CHAPTER 5

P5.11*

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

$$= \sqrt{\int_0^{0.5} [5\sin(2\pi t)]^2 dt}$$

$$\sqrt{\int_0^{0.5} [12.5 + 12.5\sin(4\pi t)]dt}$$

$$= \sqrt{\left(12.5t - \frac{12.5}{4\pi}\cos(4\pi t)\right)_{t=0}^{t=0.5}}$$

$$= \sqrt{6.25}$$

$$= 2.5\,\text{V}$$

P5.13

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

$$= \sqrt{\int_0^{1} [3\exp(-t)]^2 dt}$$

$$\sqrt{\int_0^{1} [9\exp(-2t)]dt}$$

$$= \sqrt{[-4.5\exp(-2t)]_{t=0}^{t=1}}$$

$$= \sqrt{4.5[1-\exp(-2)]}$$

$$= 1.973\,\text{V}$$

P5.20

$$V_m = 3\ \text{V} \qquad T = 0.5\ \text{s}$$

$$f = \frac{1}{T} = 2\ \text{Hz} \qquad \omega = 2\pi f = 4\pi\ \text{rad/s}$$

$$\theta = -360^\circ \frac{t_{\max}}{T} = -45^\circ$$

$$v(t) = 3\cos(4\pi t - 45^\circ)\ \text{V}$$

$$\mathbf{V} = 3\angle -45^\circ\ \text{V}$$

$$V_{rms} = \frac{3}{\sqrt{2}} = 2.121\ \text{V}$$

P5.23* We are given the expression

$$5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t)$$

Converting to phasors we obtain

$$5\angle 75^\circ - 3\angle -75^\circ + 4\angle -90^\circ =$$
$$1.2941 + j4.8296 - (0.7765 - j2.8978) - j4 =$$
$$0.5176 + j3.7274 = 3.763\angle 82.09^\circ$$

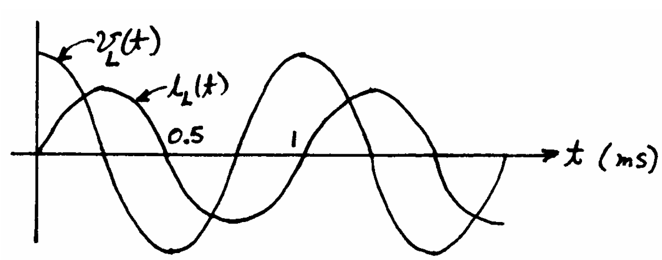
Thus, we have

$$5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t) =$$
$$3.763\cos(\omega t + 82.09^\circ)$$

P5.28*

$$v_L(t) = 10\cos(2000\pi t)$$
$$\omega = 2000\pi$$
$$Z_L = j\omega L = j200\pi = 200\pi\angle 90^\circ$$
$$\mathbf{V}_L = 10\angle 0^\circ$$
$$\mathbf{I}_L = \mathbf{V}_L / Z_L = (1/20\pi)\angle -90^\circ$$
$$i_L(t) = (1/20\pi)\cos(2000\pi t - 90^\circ) = (1/20\pi)\sin(2000\pi t)$$

$i_L(t)$ lags $v_L(t)$ by 90°



P5.41*

$$\mathbf{I}_s = 10\angle 0^\circ \text{ mA}$$
$$\mathbf{V} = \mathbf{I}_s \frac{1}{1/R + 1/j\omega L + j\omega C}$$
$$= 10^{-2} \frac{1}{1/1000 - j0.005 + j0.005}$$
$$= 10\angle 0^\circ \text{ V}$$
$$\mathbf{I}_R = \mathbf{V}/R = 10\angle 0^\circ \text{ mA}$$
$$\mathbf{I}_L = \mathbf{V}/j\omega L = 50\angle -90^\circ \text{ mA}$$
$$\mathbf{I}_C = \mathbf{V}(j\omega C) = 50\angle 90^\circ \text{ mA}$$

The peak value of $i_L(t)$ is five times larger than the source current! This is possible because current in the capacitance balances the current in the inductance (i.e., $\mathbf{I}_L + \mathbf{I}_C = 0$).

P5.45* First we write the KVL equation:

$$\mathbf{V}_1 - \mathbf{V}_2 = 10\angle 0^\circ$$

Then we enclose nodes 1 and 2 in a closed surface to form a supernode and write a KCL equation:

$$\frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1}{j20} + \frac{\mathbf{V}_2}{15} + \frac{\mathbf{V}_2}{-j5} = 0$$

The solution to these equations is:

$$\mathbf{V}_1 = 9.402\angle 29.58^\circ$$

$$\mathbf{V}_2 = 4.986\angle 111.45^\circ$$

P5.46* Writing KVL equations around the meshes, we obtain

$$5\mathbf{I}_1 + j15(\mathbf{I}_1 - \mathbf{I}_2) = 20$$

$$-j10\mathbf{I}_2 + j15(\mathbf{I}_2 - \mathbf{I}_1) = 10$$

Solving, we obtain:

$$\mathbf{I}_1 = 1.644\angle 80.54^\circ$$

$$\mathbf{I}_2 = 2.977\angle 74.20^\circ$$

P5.53* This is a capacitive load because the reactance is negative.

$$P = I_{rms}^2 R = (15)^2 100 = 22.5 \text{ kW}$$

$$Q = I_{rms}^2 X = (15)^2(-50) = -11.25 \text{ kVAR}$$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}(-0.5) = 26.57^\circ$$

$$\text{power factor} = \cos(\theta) = 89.44\%$$

P5.56

$$\mathbf{I} = \frac{240\sqrt{2}\angle 50^\circ - 220\sqrt{2}\angle 30^\circ}{1 + j2} = 52.03\angle 52.71^\circ$$

$$I_{rms} = 36.79 \text{ A}$$

Delivered by Source A:

$$P_A = 240 I_{rms} \cos(50 - 52.71) = 8.820 \text{ kW}$$

$$Q_A = 240 I_{rms} \sin(50 - 52.71) = -0.418 \quad \text{kVAR}$$

Absorbed by Source *B*:

$$P_B = 220 I_{rms} \cos(30 - 52.71) = 7.467 \quad \text{kW}$$

$$Q_A = 220 I_{rms} \sin(30 - 52.71) = -3.125 \quad \text{kVAR}$$

Absorbed by resistor:

$$P_R = I_{rms}^2 R = 1.353 \ \text{kW}$$

Absorbed by inductor:

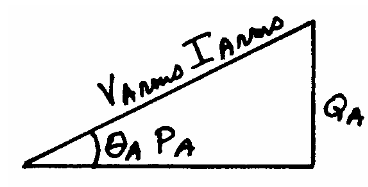
$$Q_L = I_{rms}^2 X = 2.707 \ \text{kVAR}$$

P5.62* Load A:

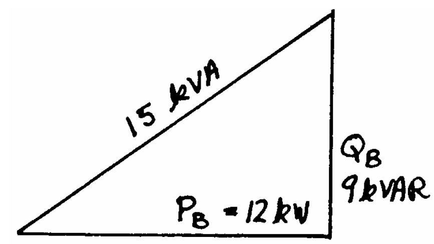
$$P_A = 10 \text{ kW}$$

$$\theta_A = \cos^{-1}(0.9) = 25.84^\circ$$

$$Q_A = P_A \tan\theta_A = 4.843 \text{ kVAR}$$



Load *B*:



$$V_{rms} I_{Brms} = 15 \text{ kVA}$$

$$\theta_B = \cos^{-1}(0.8) = 36.87^\circ$$

$$Q_B = V_{rms} I_{Brms} \sin(\theta_B) = 9 \text{ kVAR}$$

$$P_B = V_{rms} I_{Brms} \cos(\theta_B) = 12 \text{ kW}$$

Source:

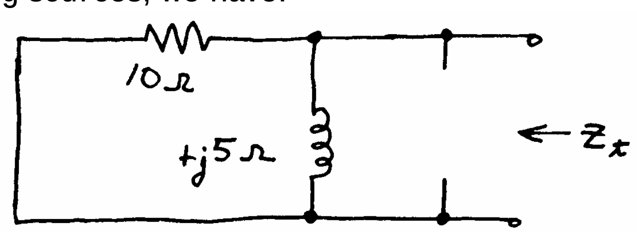
$$P_s = P_A + P_B = 22 \text{ kW}$$

$$Q_s = Q_A + Q_B = 13.84 \text{ kVAR}$$

$$\text{Apparent power} = \sqrt{(P_s)^2 + (Q_s)^2} = 26 \text{ kVA}$$

$$\text{Power factor} = \frac{P_s}{\text{Apparent power}} = 0.8462 = 84.62\% \text{ lagging}$$

P5.68 Zeroing sources, we have:



Thus, the Thévenin impedance is

$$Z_t = \frac{1}{1/10 + 1/j5} = 4.472\angle 63.43^\circ = 2 + j4$$

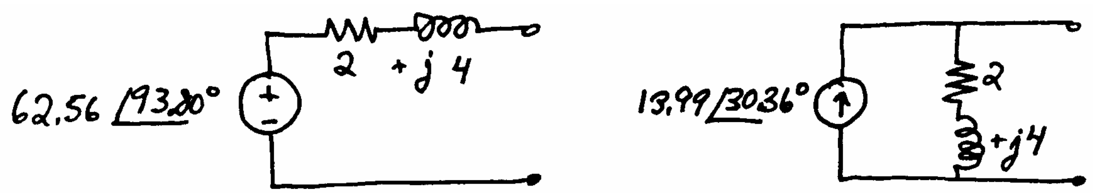
Writing a current equation for the node at the upper end of the current source under open circuit conditions, we have

$$\frac{\mathbf{V}_{oc} - 100\angle 45^\circ}{10} + \frac{\mathbf{V}_{oc}}{j5} = 5$$

$$\mathbf{V}_t = \mathbf{V}_{oc} = 62.56\angle 93.80^\circ$$

$$\mathbf{I}_n = \mathbf{V}_t / Z_t = 13.99\angle 30.36^\circ$$

Thus, the Thévenin and Norton equivalent circuits are:



For the maximum power transfer, the load impedance is

$$Z_{load} = 2 - j4$$

$$\mathbf{I}_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{62.56\angle 93.80^\circ}{2 + j4 + 2 - j4} = 15.64\angle 93.80^\circ$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 244.6 \text{ W}$$

In the case for which the load must be pure resistance, the load for maximum power transfer is

$$Z_{load} = \left|Z_t\right| = 4.472$$

$$I_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{62.56\angle 93.80^\circ}{2 + j4 + 4.472} = 8.223\angle -62.08^\circ$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 151.2 \text{ W}$$

$$Z_C = \frac{Z_a Z_b + Z_b Z_c + Z_a Z_c}{Z_c}$$