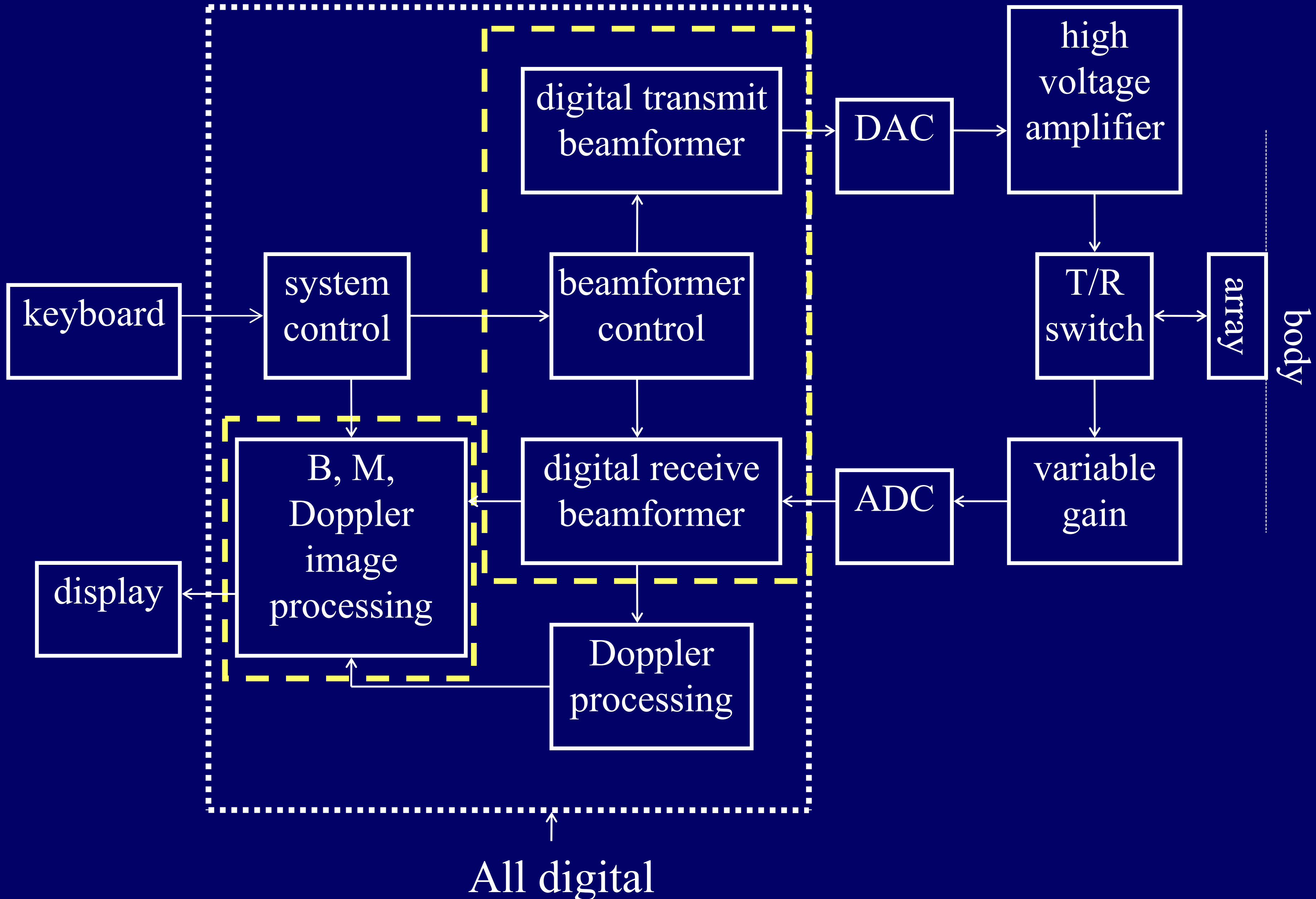
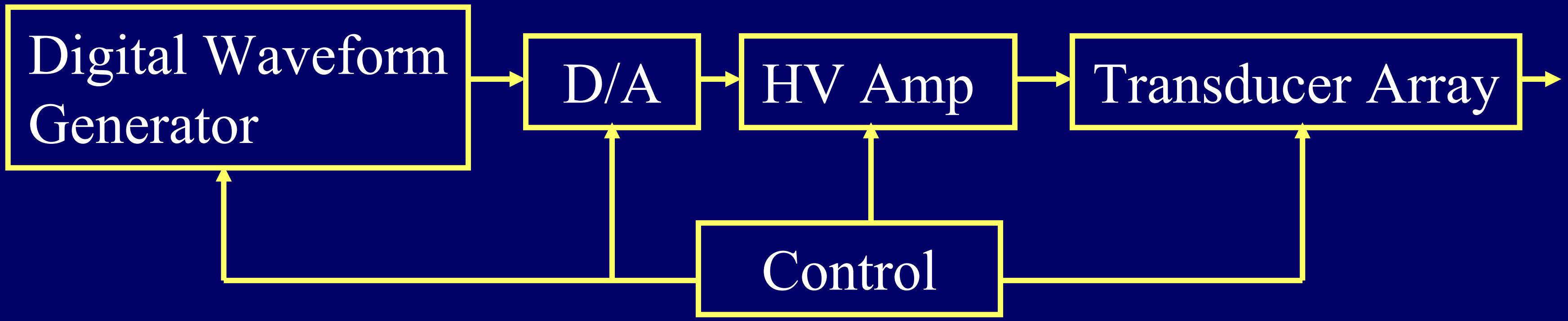
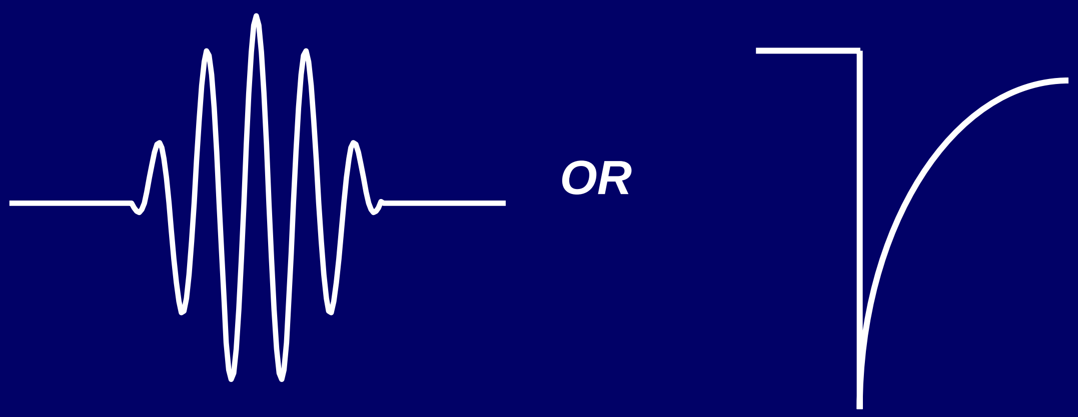
Chapter 6: Real-Time Image Formation


keyboard
system control
digital transmit beamformer
beamformer control
digital receive beamformer
DAC
high voltage amplifier
T/R switch
array
body
variable gain
ADC
B, M, Doppler image processing
display
Doppler processing
All digital

Generic Ultrasonic Imaging System

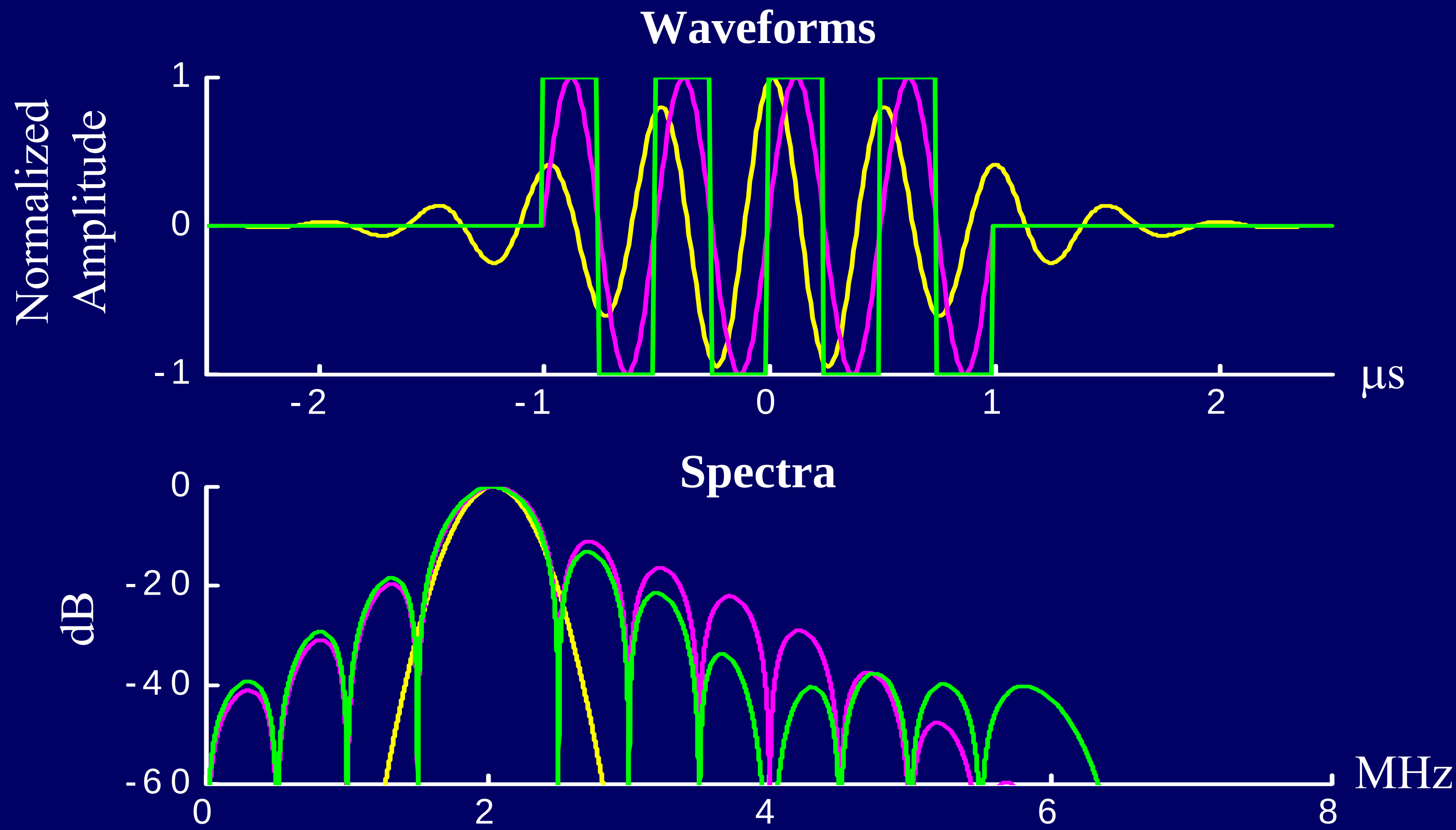
- Transmitter:
  - Arbitrary waveform.
  - Programmable transmit voltage.
  - Arbitrary firing sequence.
  - Programmable apodization, delay control and frequency control.




OR

Transmit Waveform

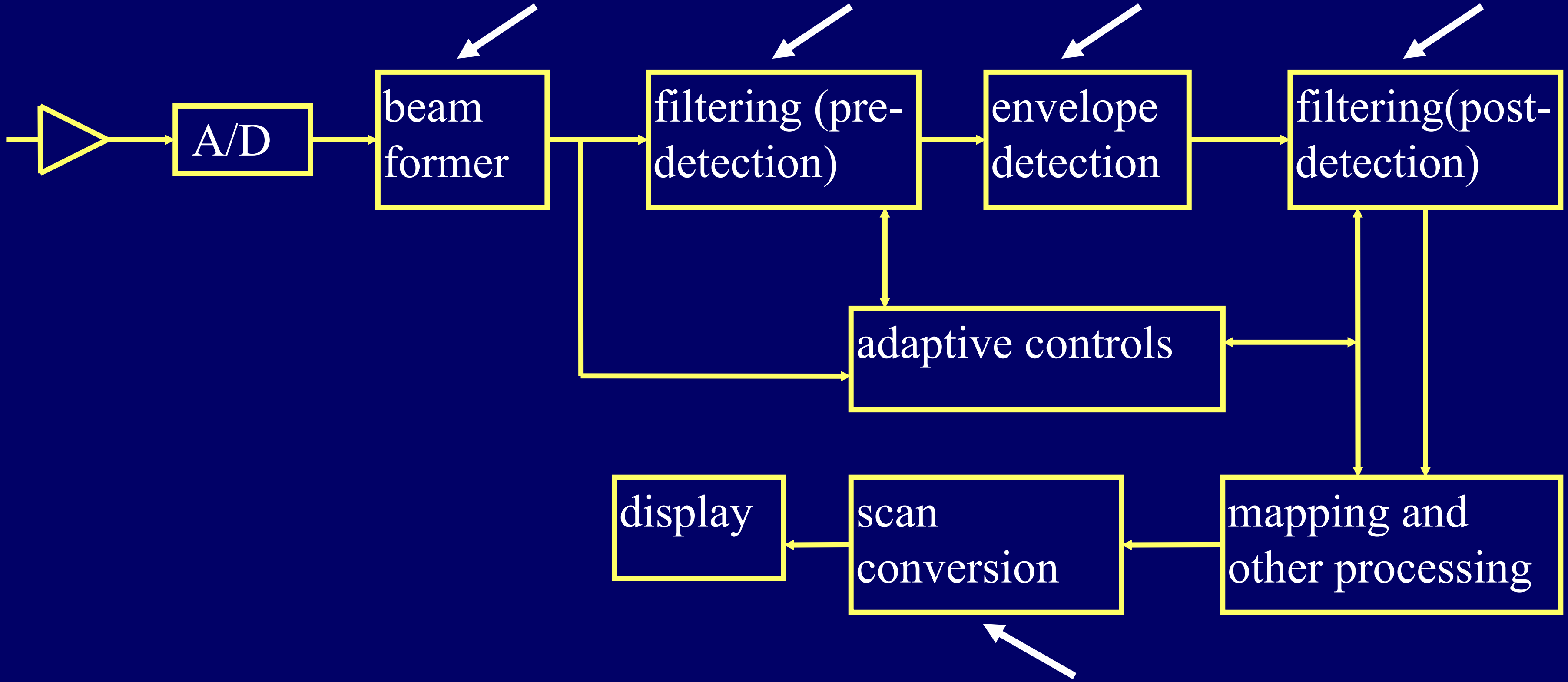
- Characteristics of transmit waveforms.



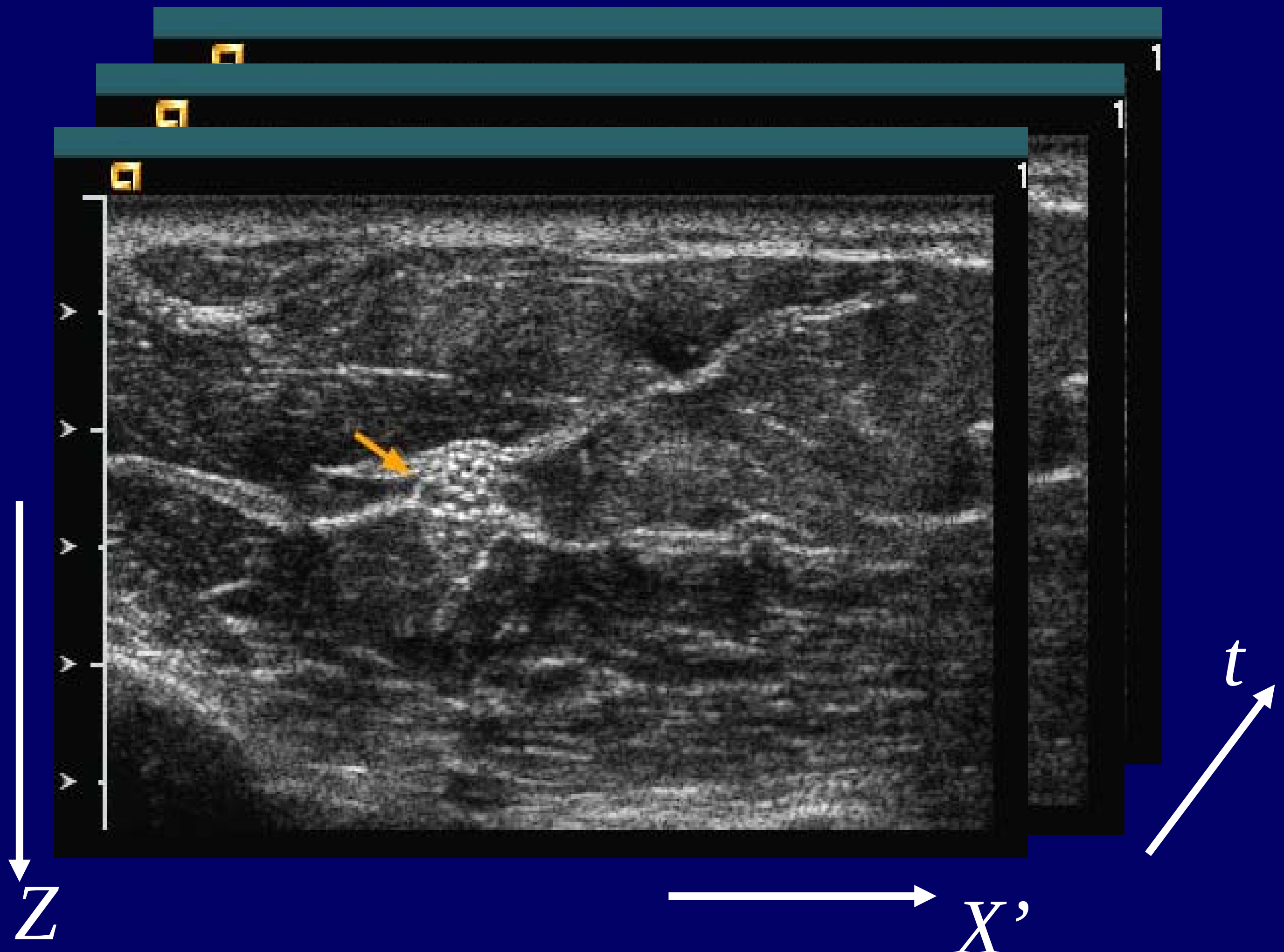
Generic Ultrasonic Imaging System

- Receiver:
  - Programmable apodization, delay control and frequency control.
  - Arbitrary receive direction.
- Image processing:
  - Pre-detection filtering.
  - Post-detection filtering.
- Full gain correction: TGC, analog and digital.
- Scan converter: various scan format.

Generic Receiver



Pre-detection Filtering

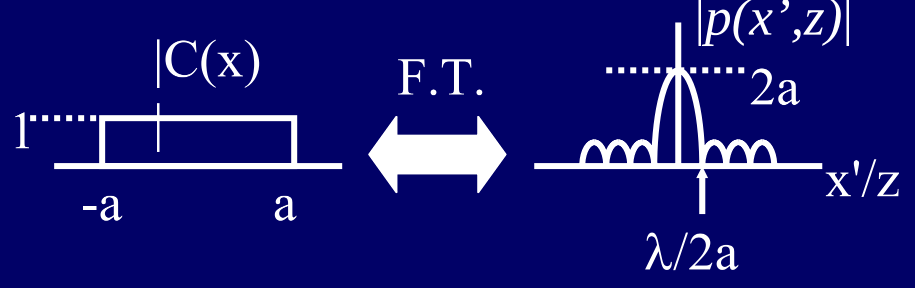


Pre-detection Filtering

- Pulse shaping. (*Z*)
- Temporal filtering. (*t*)
- Beam shaping. (*X'*)
  - Selection of frequency range. ($Z \rightarrow X'$)

$$B(x', z) = \int T(x', z, \omega)\, R(x', z, \omega)\, A(\omega)\, d\omega$$

  - Correction of focusing errors. ($X \rightarrow X'$)



Pulse-echo effective apertures

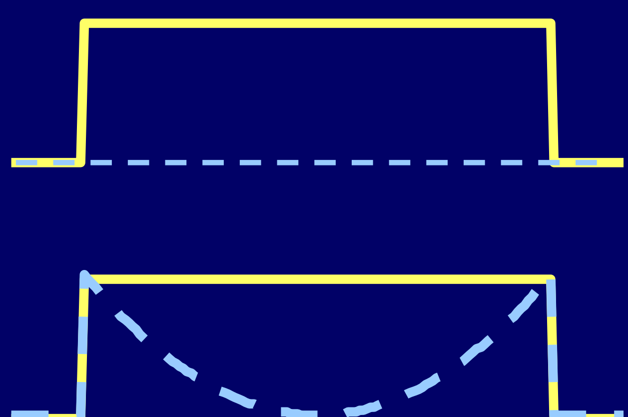
- The pulse-echo beam pattern is the multiplication of the transmit beam and the receive beam
- The pulse-echo effective aperture is the convolution of transmit and receive apertures

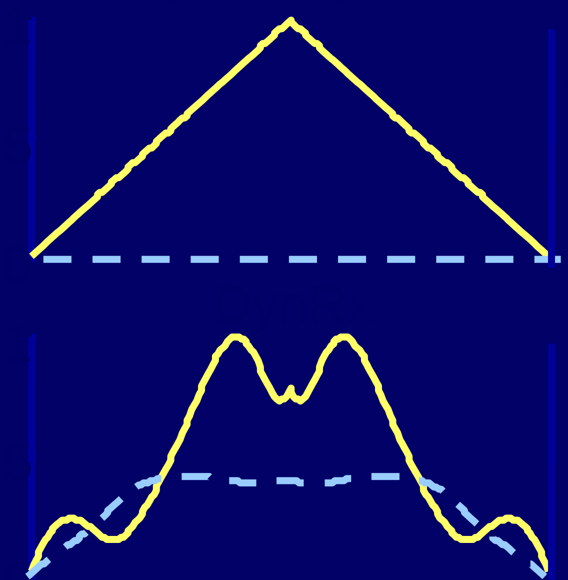
For C.W.

$$C(x) = |C(x)|\, e^{\frac{jkx^2}{2}\left(\frac{1}{R}-\frac{1}{R_0}\right)}$$

R=Ro

R≠Ro





Post-Detection Filtering

- Data re-sampling (Acoustic → Display).
- Speckle reduction (incoherent averaging).
- Feature enhancement.
- Aesthetics.
- Post-processing:
  - Re-mapping (gray scale and color).
  - Digital gain.

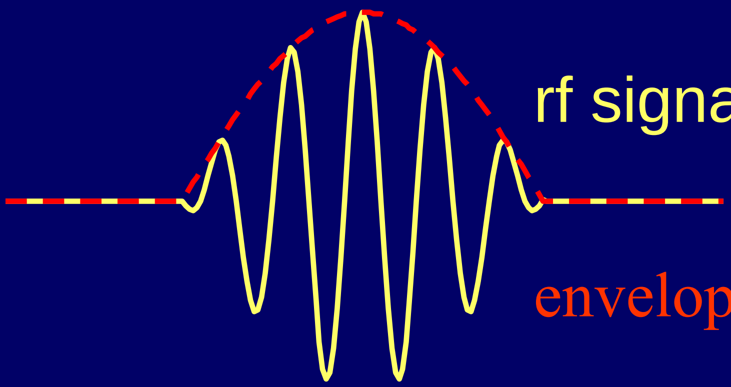
Envelope Detection

- Demodulation based:

$$S(t) = A(t)\cos 2\pi f_0 t = \mathrm{Re}\left\{A(t)\, e^{j2\pi f_0 t}\right\}$$

$$A(t) = LPF\left\{S(t)\cos 2\pi f_0\right\}$$

$$D(t) = abs(A(t))$$

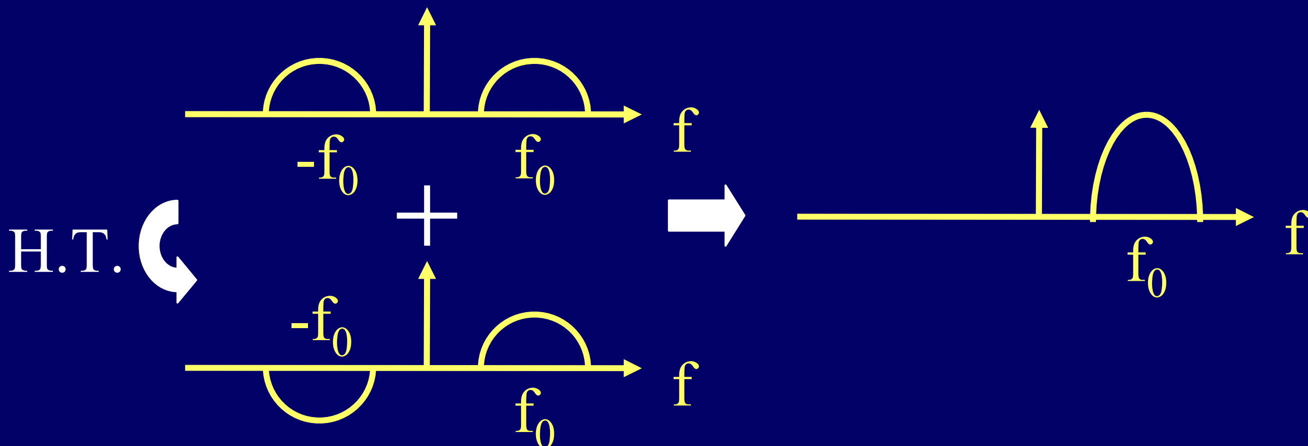


Envelope Detection

- Hilbert Transform

$$S(t) + j \cdot H.T.\{S(t)\} = 2A(t)\, e^{j2\pi f_0 t}$$

$$D(t) = abs\left(S(t) + j \cdot H.T.\{S(t)\}\right) / 2$$



$$HT(f) = -j\,\mathrm{sgn}(f)$$

$$ht(t) = -\frac{1}{\pi t}$$

Beam Former Design

Implementaiton of Beam Formation

- Delay is simply based on geometry.
- Weighting (a.k.a. apodization) strongly depends on the specific approach.

Beam Formation - Delay

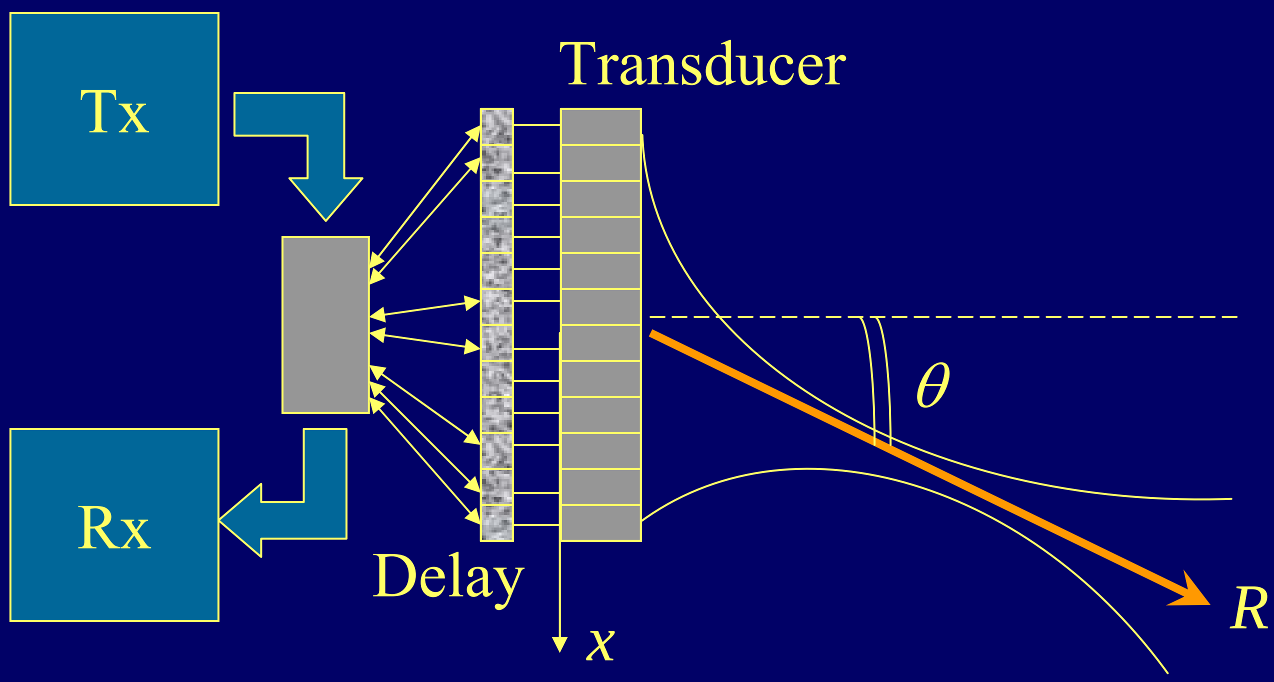
- Delay is based on geometry. For simplicity, a constant sound velocity and straight line propagation are assumed. Multiple reflection is also ignored.
- In diagnostic ultrasound, we are almost always in the near field. Therefore, range focusing is necessary.

Beam Formation - Delay

- Near field / far field crossover occurs when $f_{\#}$=aperture size/wavelength.
- The crossover also corresponds to the point where the phase error across the aperture becomes significant (destructive).

$$\frac{a^2}{2R} = \frac{\lambda}{8}$$

Phased Array Imaging

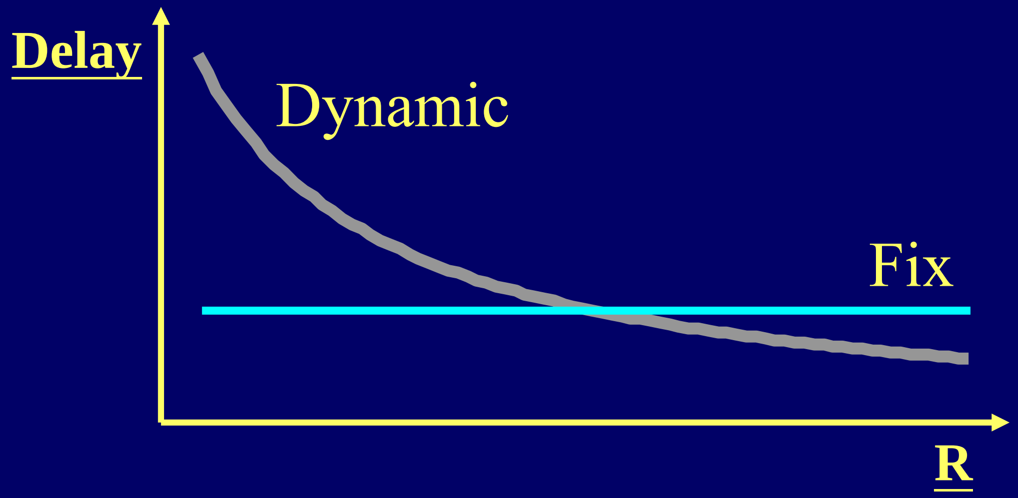


$$t_{rx}(x_i, R, \theta) = -\frac{x_i \sin\theta}{c} + \boxed{\frac{x_i^2 \cos^2\theta}{2Rc}}$$

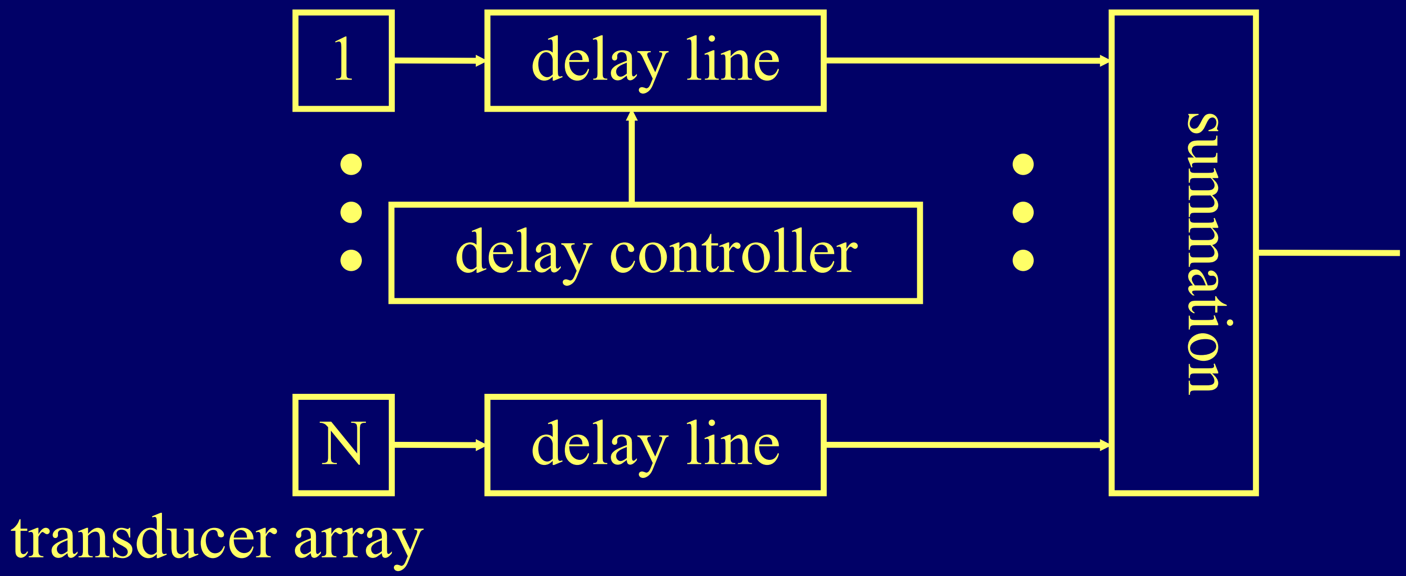
Symmetry

Dynamic Focusing

- Dynamic-focusing obtains better image quality but implementation is more complicated.



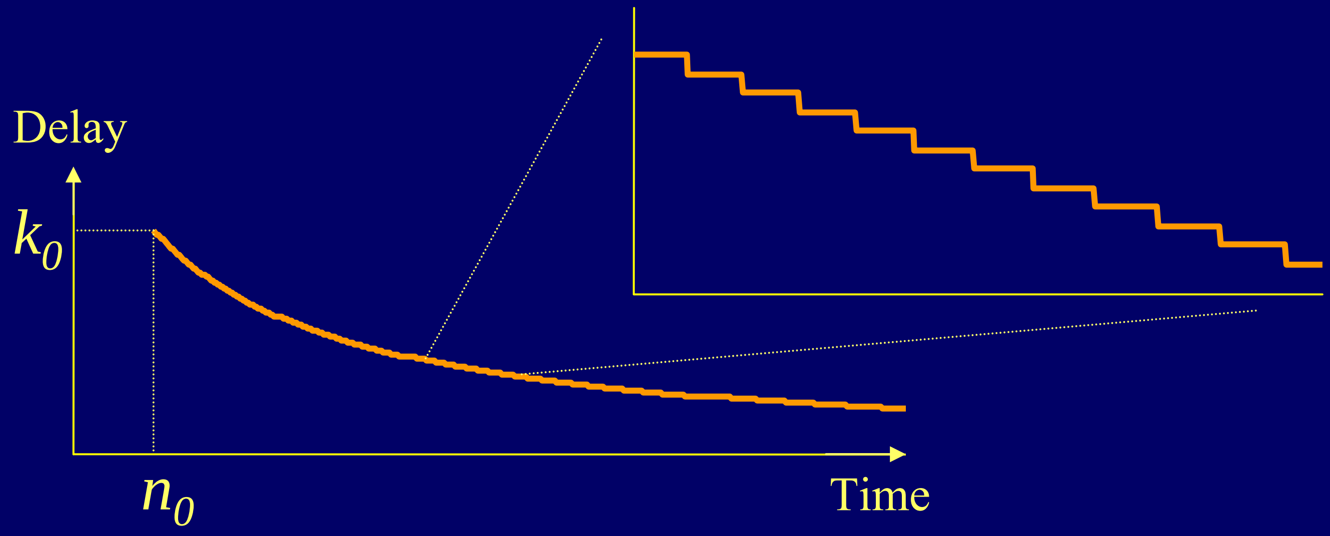
Focusing Architecture



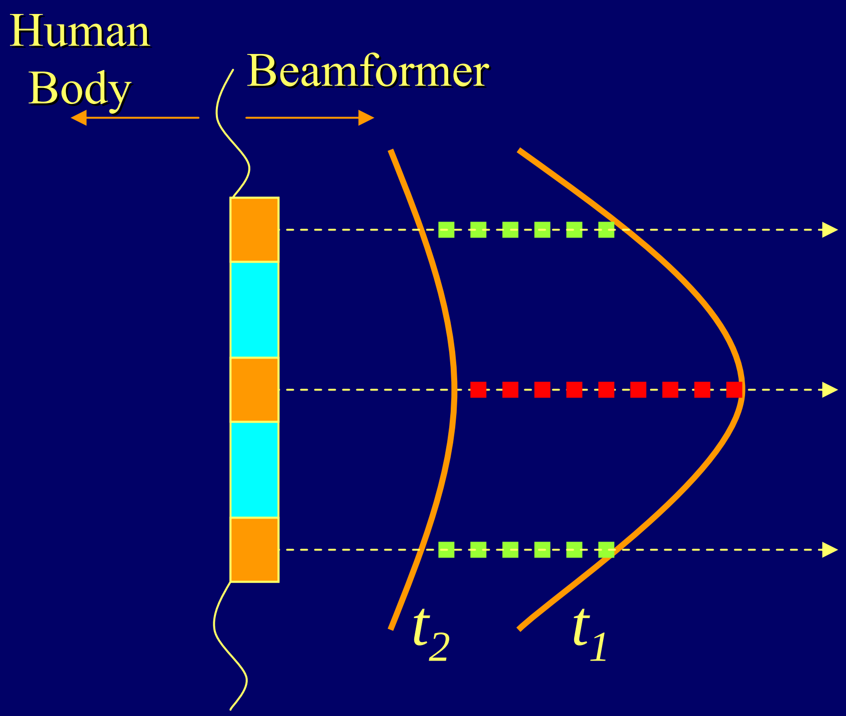
Delay Pattern

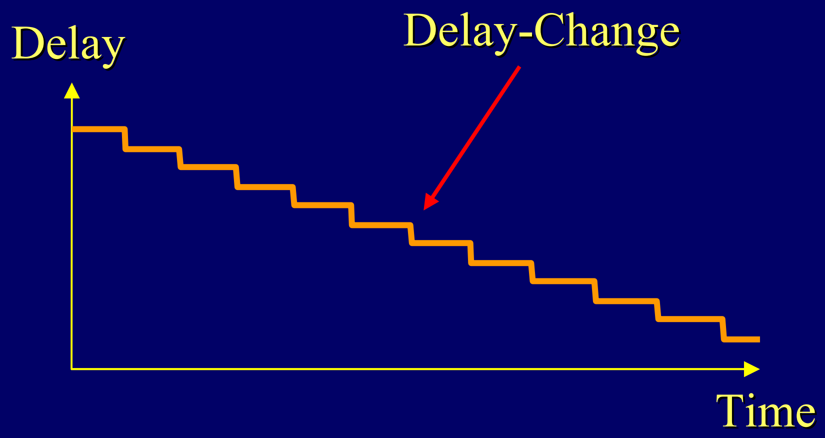
- Delays are quantized by sampling-period $\boldsymbol{t_s}$.

$$k_n = round\left(-\frac{x_i \sin\theta}{ct_s} + \frac{x_i^2 \cos^2\theta}{2Rct_s}\right) = n\Delta\tau$$

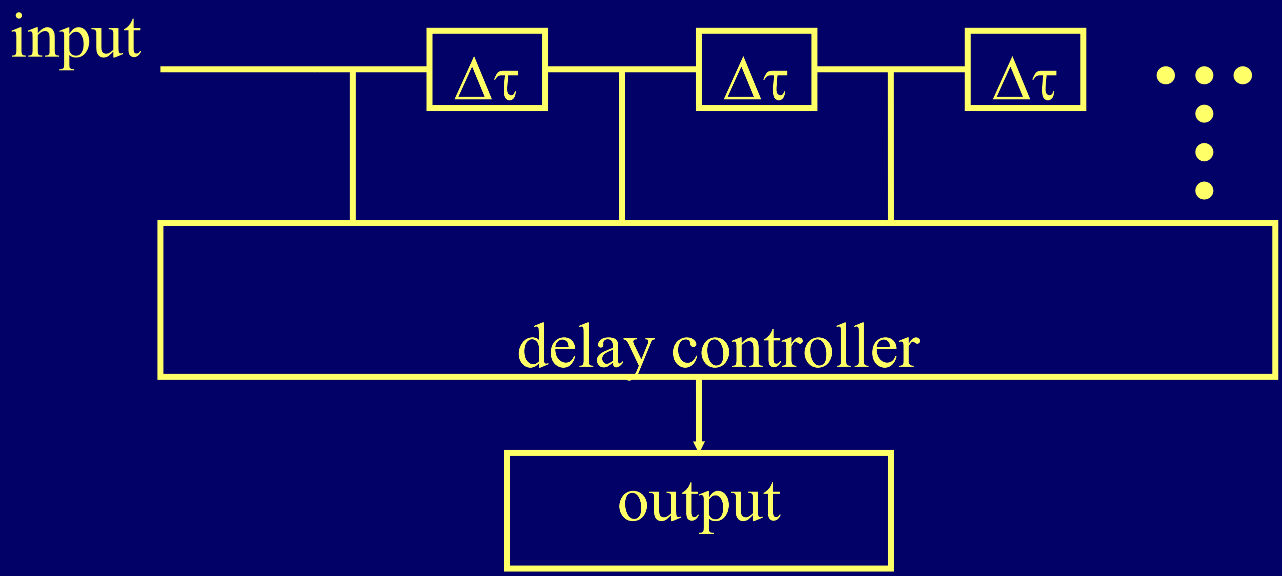


Missing Samples





Beam Formation



$$n(t) \approx -\frac{x_i \sin\theta}{c\Delta\tau} + \frac{x_i^2 \cos^2\theta}{c^2 t\Delta\tau}$$

$$n(t_1) - n(t_2) = 1 = \frac{x_i^2 \cos^2\theta}{c^2\Delta\tau}\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$$

Beam Formation - Delay

- The sampling frequency for fine focusing quality needs to be over $32*f_0$(>> Nyquist).
- Interpolation is essential in a digital system and can be done in RF, IF or BB.

$$\Delta\tau = \frac{\Delta\theta}{2\pi f_0} \leq \frac{1}{32 f_0}$$

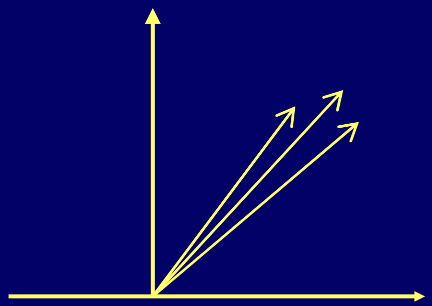
$$2\pi / 32 \approx 11.25^o$$

Delay Quantization

- The delay quantization error can be viewed as the phase error of the phasors.

$$A = \sum_{n=0}^{N-1} \cos(\phi_n)$$

$$\sigma_A^2 = \sum_{n=0}^{N-1} \left(\frac{dA}{d\phi} \right)^2 \sigma_{\phi_n}^2$$



Delay Quantization

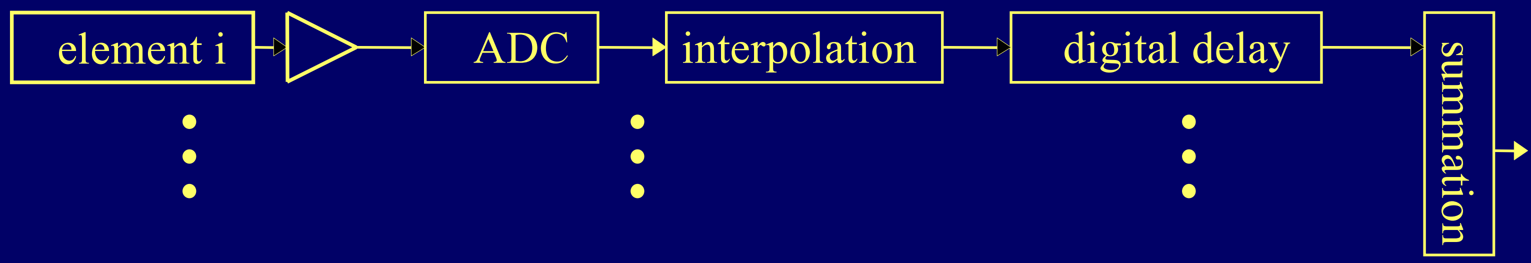
$$\left\langle \sin^2 \phi \right\rangle = \frac{1}{2}$$

$$\sigma_{\phi_n}^2 = \sigma_{\phi}^2 = \frac{\Delta\phi^2}{12}$$

$$\sigma_A^2 = \frac{N \cdot \Delta\phi^2}{24} < 1 \Rightarrow \Delta\phi < \sqrt{\frac{24}{N}}$$

- N=128, 16 quantization steps per cycles are required.
- In general, 32 and 64 times the center frequency is used.

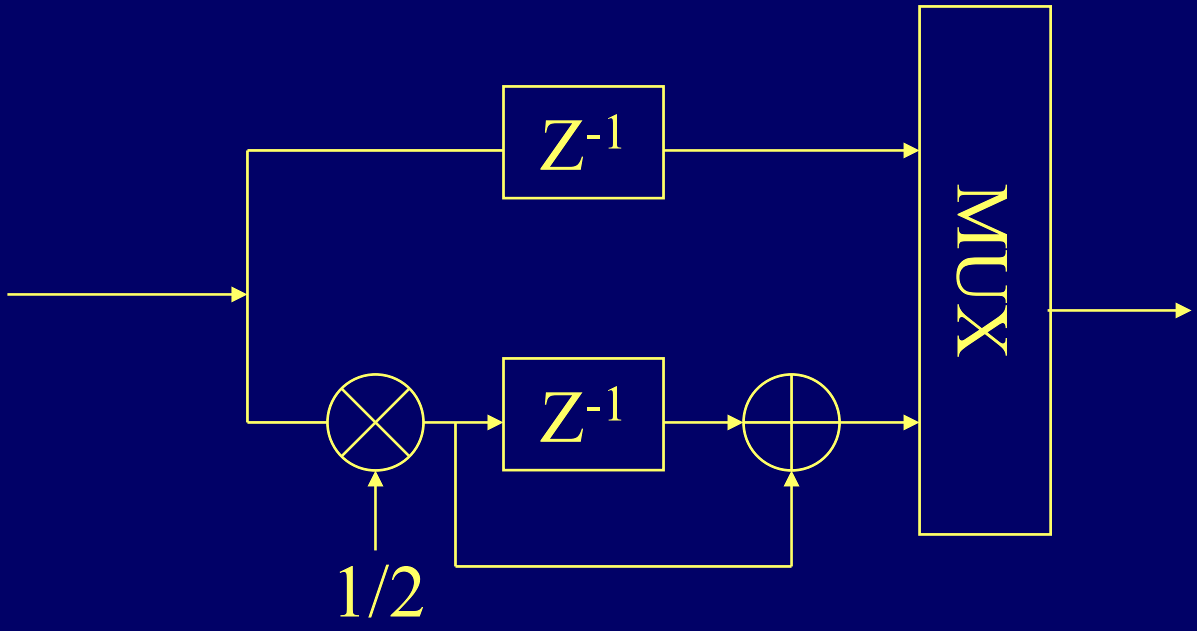
Beam Formation - Delay



- RF beamformer requires either a clock well over 100MHz, or a large number of real-time computations.
- BB beamformer processes data at a low clock frequency at the price of complex signal processing.

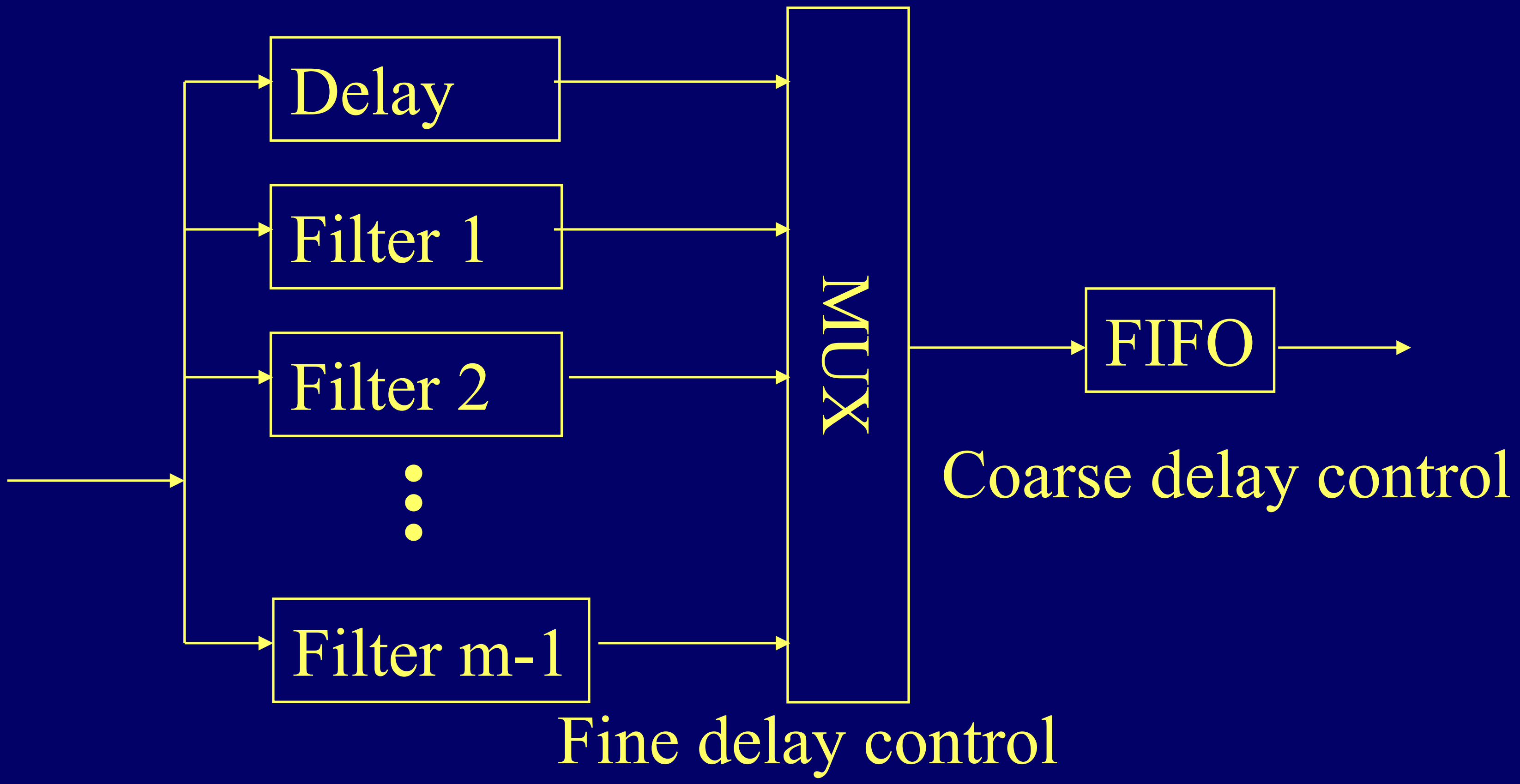
Beam Formation - RF

- Interpolation by 2:



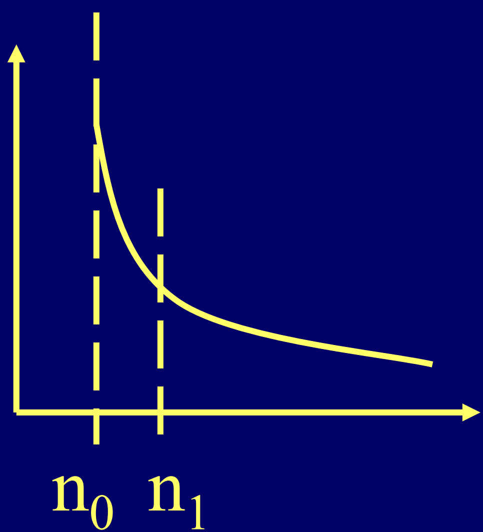
Beam Formation - RF

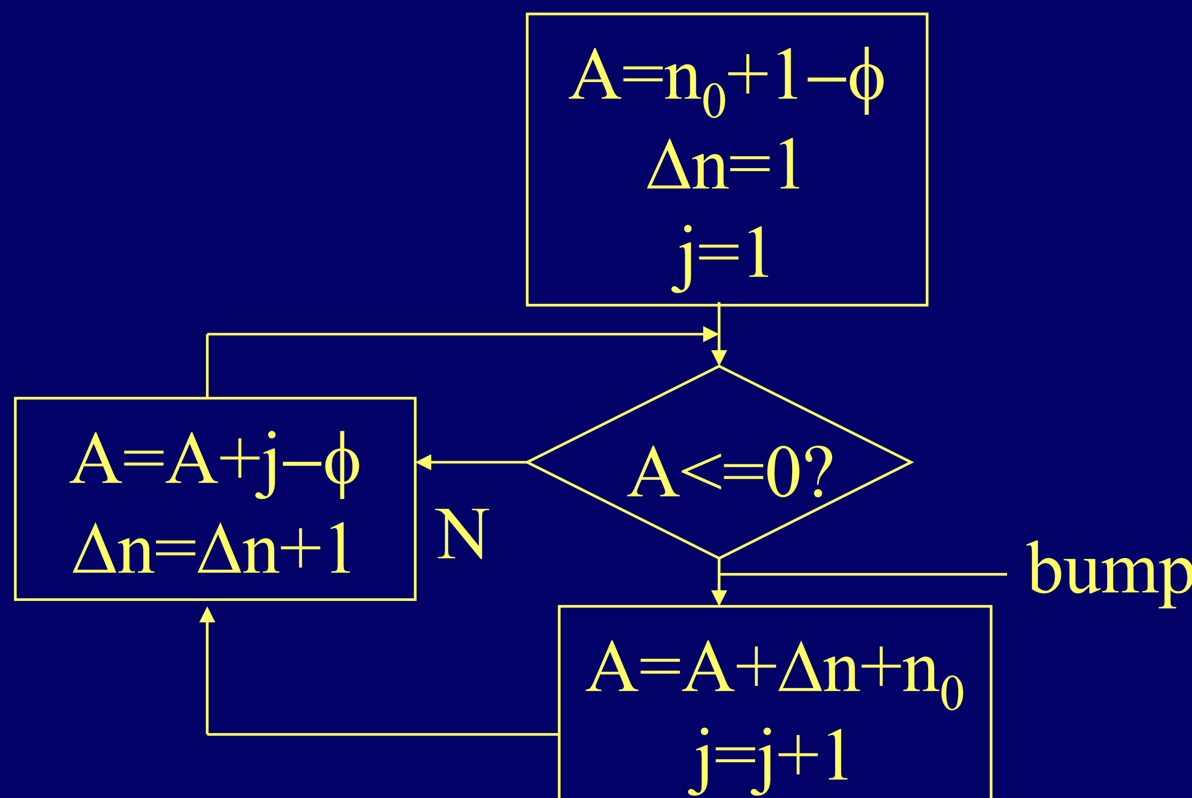
- General filtering architecture (interpolation by m):



Autonomous Delay Control

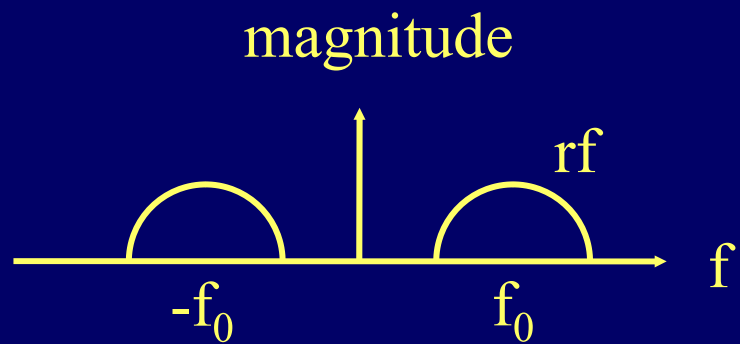
Autonomous vs. Centralized



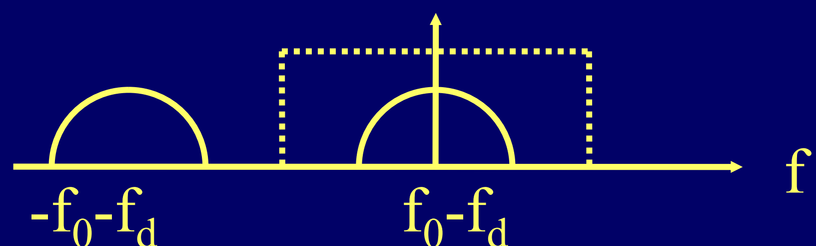


Beam Formation - BB

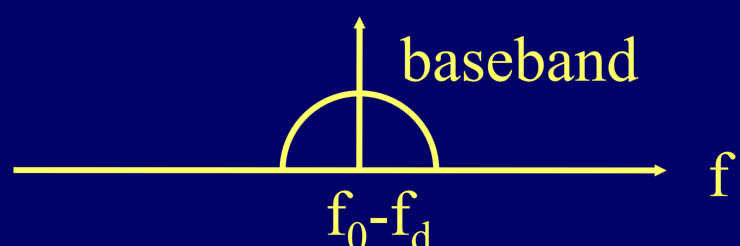
$A(t-\tau)\cos 2\pi f_0(t-\tau)$



$A(t-\tau)\cos 2\pi f_0(t-\tau)e^{-j2\pi fdt}$



$LPF(A(t-\tau)\cos 2\pi f_0(t-\tau)e^{-j2\pi fdt})$



Beam Formation - BB

$$I = LPF\left\{A(t-\tau)\cos 2\pi f_0 (t-\tau)\cos 2\pi f_d t\right\}$$

$$= LPF\left\{\frac{A(t-\tau)}{2}\Big(\cos 2\pi((f_0 - f_d)(t-\tau) - f_d\tau) + \cos 2\pi((f_0 + f_d)(t-\tau) + f_d\tau)\Big)\right\}$$

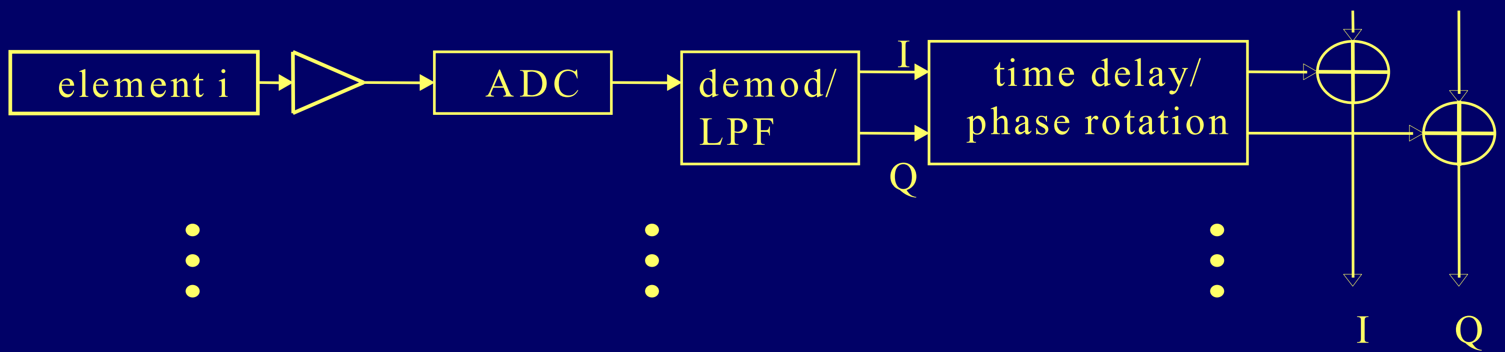
$$= \frac{A(t-\tau)}{2}\cos 2\pi((f_0 - f_d)(t-\tau) - f_d\tau)$$

$$Q = LPF\left\{- A(t-\tau)\cos 2\pi f_0 (t-\tau)\sin 2\pi f_d t\right\}$$

$$= LPF\left\{\frac{A(t-\tau)}{2}\Big(\sin 2\pi((f_0 - f_d)(t-\tau) - f_d\tau) - \sin 2\pi((f_0 + f_d)(t-\tau) + f_d\tau)\Big)\right\}$$

$$= \frac{A(t-\tau)}{2}\sin 2\pi((f_0 - f_d)(t-\tau) - f_d\tau)$$

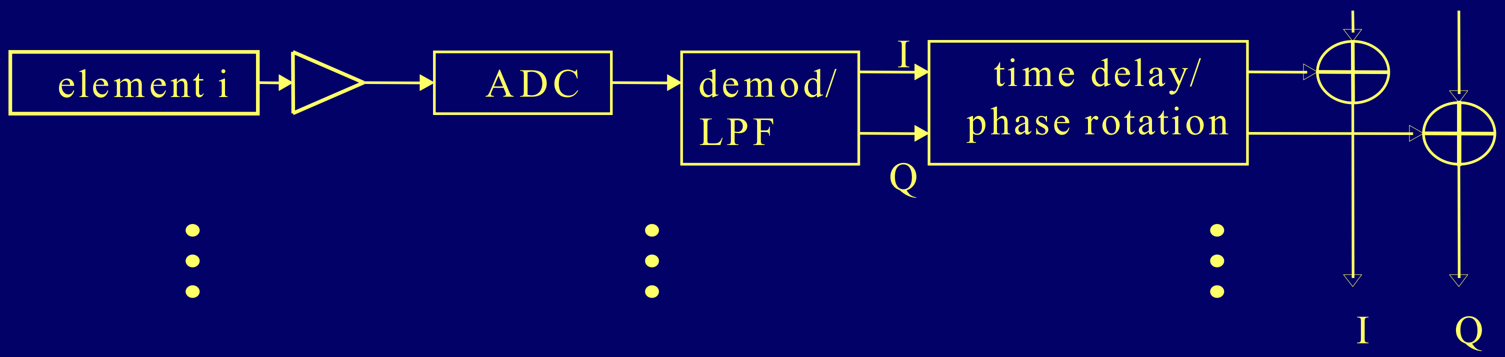
Beam Formation - BB



$$BB(t) = \frac{A(t-\tau)}{2} e^{j2\pi\Delta f(t-\tau)} e^{-j2\pi f_d \tau}$$

$$O(t) = \sum_{i=1}^{N} \frac{A(t-\tau_i+\tau_i')}{2} e^{j2\pi\Delta f(t-\tau_i+\tau_i')} e^{-j2\pi f_d(\tau_i-\theta_i)}$$

Beam Formation - BB



$$\Delta\tau = \frac{\Delta\theta}{2\pi\Delta f} \leq \frac{1}{32\Delta f}$$

- The coarse time delay is applied at a low clock frequency, the fine phase needs to be rotated accurately (e.g., by CORDIC).

ΔΣ-Based Beamformers

Why ΔΣ ?

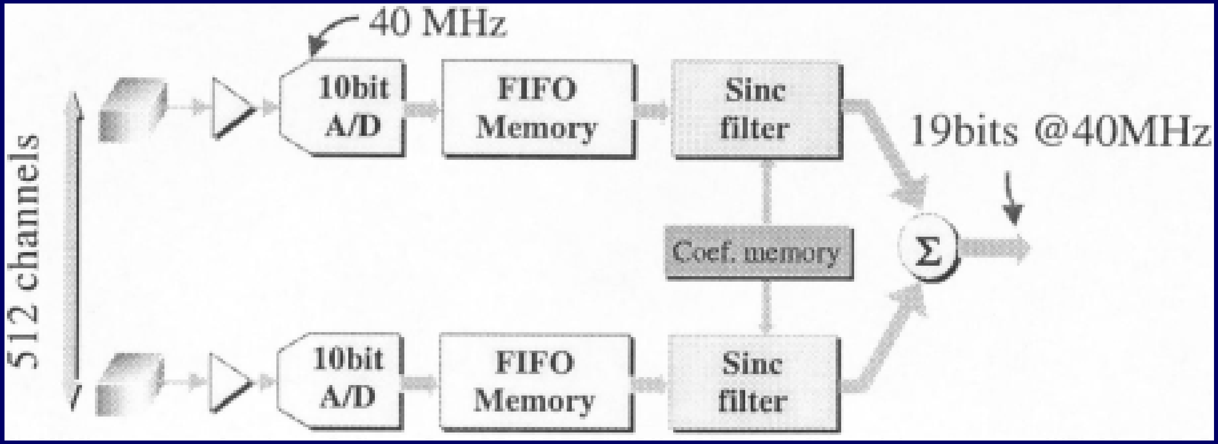
Current Problems

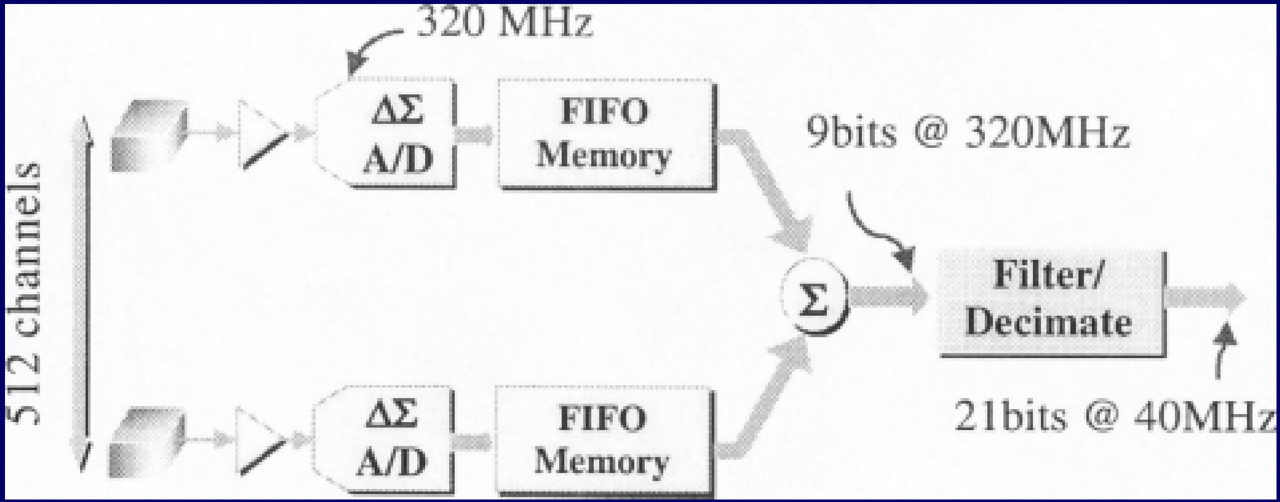
- High Delay Resolution -- 32 f_0 (requires interpolation)
- Multi-Bit Bus

ΔΣ Advantages

- High Sampling Rate -- No Interpolation Required
- Single-Bit Bus -- Suitable for Beamformers with Large Channel-Count

Conventional vs. ΔΣ





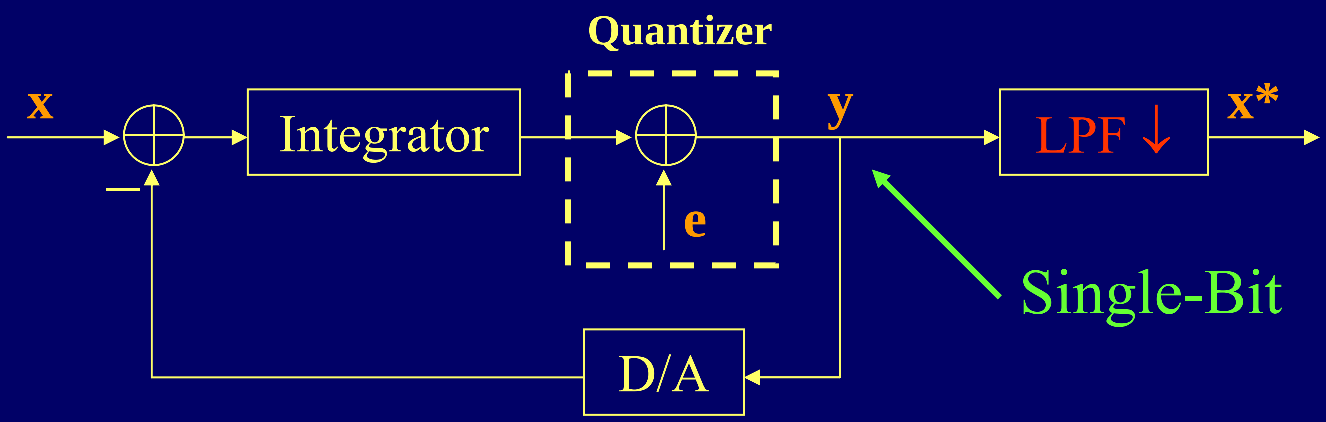
Advantages of Over-Sampling

- Noise averaging.
- For every doubling of the sampling rate, it is equivalent to an additional 0.5 bit quantization.
- Less requirements for delay interpolation.
- Conventional A/D not ideal for single-bit applications.

Advantages of ΔΣ Beamformers

- Noise shaping.
- Single-bit vs. multi-bits.
- Simple delay circuitry.
- Integration with A/D and signal processing.
- For hand-held or large channel count devices.

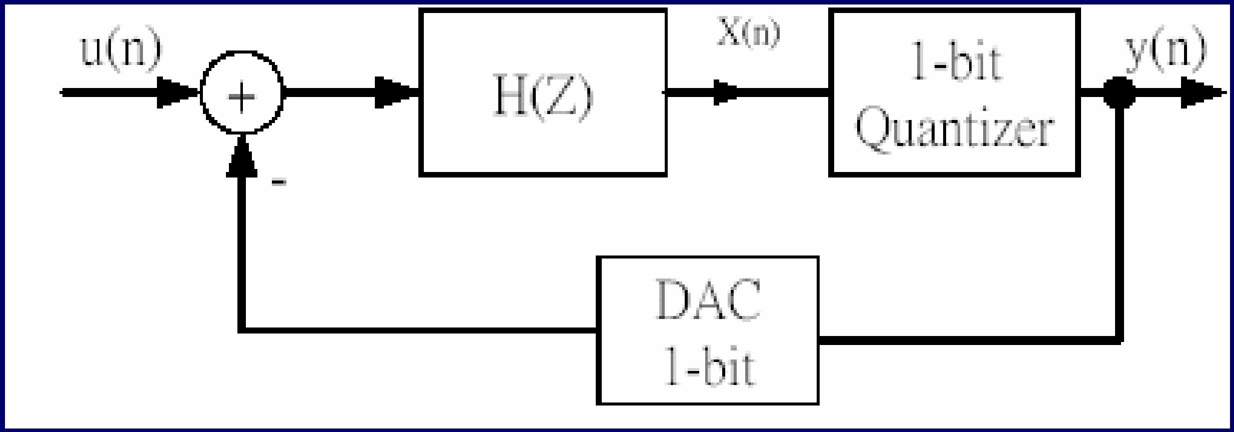
Block-Diagram of the ΔΣ Modulator

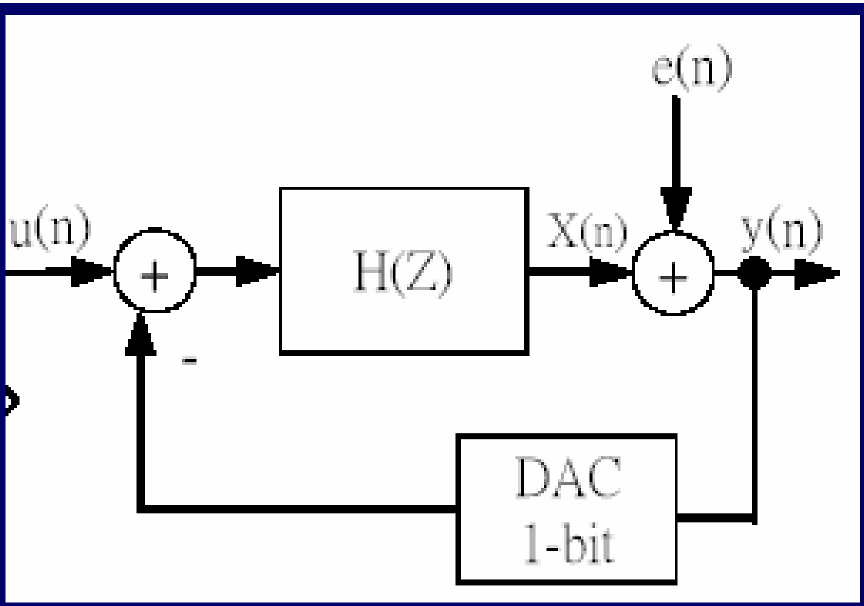


- Over-Sampling
- Noise-Shaping
- Reconstruction

- The SNR of a 32 f_0, 2nd-order, low-passed ΔΣ modulator is about 40dB.

Noise Shaped ΔΣ Modulator





Signal and Noise Transfer

Signal transfer function $S_{TF}(z) \equiv \dfrac{Y(z)}{U(z)} = \dfrac{H(z)}{1+H(z)}$

Noise transfer function $N_{TF}(z) \equiv \dfrac{Y(z)}{E(z)} = \dfrac{1}{1+H(z)}$

$H(z)=\dfrac{z^{-1}}{1-z^{-1}}$ (Noniverting Forward-Euler SC integrator)

$=> S_{TF}(z)=\dfrac{H(z)}{1+H(z)} = z^{-1}$

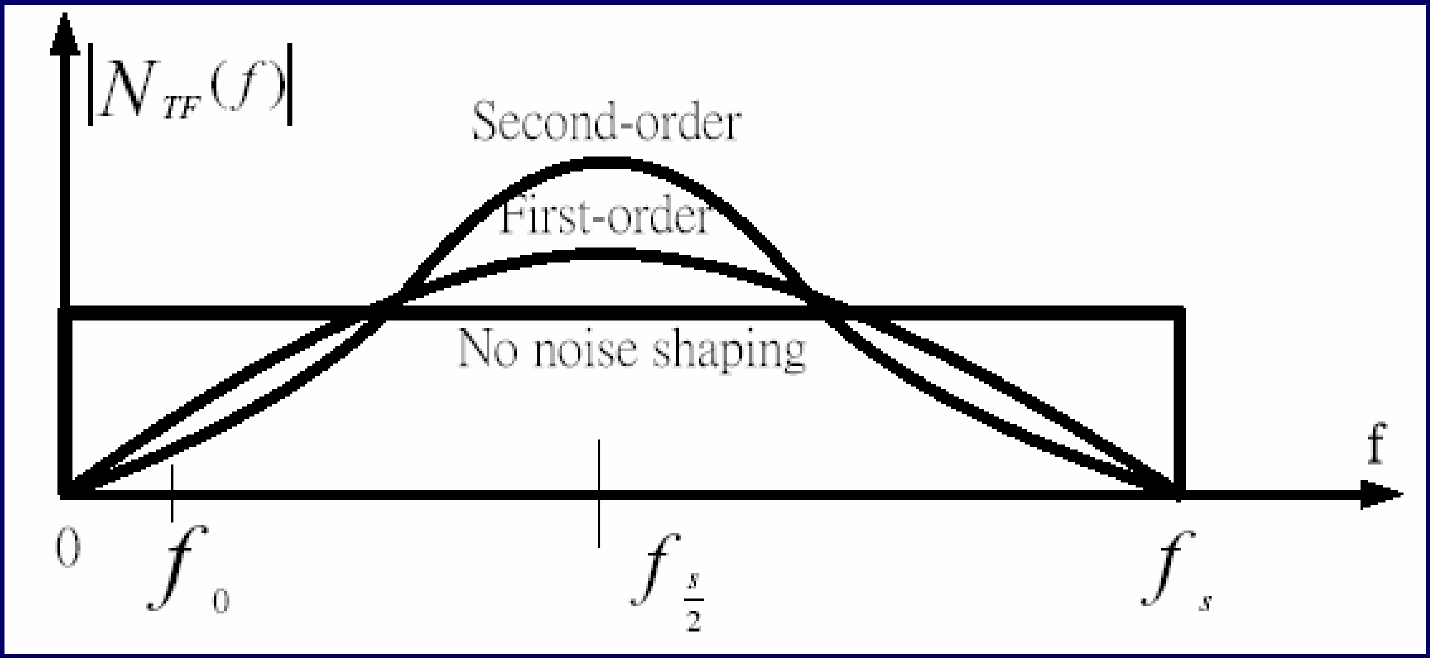
$N_{TF}(z)=\dfrac{1}{1+H(z)} = (1-z^{-1})$ $z=e^{j\omega T}=e^{j2\pi f/fs}$

$N_{TF}(f)=1-e^{-j2\pi f/fs}=\sin(\dfrac{\pi f}{f_s}) \times (2j) \times (e^{-j\pi f/fs})$

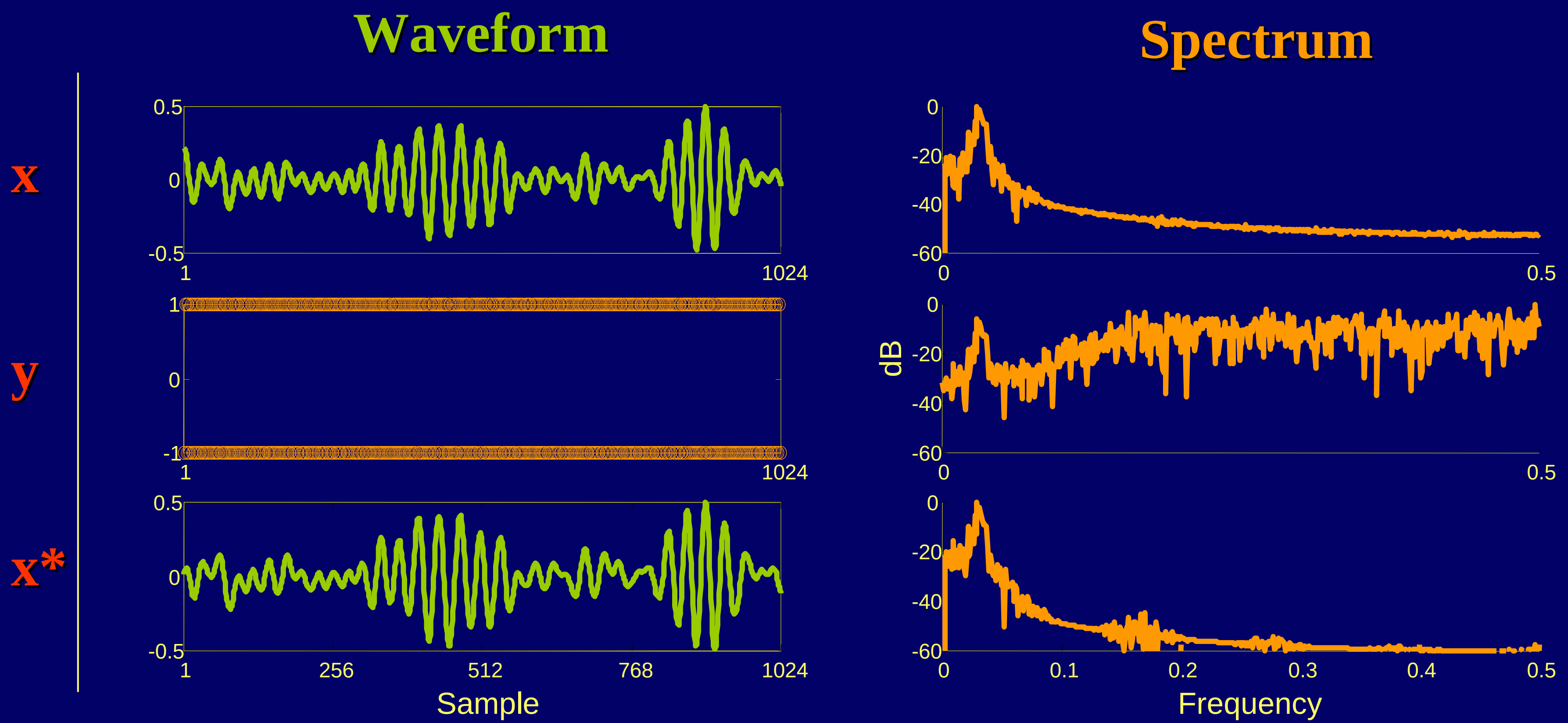
$$\left|N_{TF}(f)\right| = 2\sin(\frac{\pi f}{f_s})$$

Noise Shaping Transfer Functions

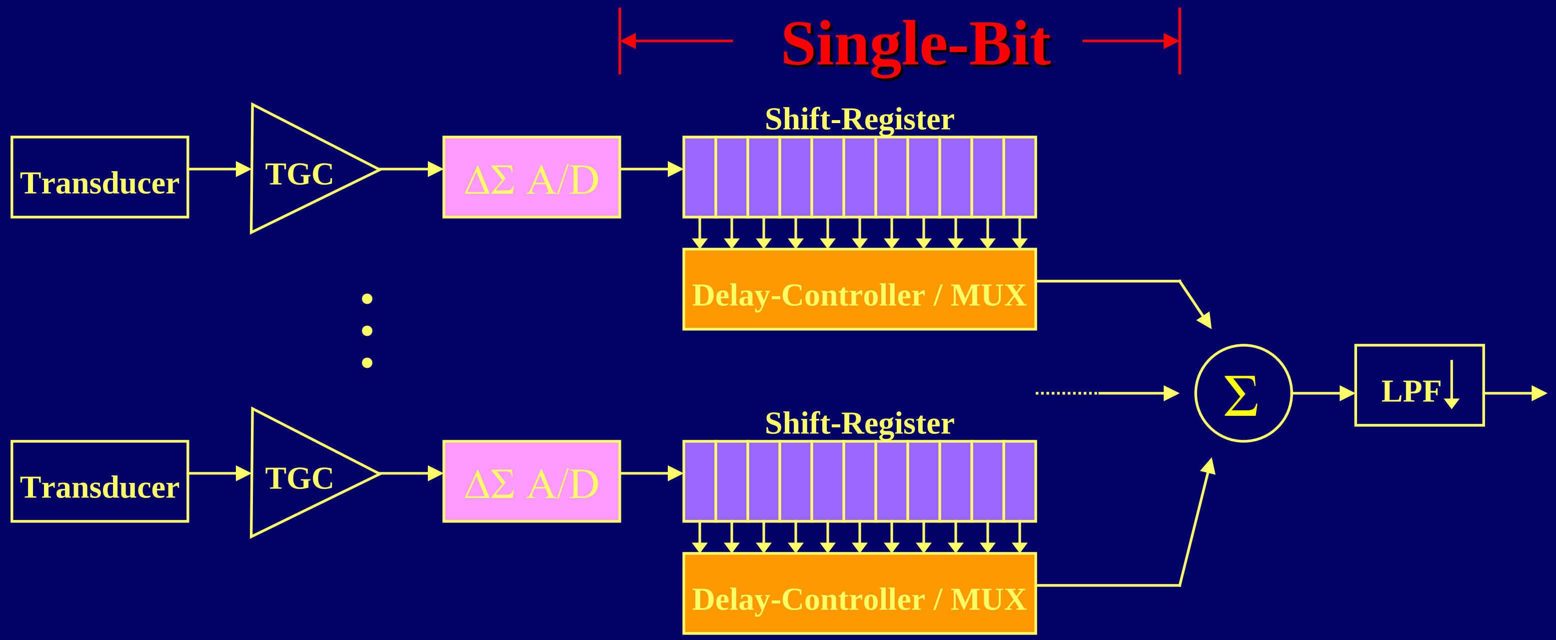
- For first order noise shaping, 1.5 bits (9 dB) is gained when the sampling frequency is doubles.
- For second order noise shaping, 2.5 bits (15 dB) is gained when the sampling frequency is doubles.



Property of a ΔΣ Modulator

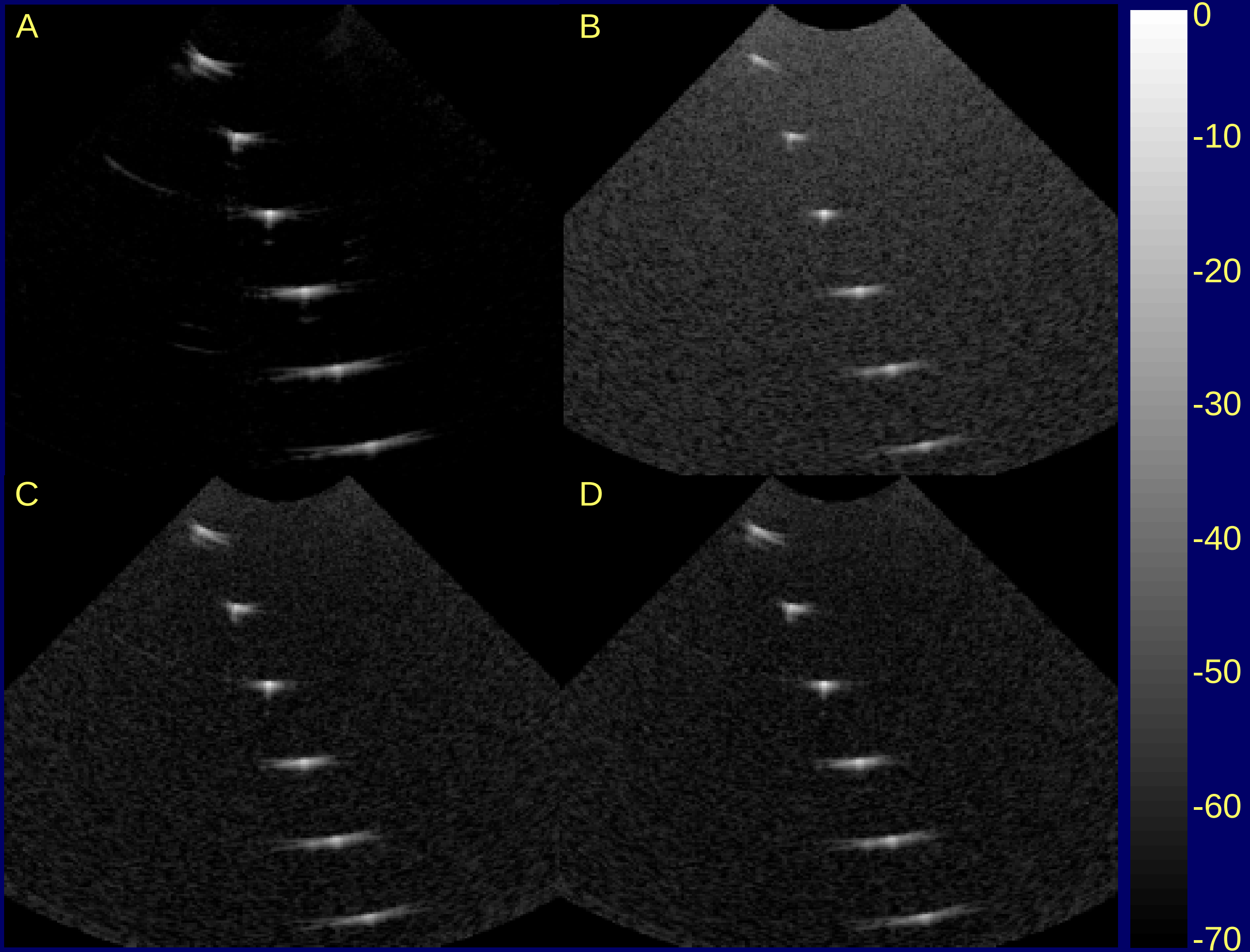


A Delta-Sigma Beamformer



- No Interpolation
- Single-Bit Bus

Results



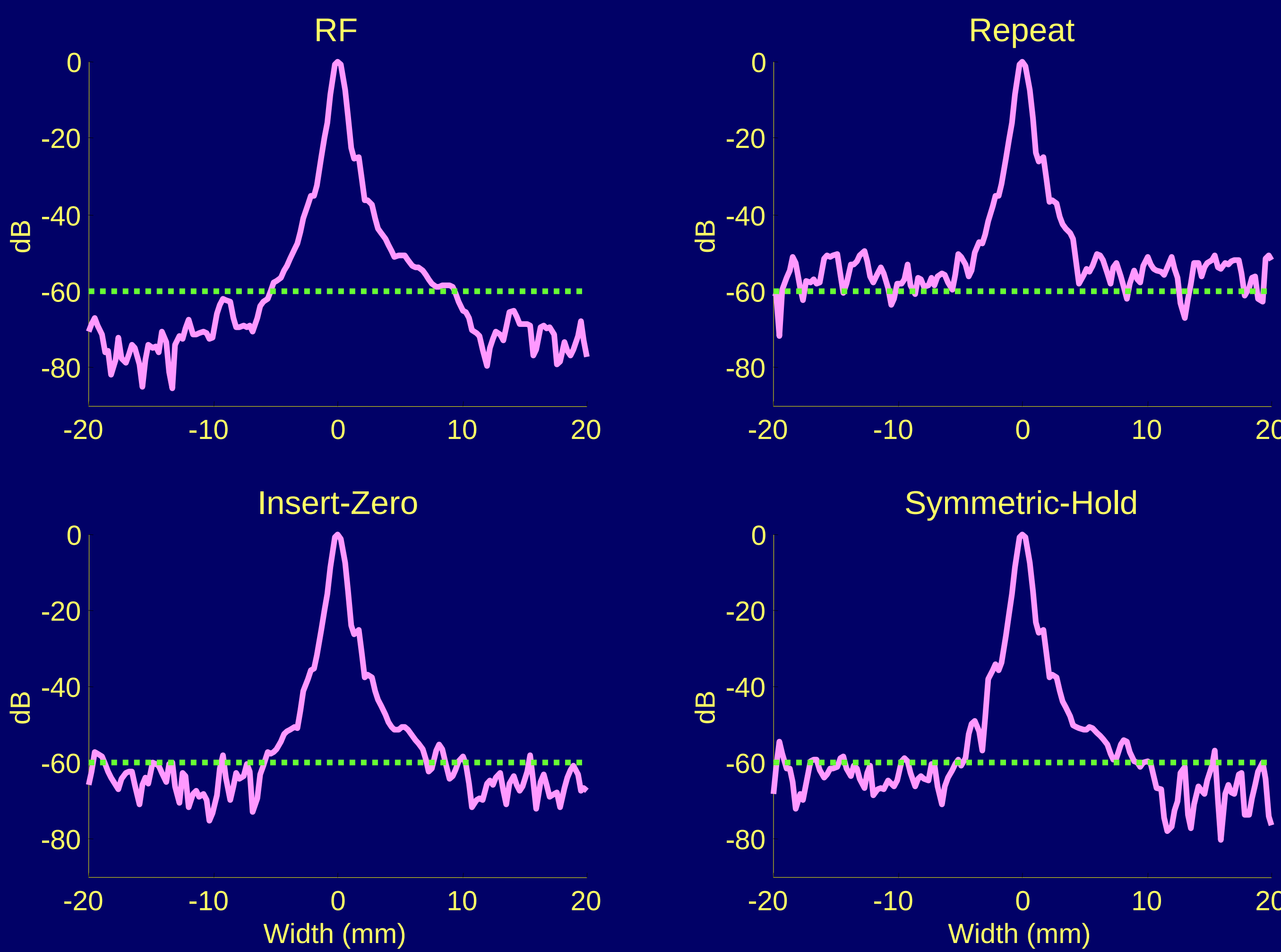
A. RF

B. Repeat

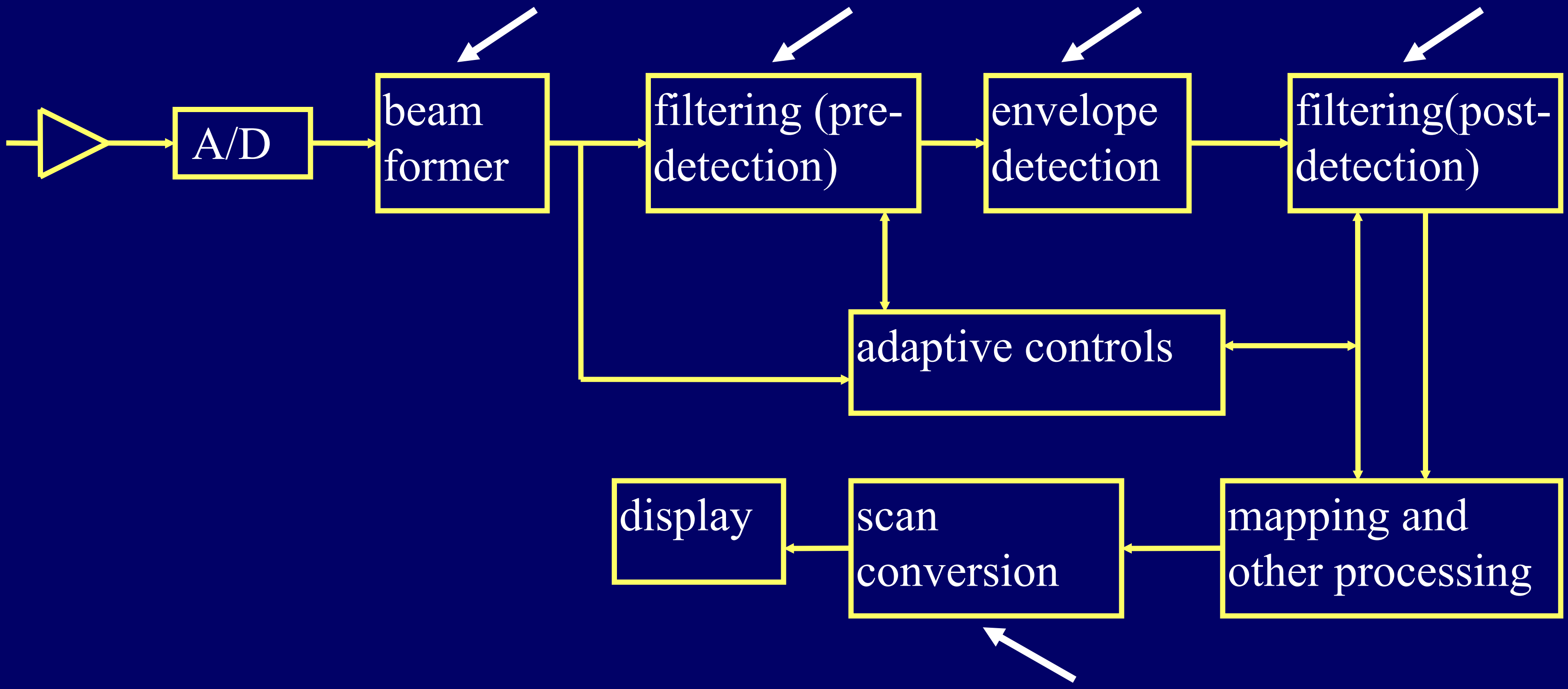
C. Insert-Zero

D. Sym-Hold

Cross-Section-Views of Peak 3

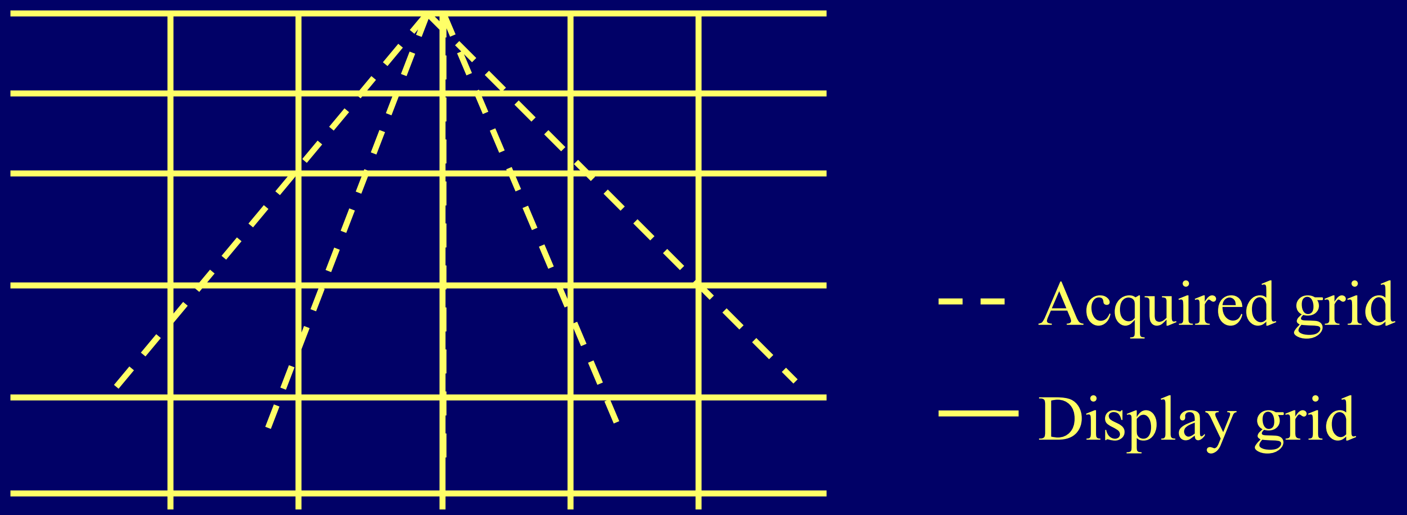


Generic Receiver

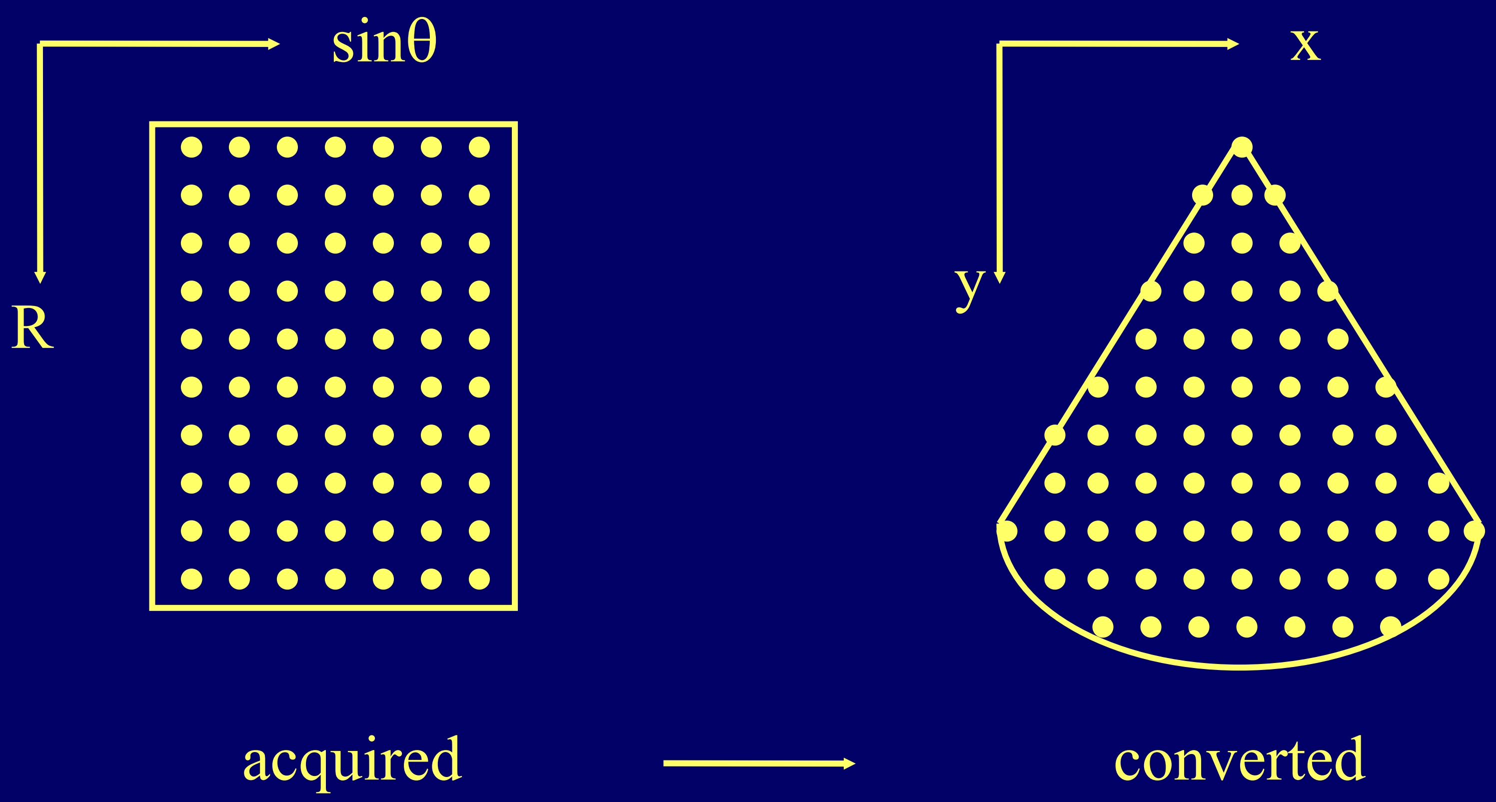


Scan Conversion

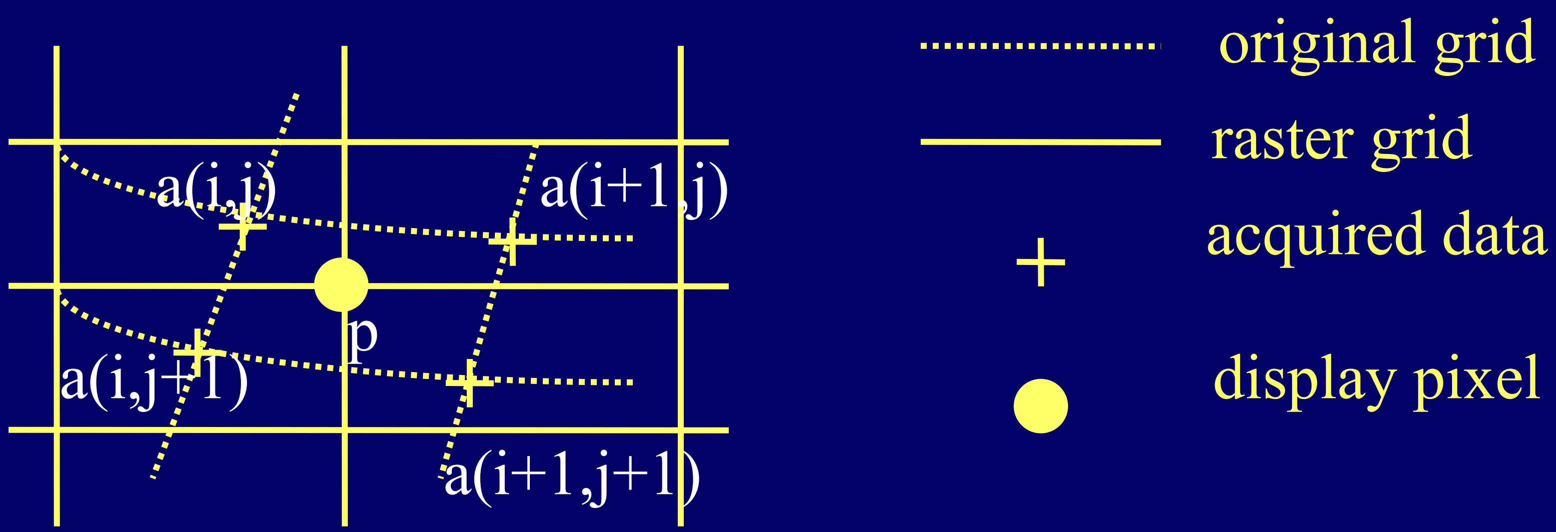
- Acquired data may not be on the display grid.



Scan Conversion

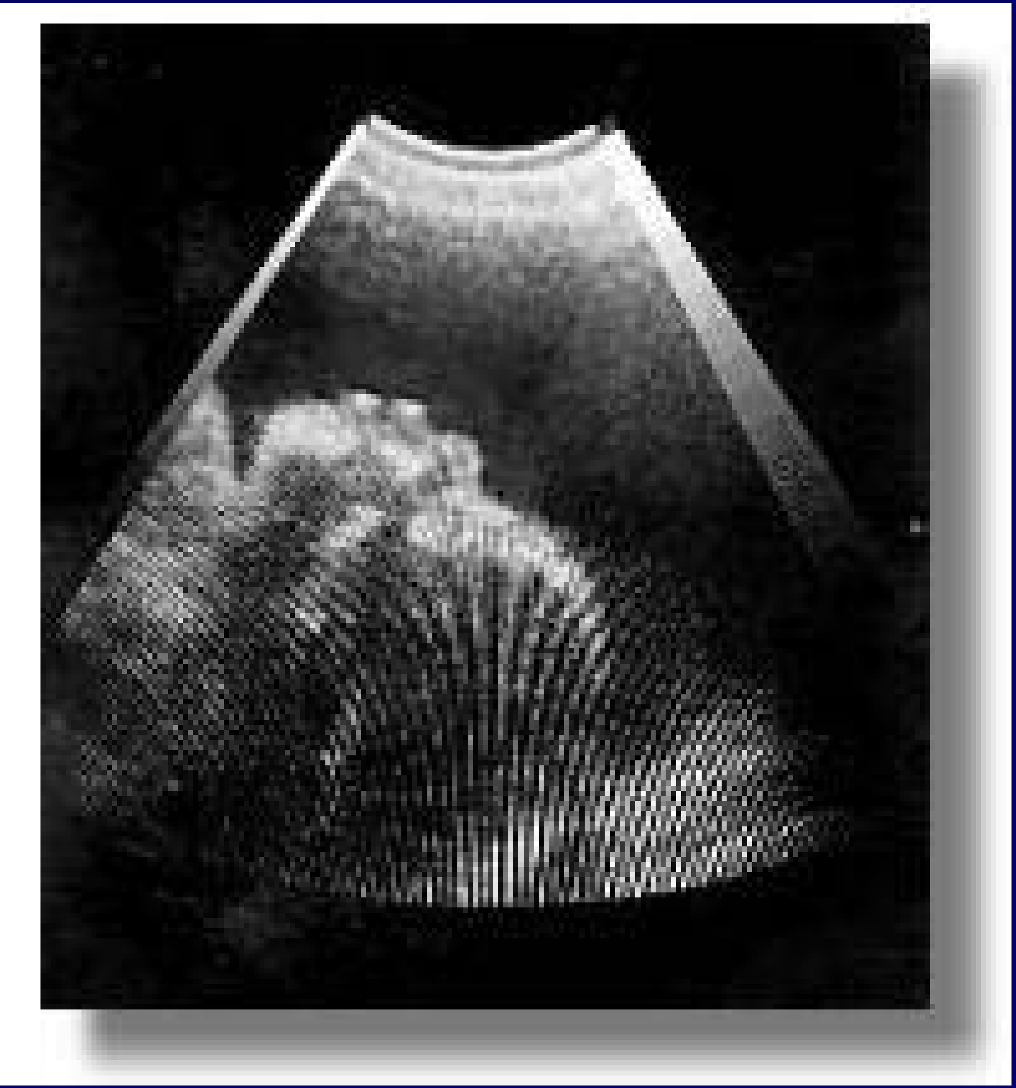


Scan Conversion

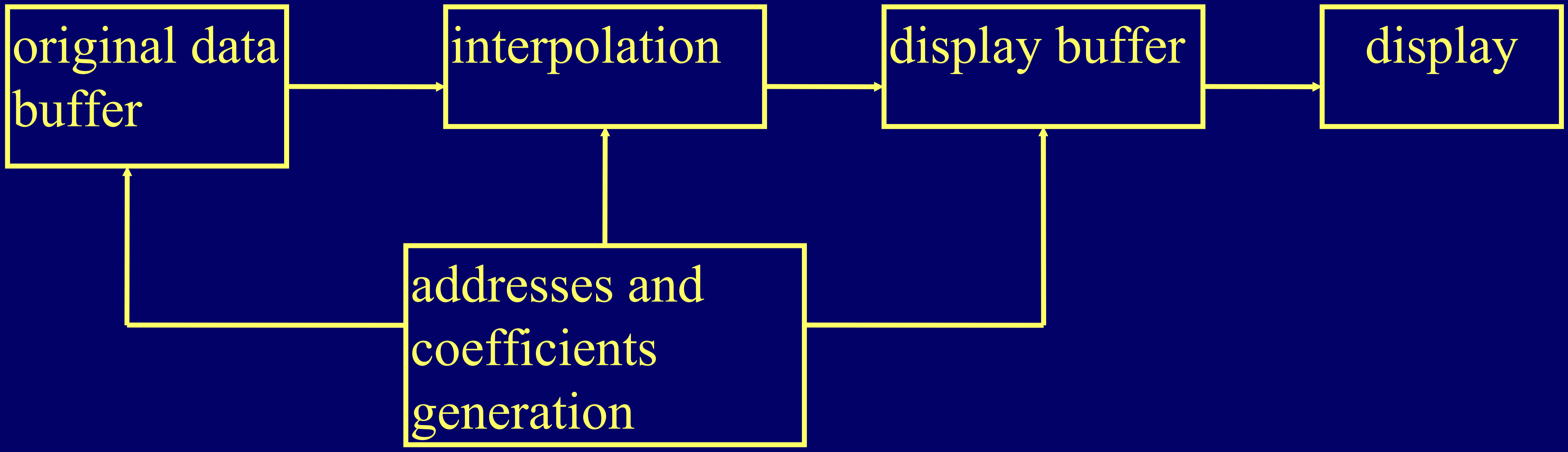


$$p(m,n) = c_{m,n,i,j}a(i,j) + c_{m,n,i+1,j}a(i+1,j) + c_{m,n,i,j+1}a(i,j+1) + c_{m,n,i+1,j+1}a(i+1,j+1)$$

Moiré Pattern



Scan Conversion



Temporal Resolution (Frame Rate)

- Frame rate=1/Frame time.
- Frame time=number of lines * line time.
- Line time=(2*maximum depth)/sound velocity.
- Sound velocity is around 1540 m/s.
- High frame rate is required for real-time imaging.

Temporal Resolution

- Display standard: NTSC: 30 Hz. PAL: 25 Hz (2:1 interlace). 24 Hz for movie.
- The actual acoustic frame rate may be higher or lower. But should be high enough to have minimal flickering.
- Essence of real-time imaging: direct interaction.

Temporal Resolution

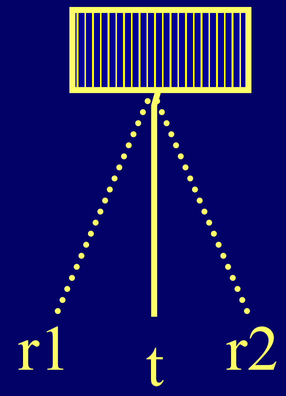
- For an actual frame rate lower than 30 Hz, interpolation is used.
- For an actual frame rate higher than 30 Hz, information can be displayed during playback.
- Even at 30 Hz, it is still possibly undersampling.

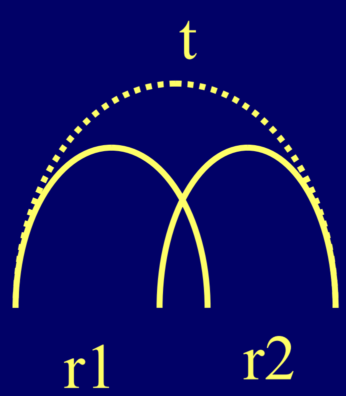
Temporal Resolution

- B-mode vs. Doppler.
- Acoustic power: peak vs. average.
- Increasing frame rate:
  - Smaller depth and width.
  - Less flow samples.
  - Wider beam width.
  - Parallel beam formation.

Parallel Beamformation

- Simultaneously receive multiple beams.
- Correlation between beams, spatial ambiguity.
- Require duplicate hardware (higher cost) or time sharing (reduced processing time and axial resolution).





Parallel Beamformation

- Simultaneously transmit multiple beams.
- Interference between beams, spatial ambiguity.