

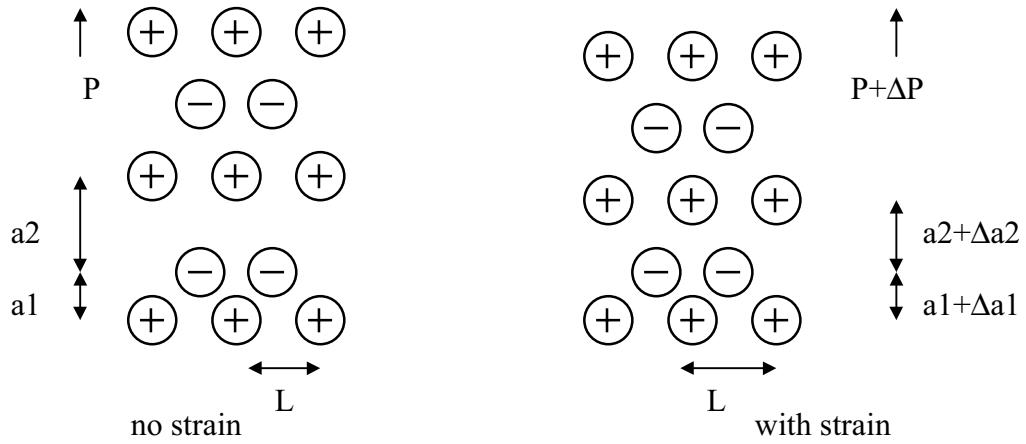
Chapter 4 : Transducers - Generation and Detection of Ultrasound

I. Introduction

- Transducers are used as both transmitters and receivers, converting electrical energy to acoustical energy and vice versa. In low frequency applications (below 20KHz), microphones and loudspeakers are two well-known examples. For diagnostic ultrasound, higher frequencies are required and piezoelectric materials are most commonly used.
- *Piezoelectricity* is defined as the generation of an electrical polarization in a substance by the application of a mechanical stress and, conversely, a change in the shape of a substance when an electric field is applied. In other words, a material is strained when an electric field is applied to it.
- Commonly used piezoelectric materials include naturally occurring crystals, such as quartz and certain man-made ceramic materials, such as lead zirconate-titanates (PZTs). Crystals such as quartz are inherently piezoelectric, with properties determined by their crystallographic features. In contrast, man-made ceramics are polarized above the Curie temperature (typically around 320-370°C for PZTs) by the application of strong electric fields to induce anisotropy responsible for their strong piezoelectric properties. PZTs are the most commonly used piezoelectric materials for diagnostic ultrasonic imaging.
- In addition to PZTs, PVDFs and composites are two other commonly seen materials in medical ultrasound. PVDF is often used for acoustic field measurements due to its broad bandwidth and sufficiently small thickness. Note that small thickness is necessary in order to minimize the interference of sound field due to the presence of the hydrophone. Composite materials, on the other hand, have gained wide interest due to the potential of improving imaging performance.

II. Piezoelectric Constitutive Relations

- Consider the following figure, in which the equilibrium spacings between neighboring rows of atoms are L , a_1 and a_2 , and q is the magnitude of the charge of the atoms,



the polarization (per unit volume) of the object is

$$P \equiv \frac{\text{dipole strength of unit cell}}{\text{volume of unit cell}} = \frac{q(a_2 - a_1)}{L^2(a_2 + a_1)}.$$

- In the presence of strain S , a_1 and a_2 change to $a_1 + \Delta a_1$ and $a_2 + \Delta a_2$, respectively, and the polarization changes to $P + \Delta P$. Since $\Delta a_1 = a_1 \cdot S$ and $\Delta a_2 = a_2 \cdot S$, under the conditions that Δa_1 and Δa_2 are small, we obtain

$$\Delta P \approx PS \equiv eS,$$

where e is defined as the *piezoelectric stress constant*. The total change in electric displacement (or electric flux density) in the presence of an electric field E is

$$D = \epsilon E + \Delta P = \epsilon E + eS,$$

where ϵ is the permittivity with zero or constant strain. We find that the electric displacement in a piezoelectric material is dependent on both the electric field and the strain.

- Similar to the above derivation, we can determine the stress in a piezoelectric medium due to an electric field E . Since the forces per unit area on the positive and negative atoms are $\pm qE/L^2$, the stresses in the regions of length a_1 and a_2 are therefore

$$\begin{aligned} T_1 &= -\frac{qE}{L^2} \\ T_2 &= \frac{qE}{L^2} \end{aligned} .$$

The average internal stress in the medium due to the electric field is

$$T_E = \frac{a1 T_1 + a2 T_2}{a1 + a2} = eE .$$

The total stress (T_{piezo}) applied to the object is the sum of the externally applied stress T (or $T_{nonpiezo}$) and the internal stress T_E (or T_{elect}). According to Hooke's law and defining c_E as the elastic constant (under a constant electric field condition), we have

$$\begin{aligned} T + T_E &= c_E S \\ T &= c_E S - eE \\ (\text{or } T_{piezo} &= T_{nonpiezo} + T_{elect}) . \end{aligned}$$

- Equations $D = \epsilon E + eS$ and $T = c_E S - eE$ are known as *piezoelectric constitutive relations*. The above derivation assumed one-dimensional situations, where a single scalar e is adequate to represent the coupling between the elastic and electric properties.

III. Wave Propagation in Piezoelectric Materials

- Equations governing the acoustic wave propagation in a piezoelectric material are obtained by using Newton's second law with the constitutive relations. Based on previous derivation of one-dimensional wave propagation in a non-piezoelectric material, we can replace the bulk modulus B by the elastic constant c_E and

$$\frac{\partial^2 w(z,t)}{\partial t^2} = (c_E / \rho) \frac{\partial T(z,t) / c_E}{\partial z} = (c_E / \rho) \left\{ \frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial (eE(z,t) / c_E)}{\partial z} \right\} .$$

Re-arranging the above equation, we obtain

$$\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\rho}{c_E} \frac{\partial^2 w(z,t)}{\partial t^2} = \frac{1}{c_E} \frac{\partial(eE(z,t))}{\partial z}.$$

- Imagine the piezoelectric material in the form of a plate with metal electrodes on each face. If the electrodes on opposite faces are short circuited, the electric field is reduced to zero and the above equation is reduced to a homogeneous wave equation. On the other hand, if the electrodes are open-circuited or if the medium is infinite long in z , there would be no free charges in the transducer medium (i.e., D is a constant in z , but may be a function of time). Under this condition, the above equation takes the following form by applying the other constitutive relation

$$\begin{aligned} \frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\rho}{B} \frac{\partial^2 w(z,t)}{\partial t^2} &= \frac{e}{\epsilon c_E} \frac{\partial D(z,t)}{\partial z} - \frac{e^2}{\epsilon c_E} \frac{\partial^2 w(z,t)}{\partial z^2} = -\frac{e^2}{\epsilon c_E} \frac{\partial^2 w(z,t)}{\partial z^2} \\ \frac{\partial^2 w(z,t)}{\partial z^2} - \left(\frac{\rho}{B(1 + e^2 / \epsilon c_E)} \right) \frac{\partial^2 w(z,t)}{\partial t^2} &= 0 \end{aligned}$$

- Note that $c_{velocity}^{nonpiezo} \equiv \sqrt{\frac{B}{\rho}}$ represents the propagation velocity in a non-piezoelectric medium. From the above equation, we have

$$c_{velocity}^{piezo} = c_{velocity}^{nonpiezo} \left(1 + \frac{e^2}{\epsilon c_E} \right)^{1/2}.$$

It is then obvious that acoustic waves propagate at a higher velocity when the material is being piezoelectrically stiffened. The elastic constant under a constant electric displacement condition (c_D) is related to that under a constant electric field condition (c_E) by

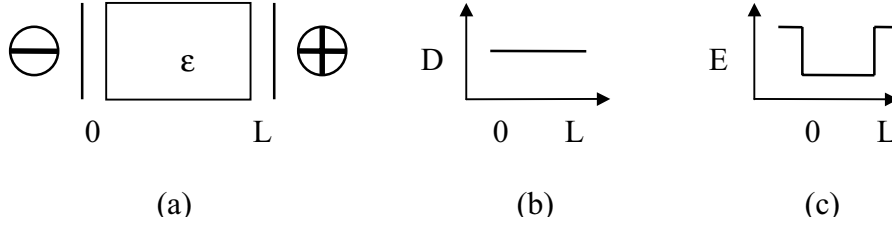
$$c_D = c_E \left(1 + \frac{e^2}{\epsilon c_E} \right).$$

The quantity k , where $k^2 \equiv \frac{e^2}{\epsilon c_E}$, is known as the *electromechanical coupling constant*.

The magnitude of the electromechanical coupling constant is a useful index of the strength of the piezoelectric effect in a particular material. Note that ϵ is the dielectric permittivity under a constant strain condition.

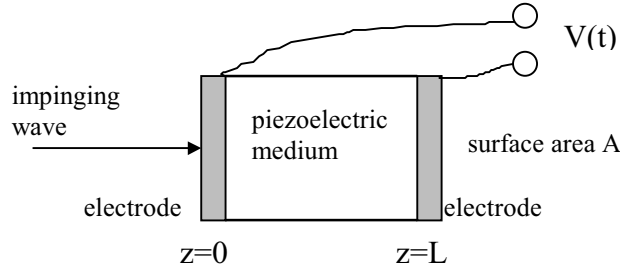
IV. Piezoelectric Generation and Detection of Ultrasound

- Ultrasonic waves are generated by the application of an external electric field to a piezoelectric material. A previously derived inhomogeneous wave equation indicates that a gradient in eE (the product of the piezoelectric stress constant and the electric field) is the source for the generation of mechanical disturbances. Usually the surfaces of a piezoelectric material offer the sharpest discontinuity in both e and E , hence they are the strongest sources of sound. This can be shown by the following simplified figure. Assuming a parallel plate capacitor containing a piezoelectric material of dielectric constant ϵ .



- Consider the application of a charge density to the capacitor shown in figure (a). Since there are no free charge between the plates, we have $\frac{\partial D}{\partial z} = 0$ and hence D is a constant as illustrated in the figure (b). Additionally, the permittivity is typically higher inside the piezoelectric material, thus making the electric field E smaller than E in the outside. Therefore, figure (c) can be used to represent E and it becomes apparent that the only gradient in the electrical field occurs at the surfaces of the material. Consequently, the surfaces of the piezoelectric material are the predominant sources for the generation of ultrasound.
- Piezoelectric detection of ultrasonic waves is reciprocal to the process of wave generation. In other words, the conversion of mechanical energy into electrical energy is also a phenomenon dominated by the behavior at the surfaces of the piezoelectric material. As shown below, the voltage measured across a piezoelectric plate is the integral of the electric field over the thickness of the crystal

$$V(t) = \int_0^L E(z, t) dz$$



Using the constitutive relation relating the electric field to the strain and the electric displacement, the voltage becomes

$$V(t) = -\int_0^L \frac{e}{\epsilon} S(z,t) dz + \int_0^L \frac{D(z,t)}{\epsilon} dz$$

From Gauss's law, we have $D(z,t) = \frac{q(t)}{A}$ where $q(t)$ is the total charge on area A . Therefore, the above equation reduces to

$$\begin{aligned} V(t) &= -\frac{e}{\epsilon} \int_0^L \frac{\partial w(z,t)}{\partial z} dz + \frac{q(t)}{\epsilon A / L} \\ &\equiv -\frac{e}{\epsilon} (w(L,t) - w(0,t)) + \frac{q(t)}{C_0} \end{aligned}$$

where $C_0 \equiv \frac{\epsilon A}{L}$ is the capacitance of the piezoelectric plate. The magnitude of the net displacement can be calculated from the boundary conditions placed at the surfaces of the material. In addition, the above equation shows that under open circuit situations, i.e., $q(t)$ is not time-varying, the voltage developed across the plate is directly related to the relative displacements of the front and back surfaces of the material. If the thickness of the plate corresponds to an odd integral number of half-wavelengths of the ultrasonic wave impinging on the material (i.e., the two surfaces oscillate 180° out of phase), then the relative displacement of the front and back surfaces, $w(L,t) - w(0,t)$, is the largest. In contrast, if the thickness of the crystal is an even number of half-wavelengths, then the amplitude of oscillation of the two surfaces is in phase, and therefore, $w(L,t) - w(0,t) = 0$.

V. Equivalent Circuits

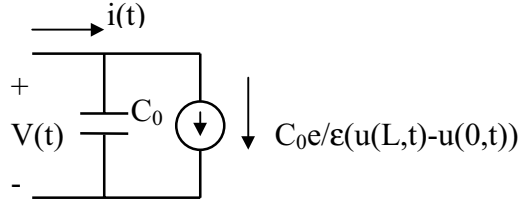
- Based on the piezoelectric detection equation that we previously derived, we have

$$V(t) = -\frac{e}{\epsilon} (w(L, t) - w(0, t)) + \frac{q(t)}{C_0}.$$

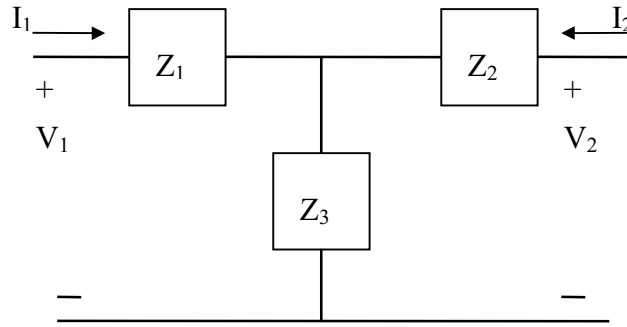
Taking the partial derivative with respect to time on both sides, we obtain

$$i(t) = C_0 \frac{\partial V(t)}{\partial t} + C_0 \frac{e}{\epsilon} (u(L, t) - u(0, t))$$

which can be represented by the following equivalent circuit.



- Defining force as we do voltage in electrical circuits, and particle velocity as we do current in electrical circuits, the non-piezoelectric component can be described using the other constitutive relation (i.e., $T(z, t)_{piezo} = T(z, t)_{nonpiezo} + \frac{e \cdot q(t)}{\epsilon \cdot A}$) and represented by the following circuit (a transmission line)



where $Z_1 = Z_2$ due to symmetry. Let Z_0 represent the characteristic impedance of the piezoelectric material, we can obtain the following equations using methods similar to those used by deriving the acoustic wave equations

$$V(z, \omega) = V_1(\omega) e^{-j\omega z / c} + V_2(\omega) e^{j\omega z / c}$$

$$I(z, \omega) = \frac{1}{Z_0} (V_1(\omega) e^{-j\omega z / c} - V_2(\omega) e^{j\omega z / c})$$

- In order to obtain Z_1 and Z_3 , the following relations need to be used.

$$Z_3 = \frac{V_2}{I_1} \Big|_{I_2=0} = \frac{V(L, \omega)}{I(0, \omega)} \Big|_{I(L, \omega)=0}$$

$$Z_1 + Z_3 = \frac{V_1}{I_1} \Big|_{I_2=0}$$

Letting $I_2 = 0$, the following equation yields

$$Z_0 I(L, \omega) = 0 = V_1(\omega) e^{-j\omega L/c} - V_2(\omega) e^{j\omega L/c}.$$

Therefore

$$V_2(\omega) = V_1(\omega) e^{-2j\omega L/c}.$$

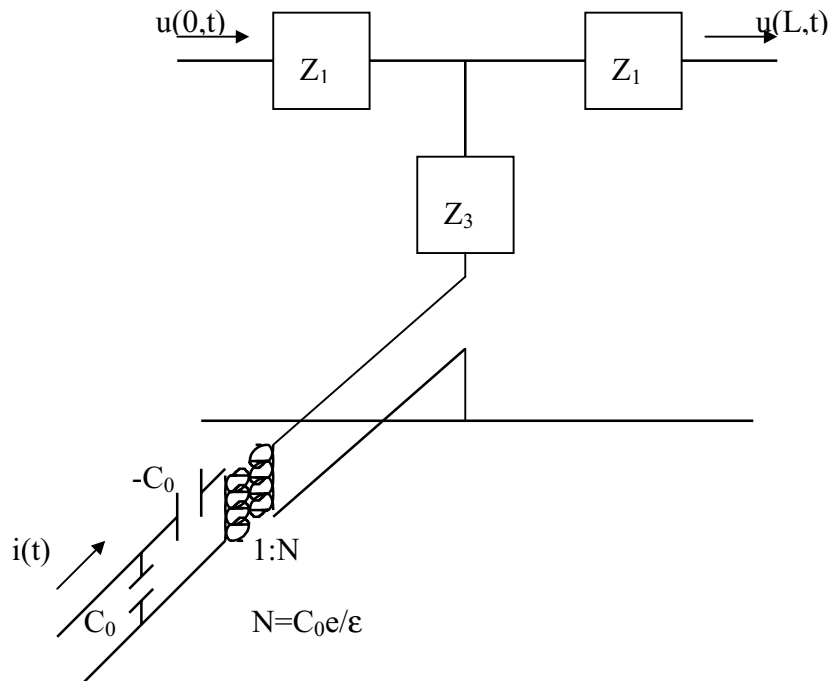
and

$$Z_3 = Z_0 \frac{V_1(\omega)(e^{-j\omega L/c} + e^{-j\omega L/c})}{V_1(\omega)(1 - e^{-2j\omega L/c})} = \frac{Z_0}{j \sin 2\pi L/\lambda} = -jZ_0 \cot 2\pi L/\lambda$$

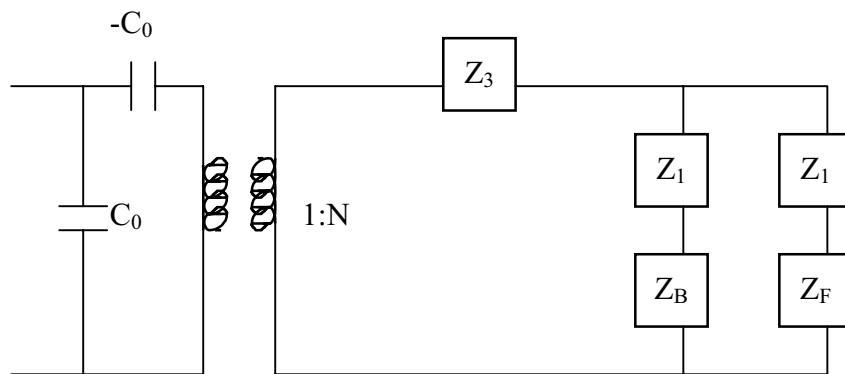
Furthermore, $Z_1 (=Z_2)$ can be obtained as the following:

$$Z_1 = \frac{V(0, \omega)}{I(0, \omega)} \Big|_{I_2=0} - Z_3 = Z_0 \frac{\cos 2\pi L/\lambda - 1}{j \sin 2\pi L/\lambda} = jZ_0 \tan \pi L/\lambda$$

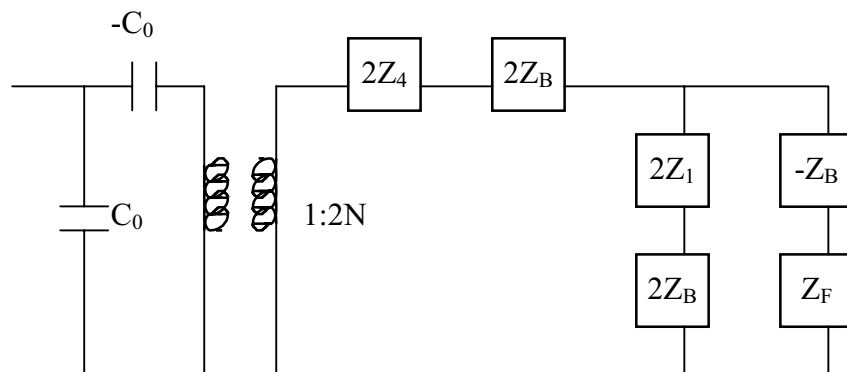
- By using a transformer to couple the electrical components with the acoustic components, we then obtain the following equivalent circuit (a.k.a. *Mason Equivalent Circuit*)



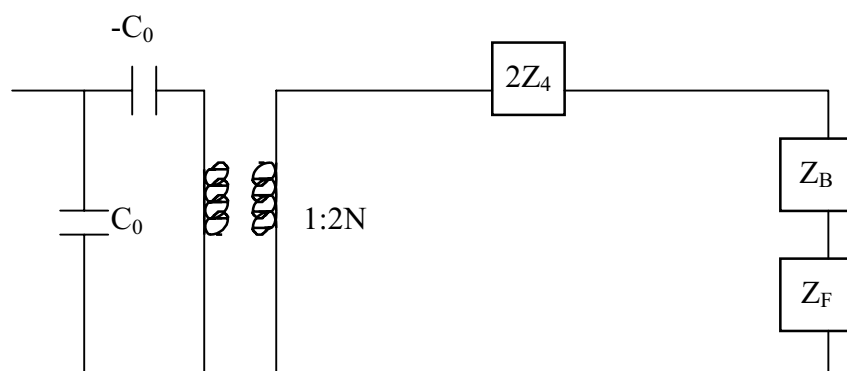
- With the front and back faces of the transducer being loaded by mechanical impedances Z_F and Z_B , the equivalent circuit can be re-drawn as the following:



- The above circuit can be further transformed to the following circuit identity with $Z_4 = 1/Z_1$.



- In the neighborhood of a mechanical resonance, impedance of the branch containing $2Z_1$ is large and the branch can be neglected. Therefore, the model can be approximated with the following circuit. Such an approximation is adequate to describe the operation of a transducer near resonance driving a wide variety of mechanical loads.

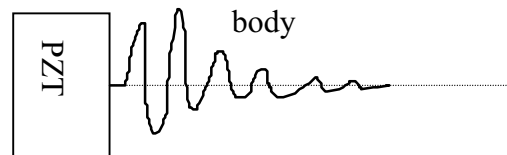


- An alternative equivalent circuit (*KLM Equivalent Circuit*) has been developed to be better suited for broadband operations. Please refer to the following paper if interested.

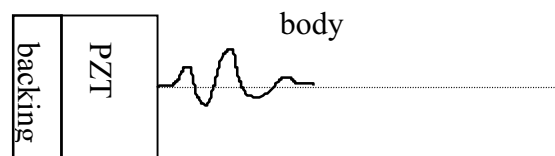
- D. Leedom, R. Krimholtz and G. Matthael, “Equivalent circuits for transducers having even or odd symmetry piezoelectric excitation”, *IEEE Trans. on Sonics and Ultrasonics*, vol. SU-18, No. 3, pp. 128-141, July 1971.

VI. Design Considerations for Broadband Transducers

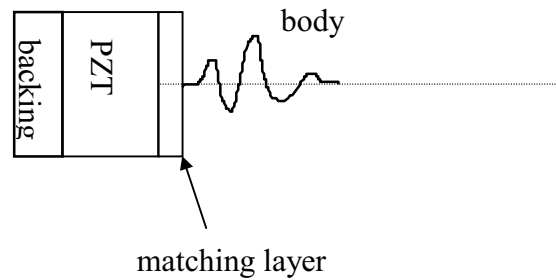
- Broadband transducers are necessary for pulse-echo imaging applications in order to achieve high range resolution, which is inversely proportional to the pulse bandwidth. However, a short pulse is usually achieved by sacrificing sensitivity. Considering the following piezoelectric transducer with no matching or damping layers, it rings and produces an unacceptable long pulse.



By placing a lossy material (highly attenuating) which has a similar acoustic impedance as the PZT, the reflection at the back of the transducer can be reduced and therefore the pulse can be shortened. Apparently, sensitivity is degraded due to attenuation.



The acoustic impedance of a typical PZT material is around twenty times higher than that in the body, therefore, part of the sensitivity loss can be recovered by adding one or multiple quarter wave matching layers at the front surface of the transducer.



- *Two-way insertion loss*, defined as the ratio of the available electrical power generated by the device as a receiver to the electrical power dissipated in the device as a transmitter under the conditions in which the acoustic wave produced is reflected from a perfectly reflecting interface and received by the same transducer, is often used as a measure of the electromechanical efficiency of the transducer. The lower the insertion loss is, the higher the sensitivity can be achieved.
- Equivalent circuits can be used to match both the mechanical impedance and the electrical impedance. Ideally, real part of the electrical impedance should be 50 Ohms in order to match a typical transmitter output impedance. Imaginary part of the impedance, on the other hand, should be to zero in order to obtain optimal efficiency. This is typically done by placing an inductor (as a tuning element) to cancel the capacitance of the transducer (and sometimes the transducer cable). However, the improvement in sensitivity is often gained at the price of bandwidth of the spectrum.
- An acoustic lens is often placed on the front of the transducer in order to provide a fixed geometric focusing. This is particularly important for imaging using one-dimensional arrays, in which case the geometric focusing is provided along the non-scan direction.
- A transducer array consists of many piezoelectric elements. The elements are arranged depending on specific scan formats. A typical diagram of a one-dimensional transducer array is shown in the following.

