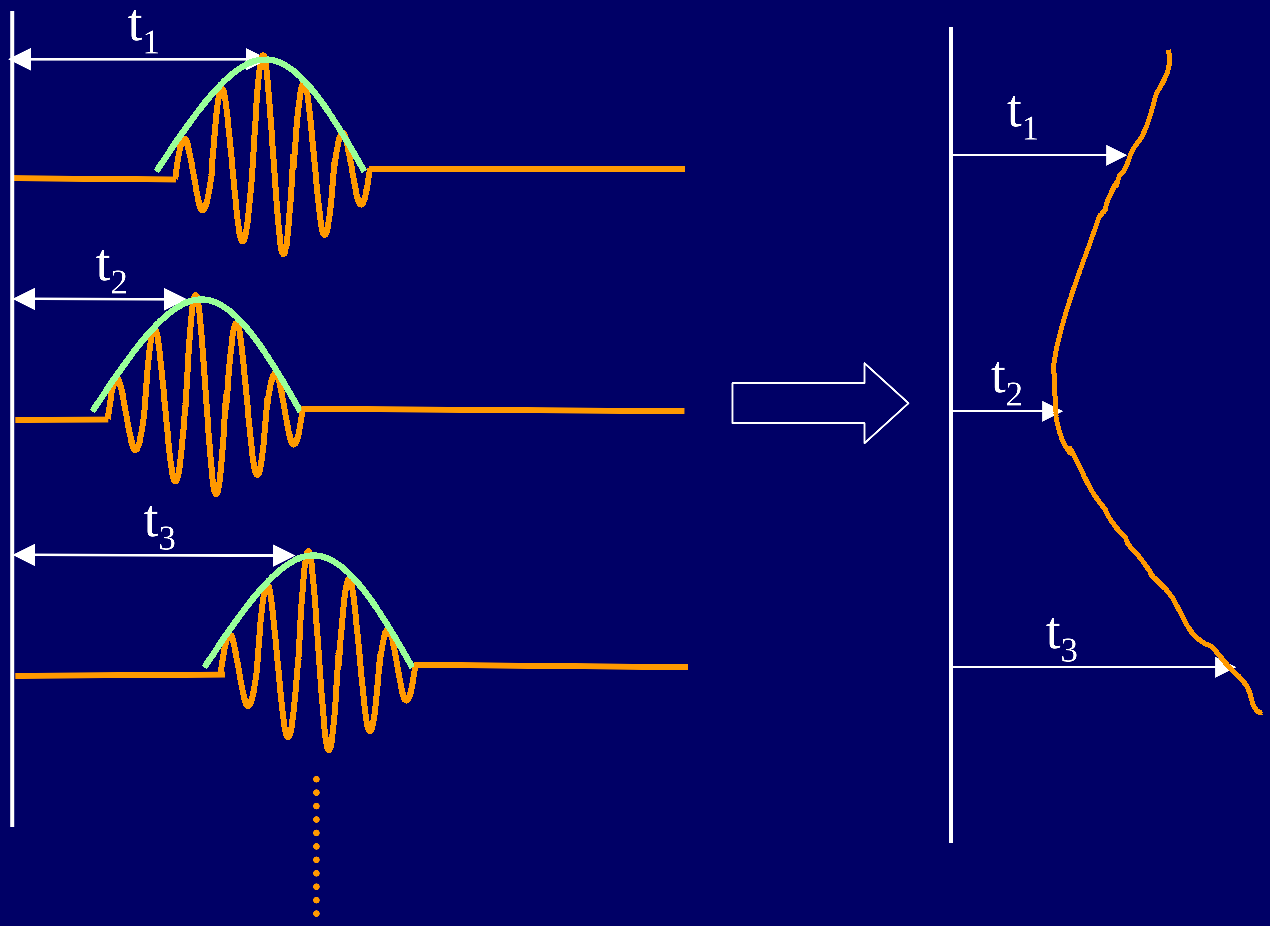
Eraser Phantom



Rubber Ball



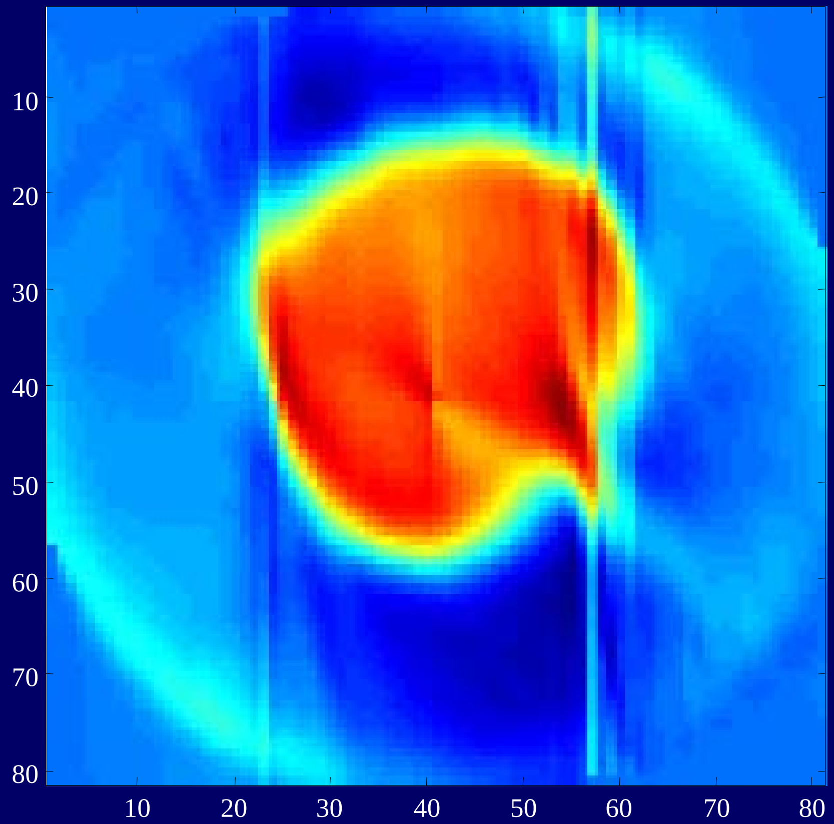
Time-of-flight



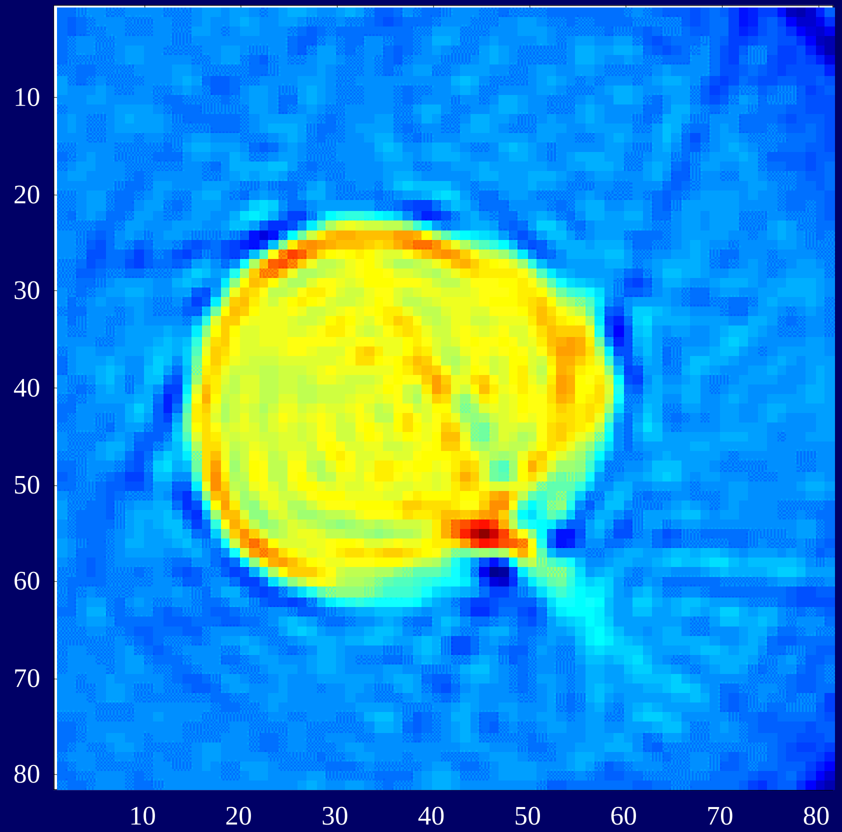
Reconstructed Image

- Breast phantom

ART

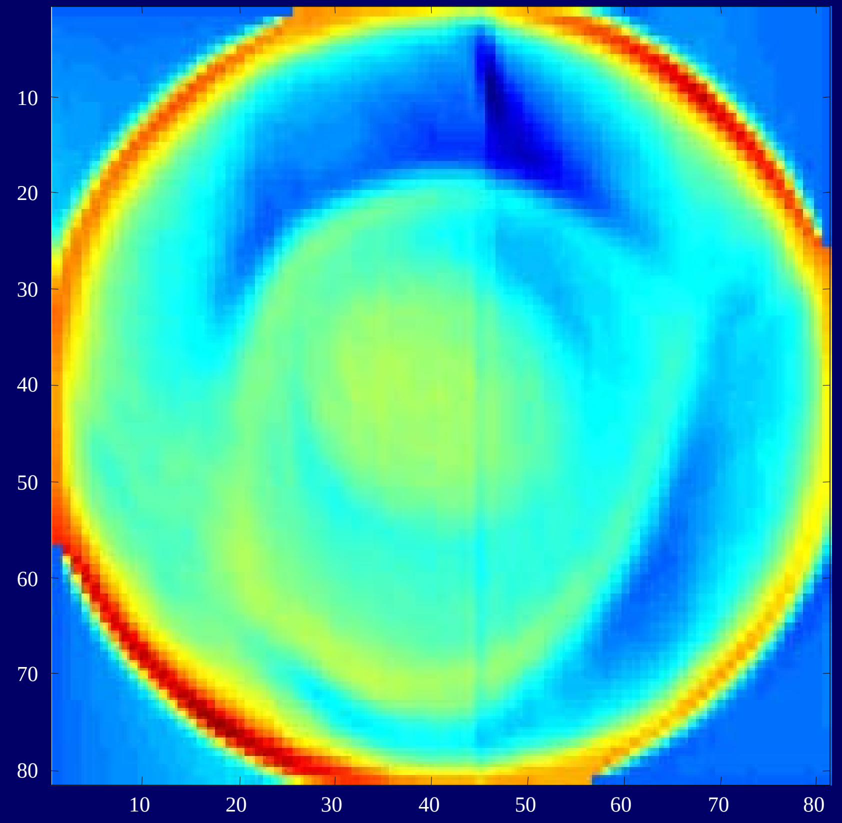


Direct Fourier Transform



Reconstructed Image

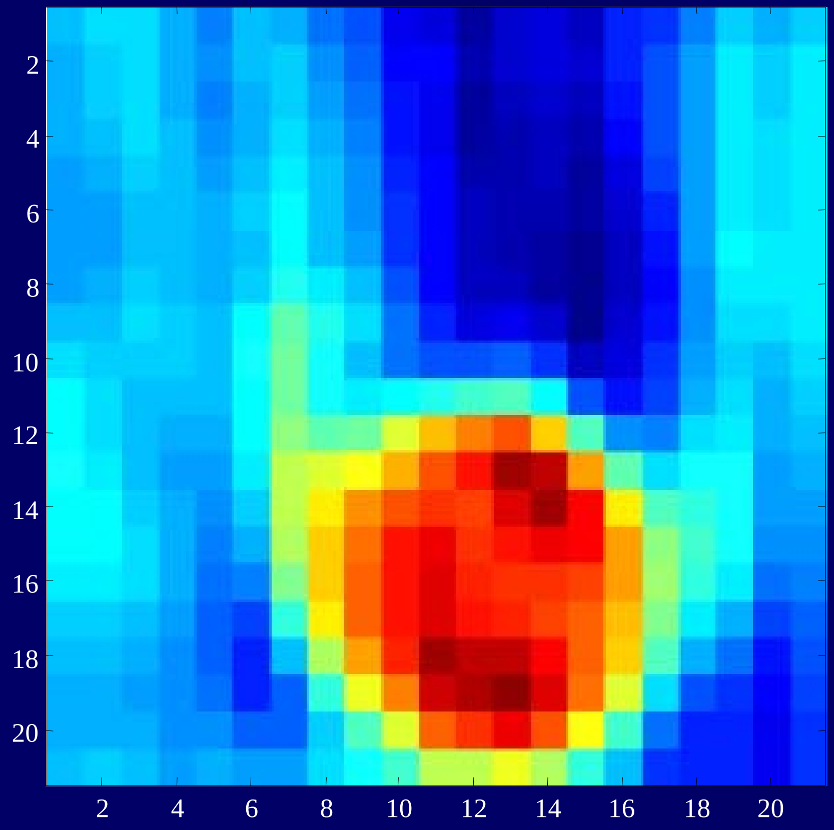
- Breast phantom, using ART



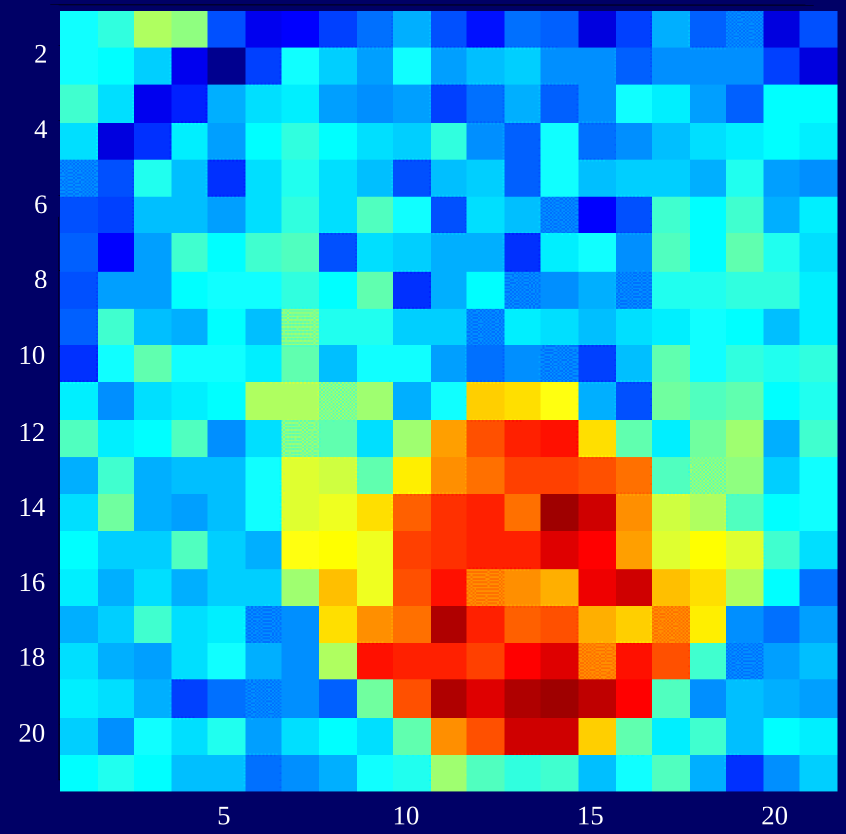
Reconstructed Image

- Rubber ball

ART



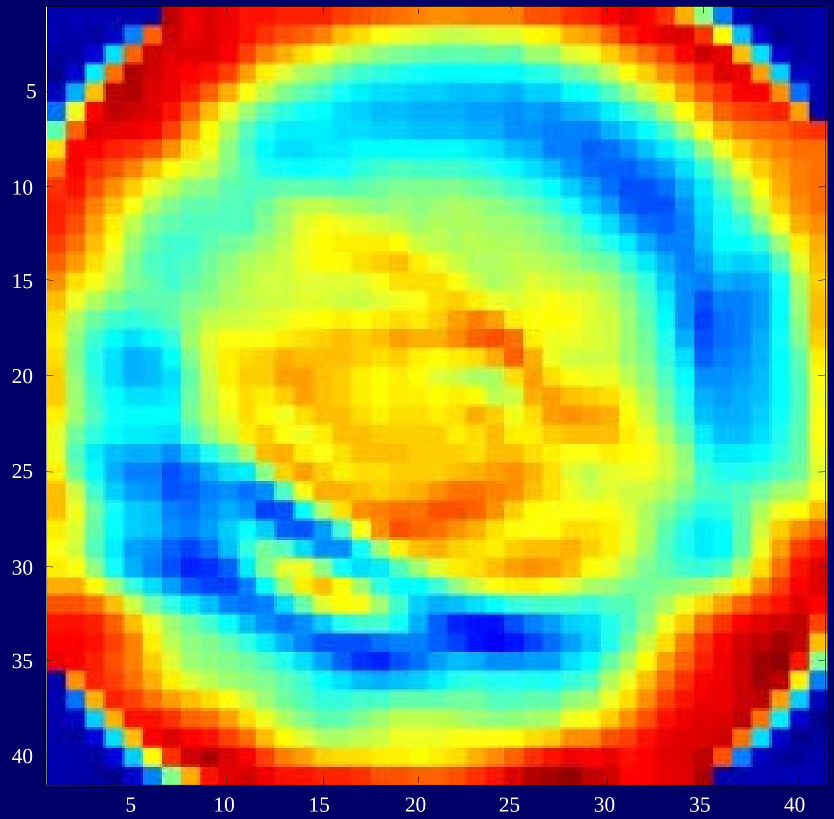
Direct Fourier Transform



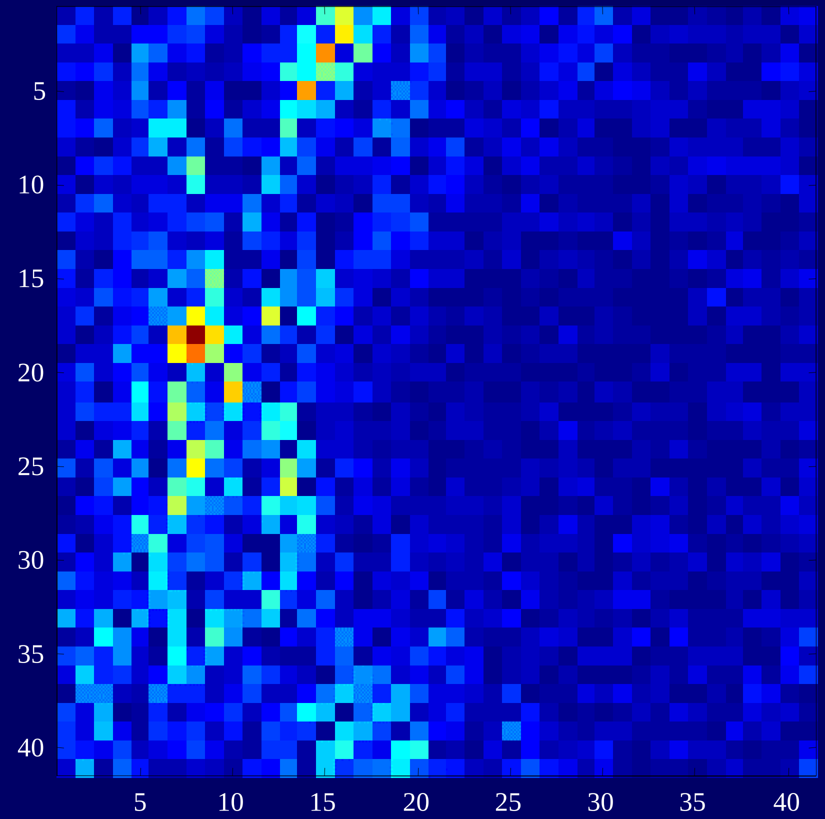
Reconstructed Image

- Eraser

ART



Direct Fourier Transform



Conclusions

- ART reconstruction is much faster than other reconstruction method but must notice the velocity profile normalization.
- Direct fourier reconstruction is effective but less efficiency.
- Backprojection reconstruction needed additional filter to cancel integration errors.

Future Works

- Correct velocity profile estimation
- Discuss backprojection method errors and additional filter design method
- Discuss diffraction errors in ultrasound

References

- [1] Albert Macovski, “Medical Imaging Systems”
- [2] Matthew O’Donnell, “X-Ray Computed Tomography (CT)”, University of Michigan
- [3] Douglas C. Noll, “Computed Tomography Notes”, University of Michigan, 2001
- [4] Brian Borchers, “Tomography Lecture Notes”, New Mexico Tech., March 2000
- [5] James F. Greenleaf et al., “Clinical Imaging with Transmissive Ultrasonic Computerized Tomography”, IEEE trans.on Biomedical Engineering vol. 28 no. 2 1981