電工學期中考參考解答

12/06/2006

1. (a)

We can write the following expression for the resistance seen by the 2-V source.

$$R_{eq} = 1 + \frac{1}{1/R_{eq} + 1/2}$$

The solutions to this equation are $R_{eq} = 2\Omega$ and $R_{eq} = -1\Omega$. However, we reason that the resistance must be positive and discard the negative root. Then, we have

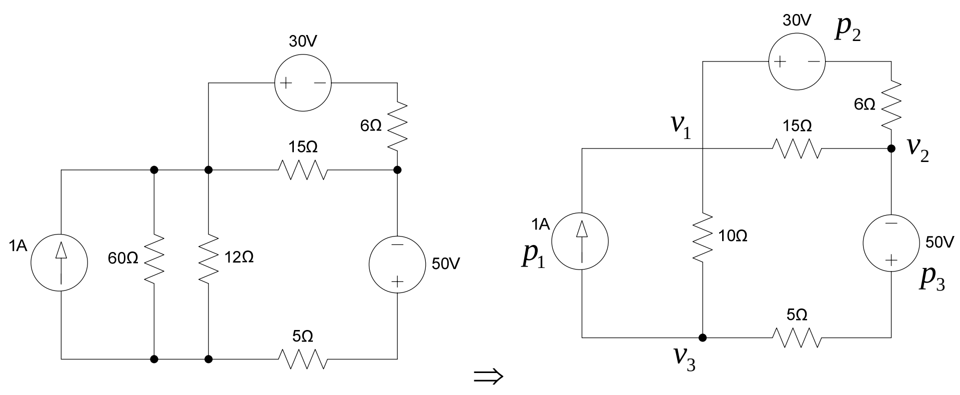
$$i_1 = \frac{2V}{R_{eq}} = 1A, \quad i_2 = i_1 \frac{R_{eq}}{2 + R_{eq}} = \frac{i_1}{2} = 0.5A, \quad i_3 = \frac{i_1}{2} = 0.5A, \quad \text{and} \quad i_6 = 0.125A.$$

(b)

$$P = IV = I^2 R, \quad P_3 = i_3{}^2 R_3 = 0.25W, \quad P_6 = i_6{}^2 R_6 = 1/32W = 0.03125W.$$

2. (a)

Node voltage analysis:



$$\begin{cases} -1 + \dfrac{v_1 - v_3}{10} + \dfrac{v_1 - 30 - v_2}{6} + \dfrac{v_1 - v_2}{15} = 0 \\ \dfrac{v_2 - v_1 + 30}{6} + \dfrac{v_2 - v_1}{15} + \dfrac{v_2 + 50 - v_3}{5} = 0 \end{cases}$$

Solving, we obtain $v_1 - v_2 = 30V$ and $v_2 - v_3 = -40V$

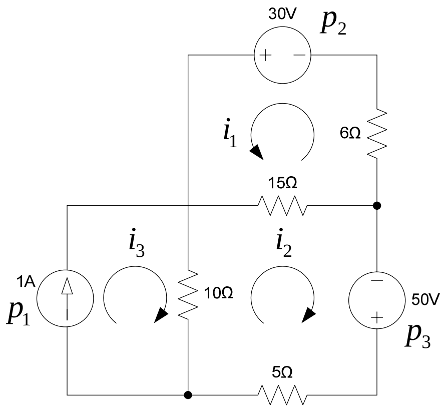
So the power supplied by each independent source is

$$p_1 = 1A \cdot (v_3 - v_1) = 10W, \quad p_2 = \left(\frac{v_2 - v_1 + 30}{6}\right) \cdot 30V = 0, \quad \text{and}$$

$$p_3 = \left(\frac{v_2 + 50 - v_3}{5}\right) \cdot 50V = 100W$$

(b)

Mesh current analysis:



$$\begin{cases} i_3 = 1 \\ 15(i_1 + i_2) + 6i_1 = 30 \\ 5i_2 + 10(i_2 - i_3) + 15(i_2 + i_1) = 50 \end{cases}$$

Solving, we obtain $i_1 = 0$ and $i_2 = 2A$

So the power supplied by each independent source is

$p_1 = 1A \cdot (10i_3) = 10W$, $p_2 = i_1 \cdot 30V = 0$, and $p_3 = i_2 \cdot 50V = 100W$

3. Applying KVL to the circuit, we obtain $L\frac{di(t)}{dt} + Ri(t) + v_C(t) = v_s = 50.$

For the capacitance, we have $i(t) = C\frac{dv_C(t)}{dt}.$

So $\frac{d^2 v_C(t)}{dt^2} + \frac{R}{L}\frac{dv_C(t)}{dt} + \frac{1}{LC}v_C(t) = \frac{50}{LC}.$ (1)

In dc steady state, since the capacitance acts as an open circuit, the steady-state voltage across it is 50 V. That is, $v_{Cp}(t) = 50.$

Comparing Equation (1) with Equation 4.67 in the text, we find

$\alpha = \frac{R}{2L} = 10^4, \quad \omega_0 = \frac{1}{\sqrt{LC}} = 10^4, \quad \zeta = \frac{\alpha}{\omega_0} = 1.$

Since we have $\zeta = 1$, this is the critically damped case. The roots are real and equal, and the complementary solution is

$v_{Cc}(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t), \quad \text{where} \quad s_1 = -\alpha.$

The complete solution is

$v_C(t) = 50 + K_1 \exp(s_1 t) + K_2 t \exp(s_1 t).$

The initial conditions are

$v_C(0+) = 0 \quad \text{and} \quad i(0+) = 0 = C\left.\frac{dv_C(t)}{dt}\right|_{t=0}.$

Thus, we have

$v_C(0+) = 50 + K_1 = 0 \quad \text{and} \quad \left.\frac{dv_C(t)}{dt}\right|_{t=0} = 0 = s_1 K_1 + K_2.$

Solving them, we find $K_1 = -50$ and $K_2 = -s_1 K_1 = \alpha K_1 = -5 \times 10^5.$

Finally, the solution is

$v_C(t) = 50 - 50\exp(s_1 t) - 5 \times 10^5 t \exp(s_1 t).$

4. Writing KVL equations around the meshes, we obtain

$$\begin{cases} 5I_1 + j15(I_1 - I_2) = 20 \\ -j10I_2 + j15(I_2 - I_1) = 10 \end{cases}$$

Solving, we obtain

$I_1 = 1.644\angle 80.54°$, $I_2 = 2.977\angle 74.20°$.

$V_1 = 20 - 5I_1 = 20.335\angle -23.50°$

$v(t) = 20.335\cos(\omega t - 23.50°)V$

$$v_{rms} = \frac{20.335}{\sqrt{2}} = 14.379V$$

5. Using the voltage-division principle, we find $\frac{v_{out}}{v_{in}} = \frac{R}{\frac{1}{j\omega C} + R}$.

The input is $v_{in}(t) = 5\cos(200\pi t) + 5\cos(2000\pi t)$,

for $\omega = 200\pi$, $\frac{v_{out}}{v_{in}} = \frac{\sqrt{101}}{101}\angle 84.29°$, and for $\omega = 2000\pi$, $\frac{v_{out}}{v_{in}} = \frac{\sqrt{2}}{2}\angle 45°$.

So the output in steady state conditions will be

$v_{out}(t) = 0.4975\cos(200\pi t + 84.29°) + 3.5355\cos(2000\pi t + 45°)$.

6. (a)

The all-pass network has the unique characteristic of unity gain at all frequencies and a frequency-dependent phase relationship between output and input.

(b)

$v_{in}(t) = \cos(\omega_1 t) + 2\cos(2\omega_1 t)$

$v_{out}(t) = v_{in}(t - \tau_0) = \cos(\omega_1 t - \omega_1 \tau_0) + 2\cos(2\omega_1 t - 2\omega_1 \tau_0)$

For the transfer function $H(f) = \frac{V_{out}}{V_{in}}$,

$\left|H(\frac{\omega_1}{2\pi})\right| = \left|H(\frac{2\omega_1}{2\pi})\right| = 1$, that means $|H(f)| = 1$ for any given frequency.

$\angle H(\frac{\omega_1}{2\pi}) = -\omega_1 \tau_0$, $\angle H(\frac{2\omega_1}{2\pi}) = -2\omega_1 \tau_0$, that means $\angle H(f) = -2\pi f \tau_0$.

That is, the transfer function of the all-pass network must have a unity gain for all frequencies and a linear frequency-dependent phase. By that way, we can thus have a relationship as $v_{out}(t) = v_{in}(t - \tau_0)$ between output and input.