CHAPTER 6

P6.7* The given input signal is

$$v_{in}(t) = 5 + 2\cos(2\pi 2500t + 30^\circ) + 2\cos(2\pi 7500t)$$

This signal has a component $v_{in1}(t) = 5$ with $f = 0$, a second component $v_{in2}(t) = 2\cos(2\pi 2500t + 30^\circ)$ with $f = 2500$, and a third component $v_{in3}(t) = 2\cos(2\pi 7500t)$ with $f = 7500$. From Figure P6.6, we find the transfer function values for these frequencies:

$$H(0) = 1, \quad H(2500) = 1.25\angle -22.5^\circ, \text{ and } H(7500) = 1.75\angle -67.5^\circ$$

The dc output is $v_{out1} = H(0)v_{in1} = 5$

The phasors for the sinusoidal input components are

$$\mathbf{V}_{in2} = 2\angle 30^\circ \quad \text{and} \quad \mathbf{V}_{in3} = 2\angle 0^\circ$$

Multiplying the input phasors by the transfer function values , results in:

$$\mathbf{V}_{out2} = \mathbf{V}_{in2} \times H(2500) = 2.5\angle 7.5^\circ \qquad \mathbf{V}_{out3} = \mathbf{V}_{in3} \times H(7500) = 3.5\angle -67.5^\circ$$

The corresponding output components are:

$$v_{out2}(t) = 2.5\cos(2\pi 2500t + 7.5^\circ)$$

$$v_{out3}(t) = 3.5\cos(2\pi 7500t - 67.5^\circ)$$

Thus, the output signal is

$$v_{out}(t) = 5 + 2.5\cos(2\pi 2500t + 7.5^\circ) + 3.5\cos(2\pi 7500t - 67.5^\circ)$$

P6.9* The phasors for the input and output are:

$$\mathbf{V}_{in} = 2\angle -25^\circ \quad \text{and} \quad \mathbf{V}_{out} = 1\angle 20^\circ$$

The transfer function for $f = 5000$ Hz is

$$H(5000) = \mathbf{V}_{out}/\mathbf{V}_{in} = 0.5\angle 45^\circ$$

P6.14* Given

$$v_{in}(t) = V_{max}\cos(2\pi ft)$$

$$v_{out}(t) = \int_0^t V_{max}\cos(2\pi ft)dt = \frac{V_{max}}{2\pi f}\sin(2\pi ft)$$

The phasors are

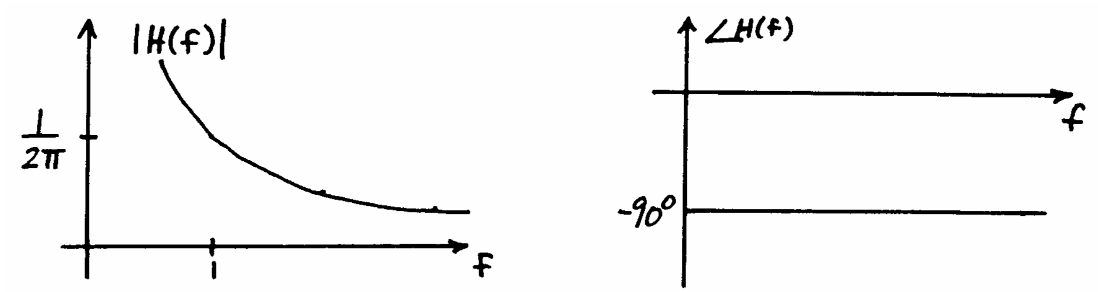
$$\mathbf{V}_{in} = V_{max}\angle 0^\circ$$

$$\mathbf{V}_{out} = \frac{V_{max}}{2\pi f}\angle -90^\circ$$

The transfer function is

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1}{2\pi f}\angle -90^\circ = \frac{-j}{2\pi f}$$

Plots of the magnitude and phase of this transfer function are:



P6.19* The half-power frequency of the filter is

$$f_B = \frac{1}{2\pi RC} = 500 \text{ Hz}$$

The transfer function is given by Equation 6.9 in the text:

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

The given input signal is

$$v_{in}(t) = 5\cos(500\pi t) + 5\cos(1000\pi t) + 5\cos(2000\pi t)$$

which has components with frequencies of 250, 500, and 1000 Hz.

Evaluating the transfer function for these frequencies yields:

$$H(250) = \frac{1}{1 + j(250/500)} = 0.8944\angle -26.57^\circ$$

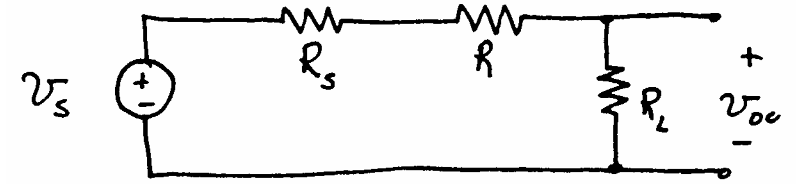
$$H(500) = 0.7071\angle -45^\circ$$

$$H(1000) = 0.4472\angle -63.43^\circ$$

Applying the appropriate value of the transfer function to each component of the input signal yields the output:

$$v_{out}(t) = 4.472\cos(500\pi t - 26.57^\circ) + 3.535\cos(1000\pi t - 45^\circ)$$
$$+ 2.236\cos(2000\pi t - 63.43^\circ)$$

P6.26 (a) First, we find the Thévenin equivalent for the source and resistances.



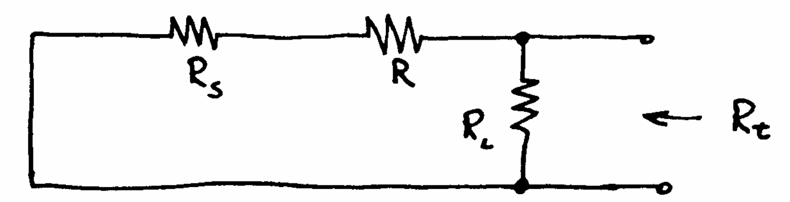
The open-circuit voltage is given by

$$v_t(t) = v_{oc}(t) = v_s(t)\frac{R_L}{R_s + R + R_L}$$

In terms of phasors, this becomes:

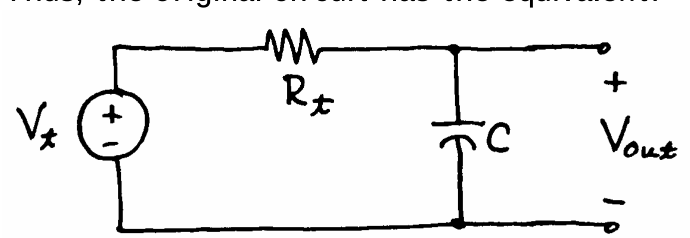
$$\mathbf{V}_t = \mathbf{V}_s \frac{R_L}{R_s + R + R_L} \qquad (1)$$

Zeroing the source, we find the Thévenin resistance:



$$R_t = \frac{1}{1/R_L + 1/(R + R_s)}$$

Thus, the original circuit has the equivalent:



The transfer function for this circuit is:

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)} \qquad (2)$$

where, $f_B = \frac{1}{2\pi R_t C}$

Using Equation (1) to substitute for $\mathbf{V}_t$ in Equation (2) and rearranging, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} = \frac{R_L}{R_s + R + R_L} \times \frac{1}{1 + j(f/f_B)} \qquad (3)$$

(b) Evaluating for the circuit components given, we have:

$$R_t = 1.5\ \text{k}\Omega$$

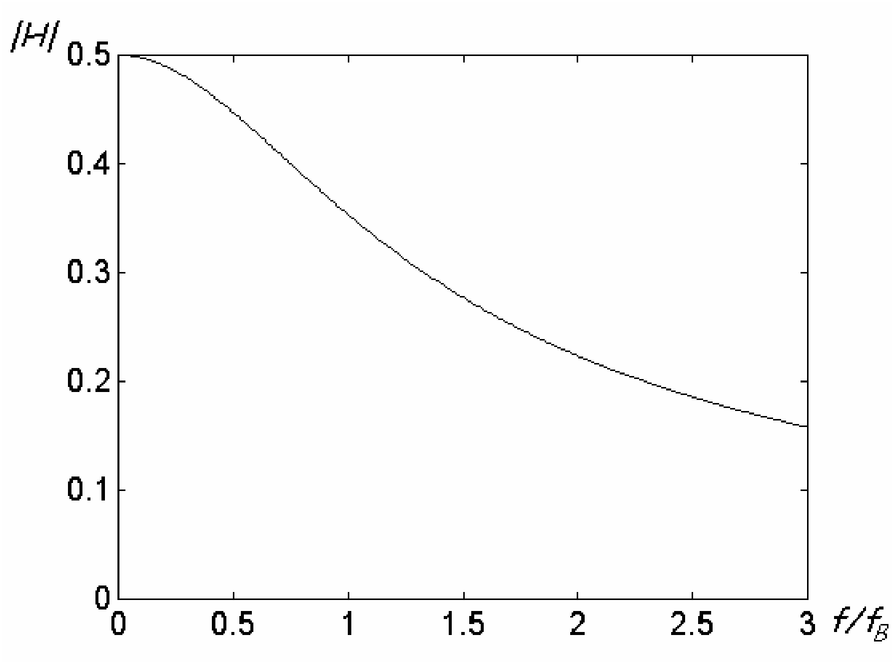
$$f_B = 106.1\ \text{Hz}$$

$$H(f) = \frac{0.5}{1 + j(f/f_B)}$$

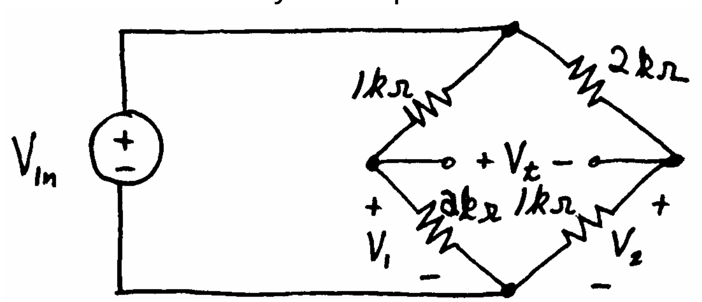
A MATLAB program to plot the transfer-function magnitude is:

```
foverfb=0:0.01:3;
Hmag=abs(0.5./(1 + i*foverfb));
plot(foverfb,Hmag)
axis([0 3 0 0.5])
```

The resulting plot is:



P6.28* The circuit seen by the capacitance is:



The open-circuit or Thévenin voltage is

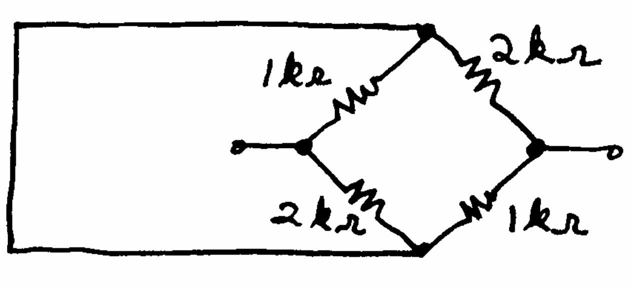
$$\mathbf{V}_t = \mathbf{V}_1 - \mathbf{V}_2$$

$$= \mathbf{V}_{in}\frac{2000}{2000+1000} - \mathbf{V}_{in}\frac{1000}{1000+2000}$$

Thus, we obtain:

$$\mathbf{V}_t = \frac{1}{3}\mathbf{V}_{in} \qquad (1)$$

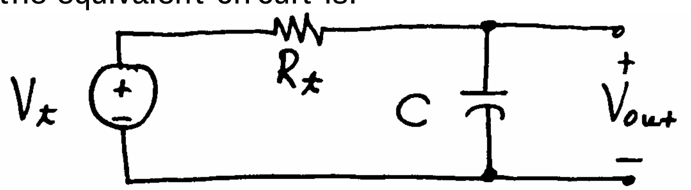
Zeroing the source, we have



The Thévenin resistance is

$$R_t = \frac{1}{1/1000 + 1/2000} + \frac{1}{1/2000 + 1/1000}$$
$$= 1333\ \Omega$$

Thus, the equivalent circuit is:



As in the text, this circuit has the transfer function:

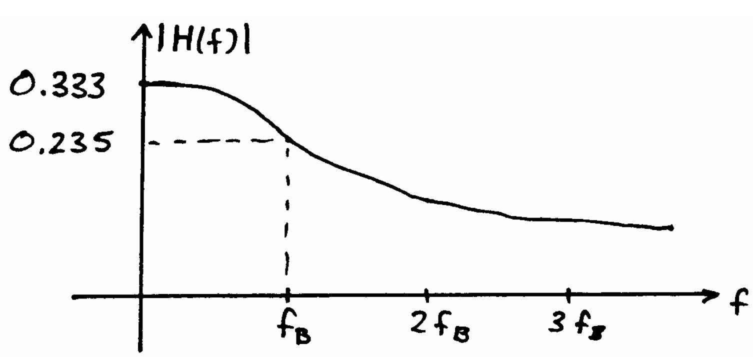
$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)} \qquad (2)$$

where $f_B = \dfrac{1}{2\pi R_t C} = \dfrac{1}{2\pi 1333 \times 10^{-5}} = 11.94\ \text{Hz}$

Using Equation (1) to substitute for $\mathbf{V}_t$ in Equation (2) and rearranging, we have

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1/3}{1 + j(f/f_B)}$$

A sketch of the transfer-function magnitude is:



P6.39* (a) The overall transfer function is the product of the transfer functions of the filters in cascade:

$$H(f) = H_1(f) \times H_2(f) = \frac{1}{[1 + j(f/f_B)]^2}$$

(b)

$$|H(f)| = \frac{1}{1 + (f/f_B)^2}$$

$$|H(f_{3dB})| = \frac{1}{\sqrt{2}} = \frac{1}{1 + (f_{3dB}/f_B)^2}$$

$$(f_{3dB}/f_B)^2 = \sqrt{2} - 1$$

$$f_{3dB} = f_B\sqrt{\sqrt{2} - 1} = 0.6436 f_B$$

P6.45*

$$H(f) = \frac{100}{1 + j(f/1000)}$$

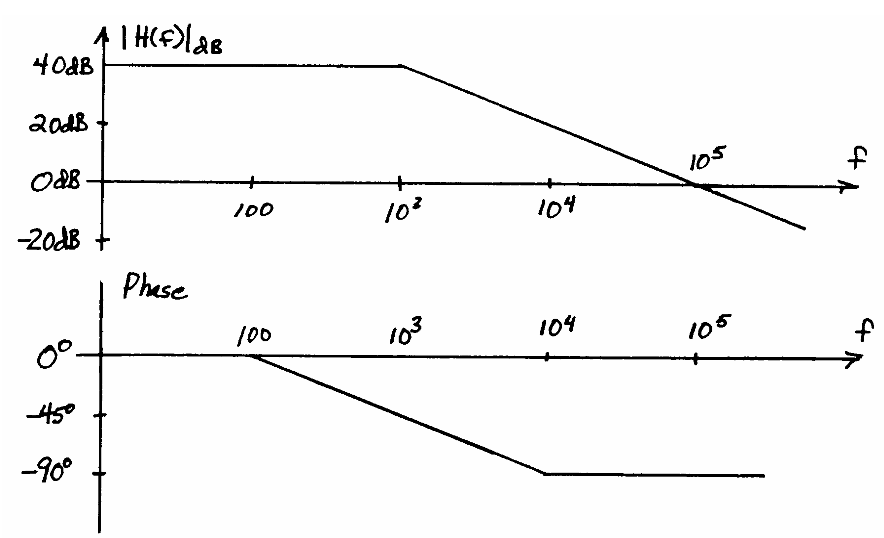
$$|H(f)| = \frac{100}{\sqrt{1 + (f/1000)^2}}$$

$$|H(f)|_{dB} = 20\log(100) - 20\log\sqrt{1 + (f/1000)^2}$$

$$= 40 - 20\log\sqrt{1 + (f/1000)^2}$$

This is similar to the transfer function treated in Section 6.4 in the text except for the additional 40 dB constant. The half-power frequency is $f_B = 1000$.

The asymptotic Bode plots are:



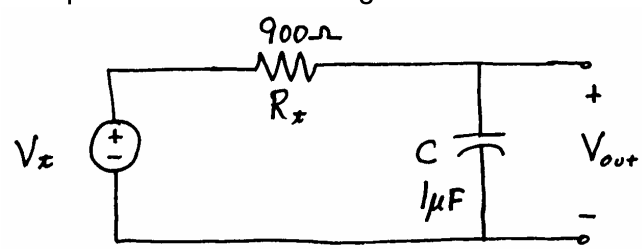
P6.52 First, we find the Thévenin equivalent for the source and the resistances. The Thévenin resistance is

$$R_t = \frac{1}{1/R_1 + 1/R_2} = 900\ \Omega$$

and the Thévenin voltage is

$$\mathbf{V}_t = \frac{R_2}{R_1 + R_2}\mathbf{V}_{in} = 0.1\mathbf{V}_{in}$$

Thus, an equivalent for the original circuit is:



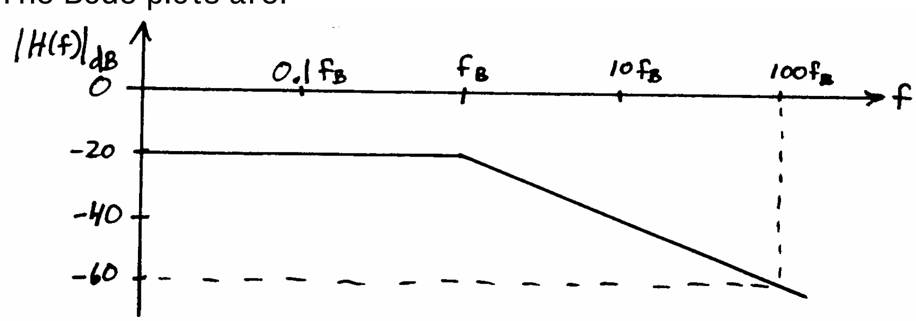
This is a lowpass filter having a transfer function given by Equation 6.8 (with changes in notation):

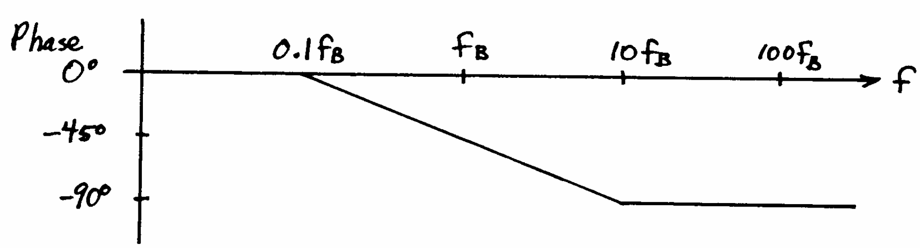
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)}$$

where $f_B = 1/(2\pi R_t C) = 176.8\ \text{Hz}$.

Using the fact that $\mathbf{V}_t = 0.1\mathbf{V}_{in}$, we have $H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{0.1}{1 + j(f/f_B)}$

The Bode plots are:





P6.57* Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}}$$

$$= \frac{R_2}{R_1 + R_2 + 1/j2\pi fC}$$

$$= \frac{R_2/(R_1 + R_2)}{1 + 1/j2\pi fC(R_1 + R_2)}$$

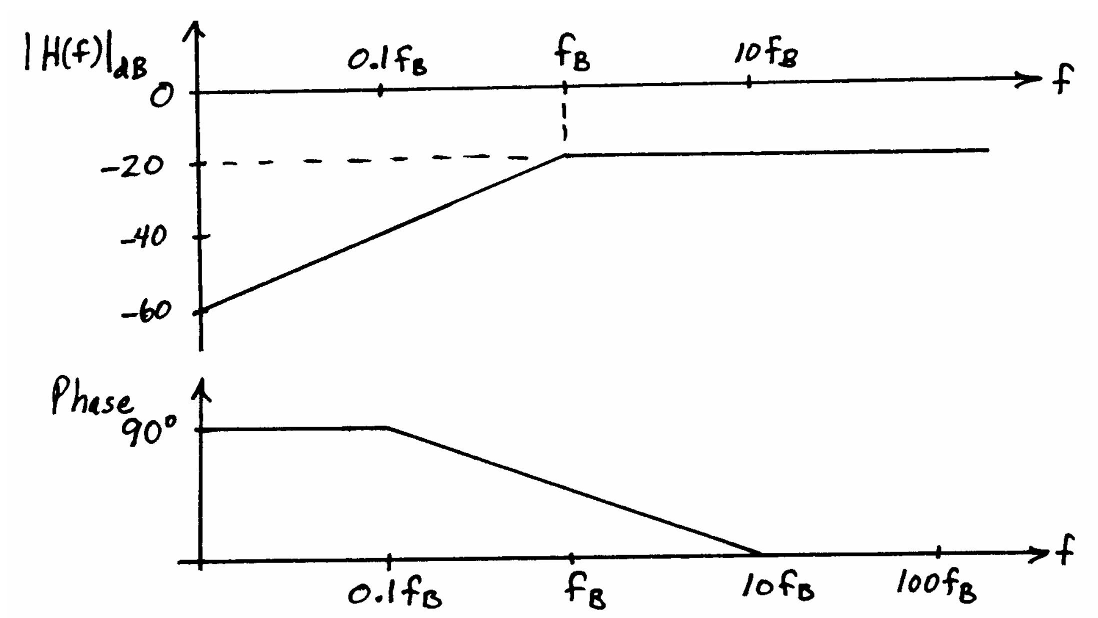
$$= \frac{R_2}{R_1 + R_2} \frac{j2\pi fC(R_1 + R_2)}{1 + j2\pi fC(R_1 + R_2)}$$

$$= \frac{R_2}{R_1 + R_2} \frac{j(f/f_B)}{1 + j(f/f_B)}$$

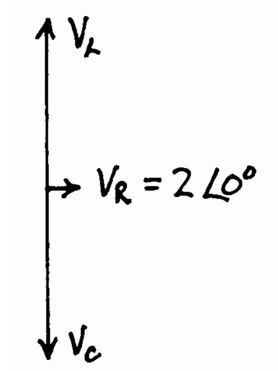
$$= 0.1 \frac{j(f/f_B)}{1 + j(f/f_B)}$$

where $f_B = 1/2\pi C(R_1 + R_2) = 15.92\text{ Hz}$.

The asymptotic Bode plots are:



P6.65* Assuming zero phase for $\mathbf{V}_R$, the phasor diagram at the resonant frequency is



Thus, $\mathbf{V}_L = 20\angle 90^\circ$ and $\mathbf{V}_C = 20\angle -90^\circ$ and

$$\mathbf{I} = \frac{\mathbf{V}_R}{R} = 40\angle 0^\circ \text{ mA}$$

$$2\pi f_0 L\angle 90^\circ = \frac{\mathbf{V}_L}{\mathbf{I}} = 500\angle 90^\circ$$

$$L = 79.57\ \mu\text{H}$$

$$\frac{1}{2\pi f_0 C}\angle -90^\circ = \frac{\mathbf{V}_C}{\mathbf{I}} = 500\angle -90^\circ$$

$$C = 318.3\text{ pF}$$

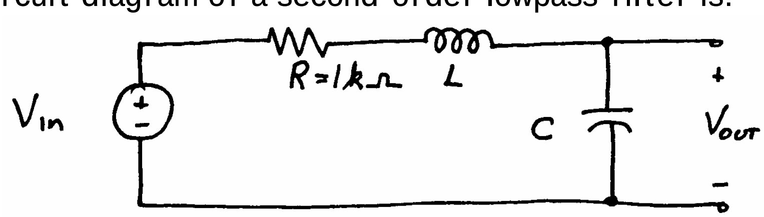
P6.69*

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.592\text{ MHz}$$

$$Q_p = \frac{R}{2\pi f_0 L} = 10.00$$

$$B = \frac{f_0}{Q_p} = 159.2\text{ kHz}$$

P6.77 The circuit diagram of a second-order lowpass filter is:



$$L = \frac{RQ_s}{2\pi f_0} = 1.592\text{ mH} \qquad C = \frac{1}{Q_s R(2\pi f_0)} = 1592\text{ pF}$$