CHAPTER 4

P4.4* The solution is of the form of Equation 4.17:

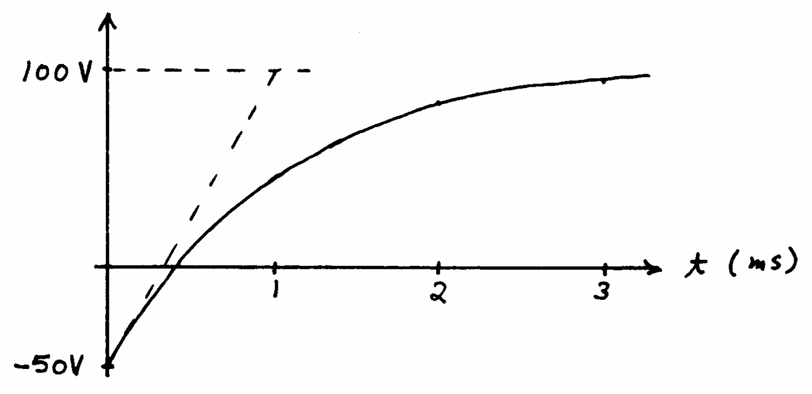
$$v_C(t) = V_s + K_2 \exp(-t/RC) = 100 + K_2 \exp(-t/10^{-3})$$

in which K_2 is a constant to be determined. At $t = 0+$, we have

$$v_C(0+) = -50 = 100 + K_2$$

Solving, we find that $K_2 = -150$ and the solution is

$$v_C(t) = 100 - 150\exp(-t/10^{-3})$$



P4.7 Prior to $t = 0$, we have $v(t) = 0$ because the switch is closed. After $t = 0$, we can write the following KCL equation at the top node of the circuit:

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} = 1\text{ mA}$$

Multiplying both sides by R and substituting values, we have

$$0.01\frac{dv(t)}{dt} + v(t) = 10 \qquad (1)$$

The solution is of the form

$$v(t) = K_1 + K_2 \exp(-t/RC) = K_1 + K_2 \exp(-100t) \qquad (2)$$

Substituting Equation (2) into Equation (1), we eventually obtain

$$K_1 = 10$$

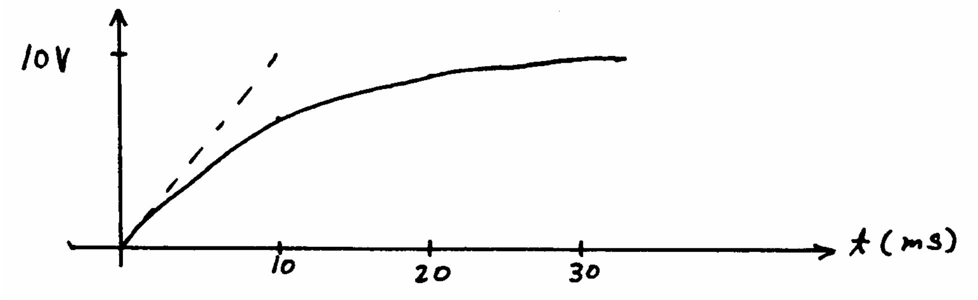
The voltage across the capacitance cannot change instantaneously, so we have

$$v(0+) = v(0-) = 0$$

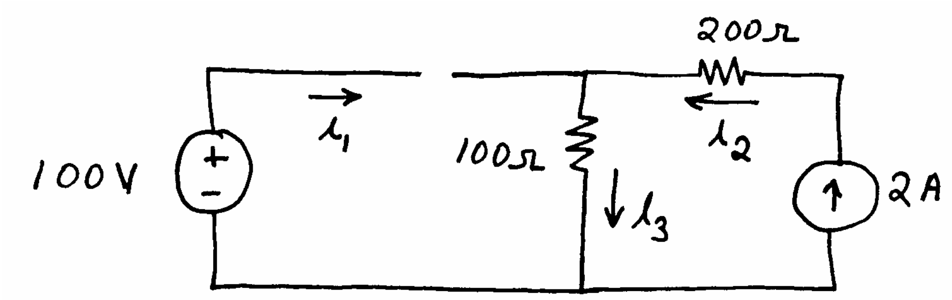
$$v(0+) = 0 = K_1 + K_2$$

Thus, $K_2 = -K_1 = -10$, and the solution is

$$v(t) = 10 - 10\exp(-100t) \text{ for } t > 0$$



P4.15* In steady state, the equivalent circuit is:



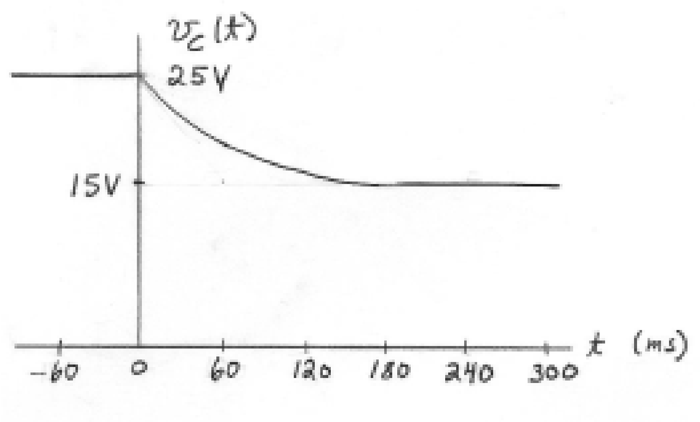
Thus, we have

$$i_1 = 0$$

$$i_3 = i_2 = 2 \text{ A}$$

P4.21 With the switch closed and the circuit in steady state prior to $t = 0$, the capacitor behaves as an open circuit, and $v_C(0-) = 25\,\text{V}$. Because there cannot be infinite current in this circuit when the switch opens, we have $v_C(0+) = 25\,\text{V}$. After switching and the circuit reaches steady state, we have $v_C(\infty) = 25\dfrac{15\text{ k}\Omega}{10\text{ k}\Omega + 15\ \Omega\text{k}} = 15\,\text{V}$. For $t \geq 0$, the Thévenin resistance seen by the capacitor is $R_t = \dfrac{1}{1/10 + 1/15} = 6\,\text{k}\Omega$, and the time constant is $\tau = R_t C = 60$ ms. The general form of the solution for $t \geq 0$ is $v_C(t) = K_1 + K_2 \exp(-t/\tau)$. However, we know that $v_C(0+) = 25 = K_1 + K_2$ and $v_C(\infty) = 15 = K_1$. Solving, we find that $K_2 = 10$. Thus, we have

$$v_C(t) = 25 \quad t \leq 0$$
$$= 15 + 10\exp(-t/\tau) \quad t \geq 0$$



P4.30* In steady state with the switch closed, we have $i(t) = 0$ for $t < 0$ because the closed switch shorts the source.

In steady state with the switch open, the inductance acts as a short circuit and the current becomes $i(\infty) = 1\,\text{A}$. The current is of the form

$$i(t) = K_1 + K_2 \exp(-Rt/L) \text{ for } t \geq 0$$

in which $R = 20\,\Omega$, because that is the Thévenin resistance seen looking back from the terminals of the inductance with the switch open. Also, we have

$$i(0+) = i(0-) = 0 = K_1 + K_2$$
$$i(\infty) = 1 = K_1$$

Thus, $K_2 = -1$ and the current (in amperes) is given by

$$i(t) = 0 \qquad \text{for } t < 0$$
$$= 1 - \exp(-20t) \qquad \text{for } t \geq 0$$

P4.35 Write a current equation at the top node:

$$5\cos(10t) = \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)dt + i_L(0)$$

Differentiate each term with respect to time to obtain a differential equation:

$$-50\sin(10t) = \frac{1}{R}\frac{dv(t)}{dt} + \frac{v(t)}{L}$$

Substitute the particular solution suggested in the hint:

$$-50\sin(10t) \equiv \frac{1}{R}[-10A\sin(10t) + 10B\cos(10t)] + \frac{1}{L}[A\cos(10t) + B\sin(10t)]$$

Equating coefficients of sine and cosine terms, we have

$$-50 = \frac{-10A}{R} + \frac{B}{L}$$

$$0 = \frac{10B}{R} + \frac{A}{L}$$

Solving for A and B and substituting values of the circuit parameters, we find $A = 25$ and $B = -25$. The time constant is $\tau = L / R = 0.1$ s, and the general form of the solution is

$$v(t) = K_1 \exp(-t / \tau) + v_p(t) = K_1 \exp(-t / \tau) + 25\cos(10t) - 25\sin(10t)$$

However, because the current in the inductor is zero at t = 0+, the 5 A supplied by the source must flow through the 10-Ω resistor and we have $v(0+) = 50$. Substituting this into the general solution we find $K_1 = 25$. Thus

$$v(t) = 25\exp(-t / \tau) + 25\cos(10t) - 25\sin(10t) \quad t \geq 0$$

P4.37* Applying KVL, we obtain the differential equation:

$$L\frac{di(t)}{dt} + Ri(t) = 5\exp(-t) \text{ for } t > 0 \qquad (1)$$

We try a particular solution of the form:

$$i_p(t) = A\exp(-t) \qquad (2)$$

in which A is a constant to be determined. Substituting Equation (2) into Equation (1), we have

$$-LA\exp(-t) + RA\exp(-t) = 5\exp(-t)$$

which yields

$$A = \frac{5}{R - L} = -1$$

The complementary solution is of the form

$$i_c(t) = K_1 \exp(-Rt/L)$$

The complete solution is

$$i(t) = i_p(t) + i_c(t) = -\exp(-t) + K_1 \exp(-Rt/L)$$

Before the switch closes, the current must be zero. Furthermore, the current cannot change instantaneously, so we have $i(0+) = 0$. Therefore, we have $i(0+) = 0 = -1 + K_1$ which yields $K_1 = 1$. Finally, the current is given by $i(t) = -\exp(-t) + \exp(-Rt/L)$ for $t \geq 0$.

P4.45* Applying KVL to the circuit, we obtain

$$L\frac{di(t)}{dt} + Ri(t) + v_C(t) = v_s = 50 \qquad (1)$$

For the capacitance, we have

$$i(t) = C\frac{dv_C(t)}{dt} \qquad (2)$$

Using Equation (2) to substitute into Equation (1) and rearranging, we have

$$\frac{d^2v_C(t)}{dt^2} + (R/L)\frac{dv_C(t)}{dt} + (1/LC)v_C(t) = 50/LC \qquad (3)$$

$$\frac{d^2v_C(t)}{dt^2} + 4\times 10^4\frac{dv_C(t)}{dt} + 10^8 v_C(t) = 50\times 10^8$$

We try a particular solution of the form $v_{Cp}(t) = A$, resulting in $A = 50$. Thus, $v_{Cp}(t) = 50$. (An alternative method to find the particular solution is to solve the circuit in dc steady state. Since the capacitance acts as an open circuit, the steady-state voltage across it is 50 V.) Comparing Equation (3) with Equation 4.67 in the text, we find

$$\alpha = \frac{R}{2L} = 2\times 10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$$

Since we have $\alpha > \omega_0$, this is the overdamped case. The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -0.2679 \times 10^4$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -3.732 \times 10^4$$

The complementary solution is

$$v_{Cc}(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

and the complete solution is

$$v_C(t) = 50 + K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

The initial conditions are

$$v_C(0) = 0 \quad \text{and} \quad i(0) = 0 = C \frac{dv_C(t)}{dt}\Big|_{t=0}$$

Thus, we have

$$v_C(0) = 0 = 50 + K_1 + K_2$$
$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 0 = s_1 K_1 + s_2 K_2$$

Solving, we find $K_1 = -53.87$ and $K_2 = 3.867$. Finally, the solution is

$$v_C(t) = 50 - 53.87 \exp(s_1 t) + 3.867 \exp(s_2 t)$$

P4.48 (a) Using Equation 4.103 from the text, the damping coefficient is

$$\alpha = \frac{1}{2RC} = 20 \times 10^6$$

Equation 4.104 gives the undamped resonant frequency

$$\omega_0 = \frac{1}{LC} = 10 \times 10^6$$

Equation 4.71 gives the damping ratio

$$\zeta = \alpha/\omega_0 = 2$$

Thus, we have an overdamped circuit.

(b) Writing a current equation at $t = 0+$, we have

$$\frac{v(0+)}{R} + i_L(0+) + Cv'(0+) = 1$$

Substituting $v(0+) = 0$ and $i_L(0+) = 0$, yields

$$v'(0+) = 1/C = 10^9$$

(c) Under steady-state conditions, the inductance acts as a short circuit. Therefore, the particular solution for $v(t)$ is:

$$v_p(t) = 0$$

(d) The roots of the characteristic equation are found from Equations 4.72 and 4.73 in the text.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2.679 \times 10^6$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -37.32 \times 10^6$$

The complementary solution is

$$v_c(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$

and (since the particular solution is zero) the complete solution is

$$v(t) = K_1 \exp(s_1 t) + K_2 (s_2 t)$$

The initial conditions are

$$v(0+) = 0 \quad \text{and} \quad v'(0+) = 10^9$$

Thus, we have

$$v(0+) = 0 = K_1 + K_2$$

$$v'(0+) = 10^9 = s_1 K_1 + s_2 K_2$$

Solving, we find $K_1 = 28.87$ and $K_2 = -28.87$. Finally, the solution is

$$v(t) = 28.87 \exp(s_1 t) - 28.87 \exp(s_2 t)$$

$$i(t) = 0.025 \cos(100t) + 0.00193 \exp(s_1 t) - 0.02693 \exp(s_2 t)$$