Chapter 5
Steady-State Sinusoidal Analysis

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1. Identify the frequency, angular frequency, peak value, rms value, and phase of a sinusoidal signal.

2. Solve steady-state ac circuits using phasors and complex impedances.

3. Compute power for steady-state ac circuits.

4. Find Thévenin and Norton equivalent circuits.

5. Determine load impedances for maximum power transfer.

6. Solve balanced three-phase circuits.

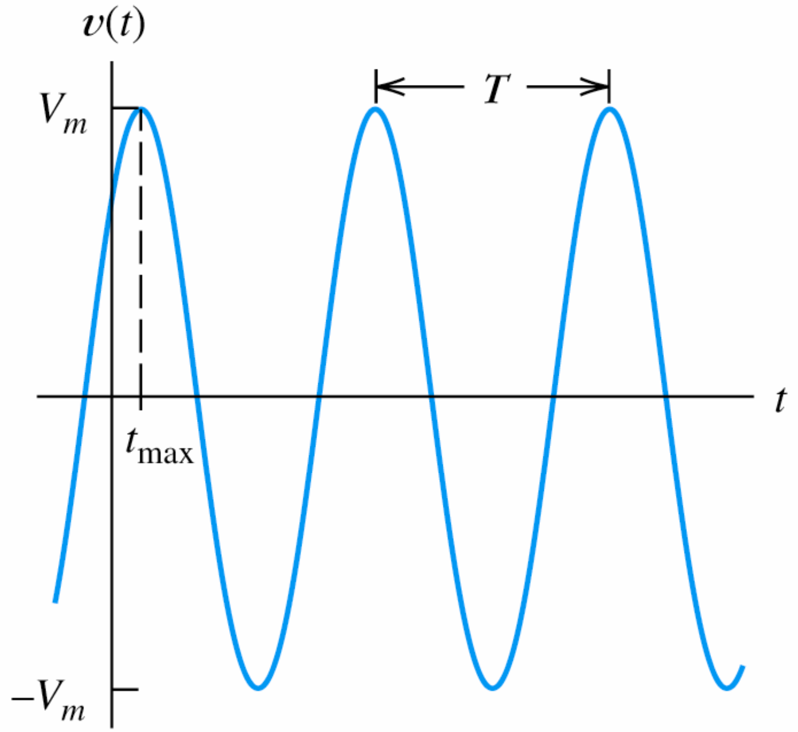


Figure 5.1 A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$.
Note: Assuming that θ is in degrees, we have $t_{max} = \frac{-\theta}{360} \times T$.
For the waveform shown, θ is -45°.

SINUSOIDAL CURRENTS AND VOLTAGES

V_m is the **peak value**

ω is the **angular frequency** in radians per second

θ is the **phase angle**

T is the **period**

Frequency

$$f = \frac{1}{T}$$

Angular frequency

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2\pi f$$

$$\sin(z) = \cos\left(z - 90^\circ\right)$$

Root-Mean-Square Values

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} \qquad P_{\text{avg}} = I_{\text{rms}}^2 R$$

RMS Value of a Sinusoid

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

The rms value for a sinusoid is the peak value divided by the square root of two. This is not true for other periodic waveforms such as square waves or triangular waves.

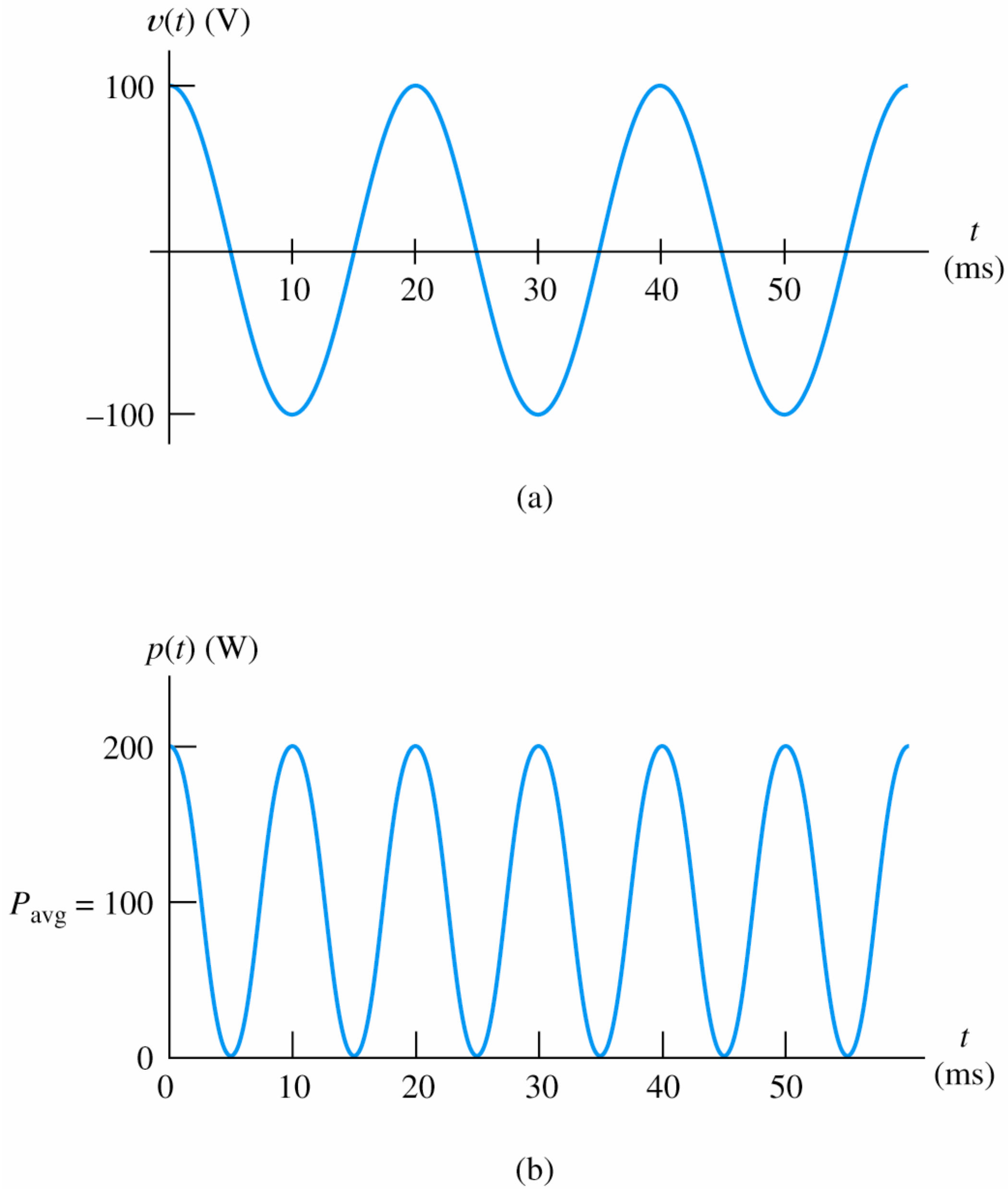


Figure 5.2 Voltage and power versus time for Example 5.1.

Phasor Definition

$$\text{Time function}: \quad v_1(t) = V_1 \cos(\omega t + \theta_1)$$

$$\text{Phasor}: \quad \mathbf{V}_1 = V_1 \angle \theta_1$$

Adding Sinusoids Using Phasors

Step 1: Determine the phasor for each term.

Step 2: Add the phasors using complex arithmetic.

Step 3: Convert the sum to polar form.

Step 4: Write the result as a time function.

Using Phasors to Add Sinusoids

$$v_1(t) = 20\cos\left(\omega t - 45^\circ\right)$$

$$v_2(t) = 10\cos\left(\omega t + 60^\circ\right)$$

$$\mathbf{V}_1 = 20\angle -45^\circ$$

$$\mathbf{V}_2 = 10\angle -30^\circ$$

$$
\begin{aligned}
\mathbf{V}_{\mathrm{s}} &= \mathbf{V}_1 + \mathbf{V}_2 \\
&= 20\angle -45^\circ + 10\angle -30^\circ \\
&= 14.14 - j14.14 + 8.660 - j5 \\
&= 23.06 - j19.14 \\
&= 29.97\angle -39.7^\circ
\end{aligned}
$$

$$
v_s(t) = 29.97\cos\left(\omega t - 39.7^\circ\right)
$$

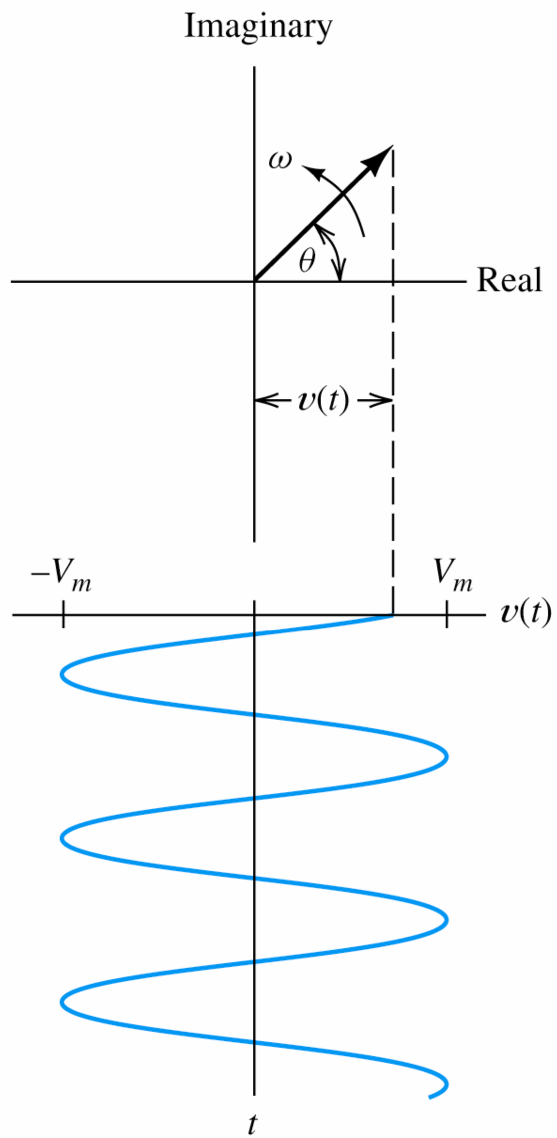


Figure 5.4 A sinusoid can be represented as the real part of a vector rotating counterclockwise in the complex plane.

Sinusoids can be visualized as the real-axis projection of vectors rotating in the complex plane. The phasor for a sinusoid is a snapshot of the corresponding rotating vector at $t = 0$.

Phase Relationships

To determine phase relationships from a phasor diagram, consider the phasors to rotate counterclockwise. Then when standing at a fixed point, if $\mathbf{V}_1$ arrives first followed by $\mathbf{V}_2$ after a rotation of θ , we say that $\mathbf{V}_1$ leads $\mathbf{V}_2$ by θ . Alternatively, we could say that $\mathbf{V}_2$ lags $\mathbf{V}_1$ by θ . (Usually, we take θ as the smaller angle between the two phasors.)

To determine phase relationships between sinusoids from their plots versus time, find the shortest time interval t_p between positive peaks of the two waveforms. Then, the phase angle is

$\theta = (t_p/T) \times 360^\circ$. If the peak of $v_1(t)$ occurs first, we say that $v_1(t)$ leads $v_2(t)$ or that $v_2(t)$ lags $v_1(t)$.

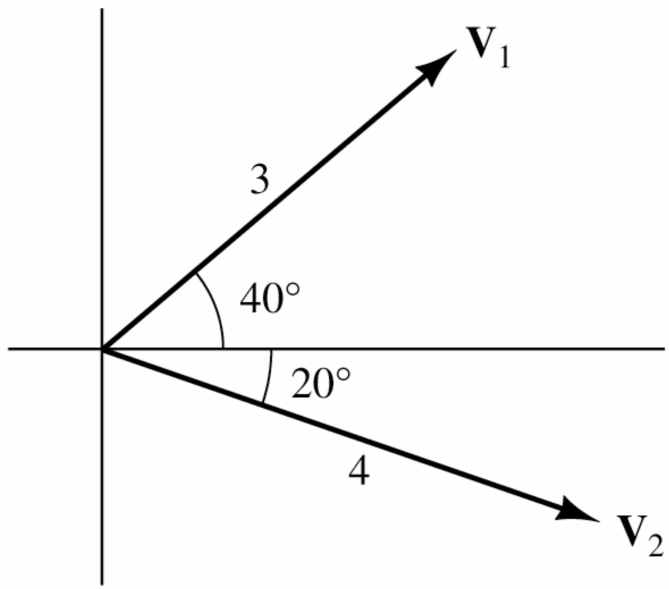


Figure 5.5 Because the vectors rotate counterclockwise, $\mathbf{v}_1$ leads $\mathbf{v}_2$ by 60° (or, equivalently, $\mathbf{v}_2$ lags $\mathbf{v}_1$ by 60°.)

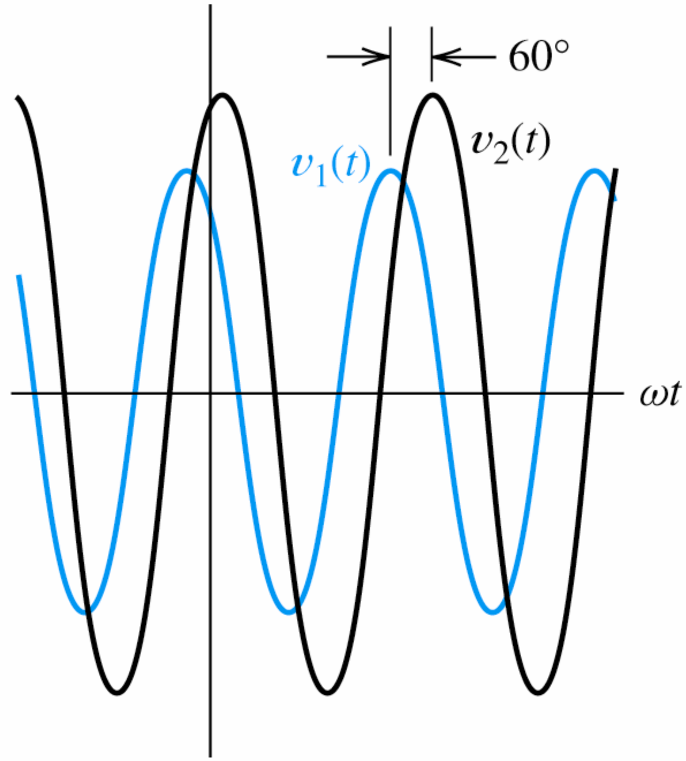
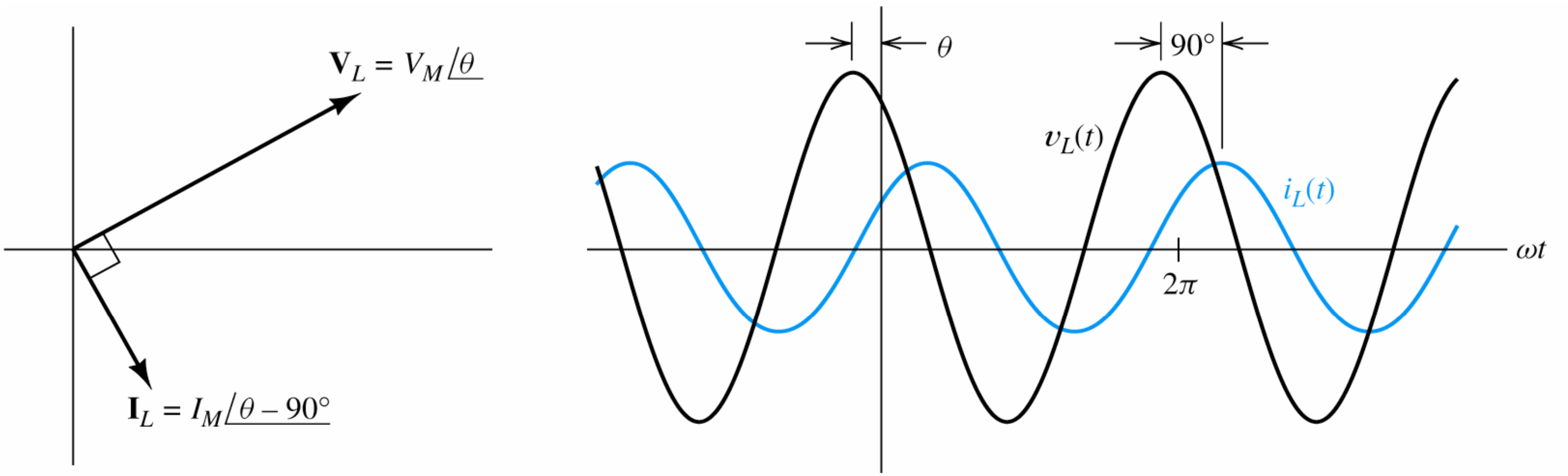


Figure 5.6 The peaks of $v_1(t)$ occur 60° before the peaks of $v_2(t)$. In other words, $v_1(t)$ leads $v_2(t)$ by 60°.



(a) Phasor diagram

(b) Current and voltage versus time

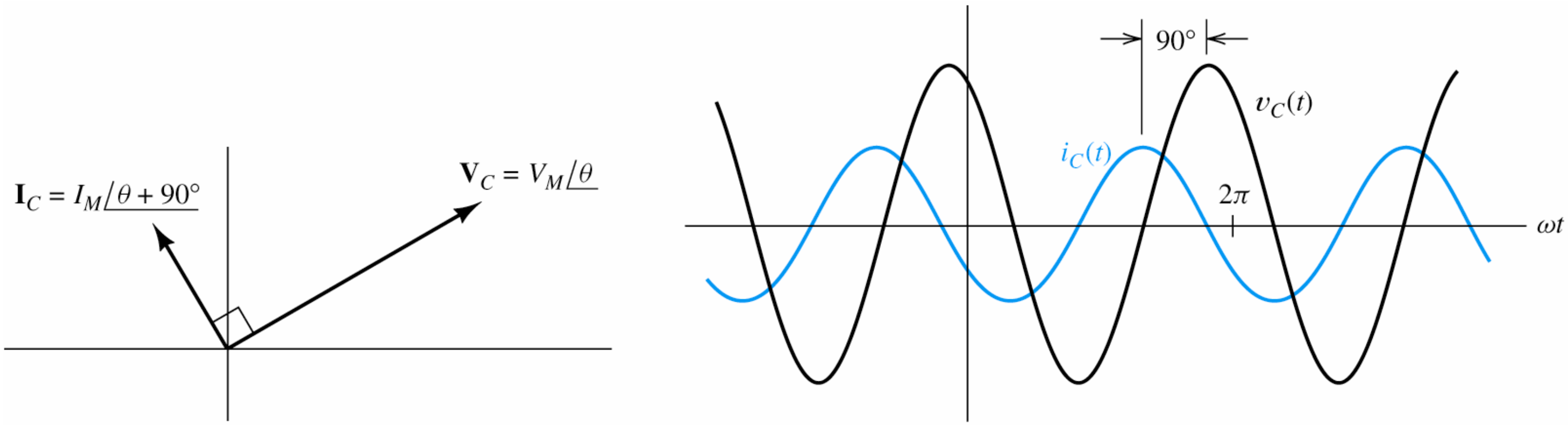
Figure 5.7 Current lags voltage by 90° in a pure inductance.

COMPLEX IMPEDANCES

$$\mathbf{V}_L = j\omega L \times \mathbf{I}_L$$

$$Z_L = j\omega L = \omega L \angle 90^\circ$$

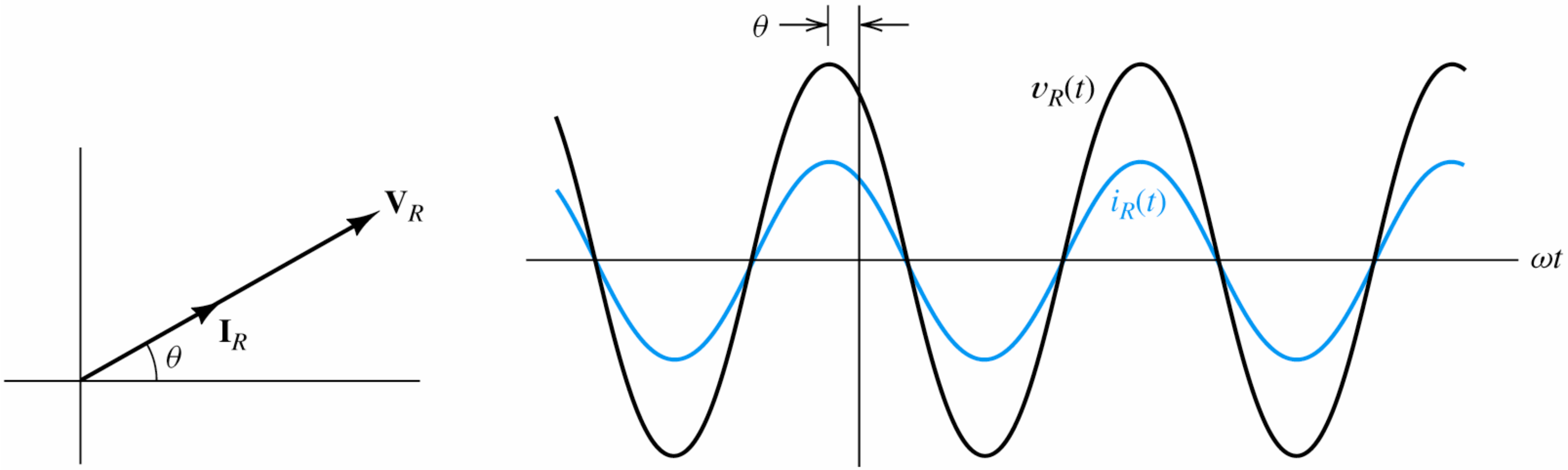
$$\mathbf{V}_L = Z_L \mathbf{I}_L$$



(a) Phasor diagram

(b) Current and voltage versus time

Figure 5.8 Current leads voltage by 90° in a pure capacitance.



(a) Phasor diagram

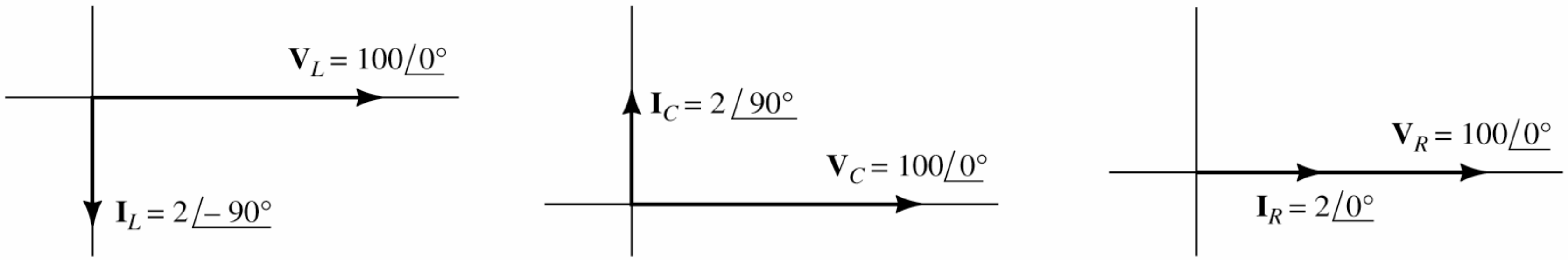
(b) Current and voltage versus time

Figure 5.9 For a pure resistance, current and voltage are in phase.

$$\mathbf{V}_C = Z_C \mathbf{I}_C$$

$$Z_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

$$\mathbf{V}_R = R\mathbf{I}_R$$



(a) Exercise 5.6 (0.25 H inductance) (b) Exercise 5.7 (100 μF capacitance) (c) Exercise 5.8 (50 Ω resistance)

Figure 5.10 Answers for Exercises 5.6, 5.7, and 5.8. The scale has been expanded for the currents compared to the voltages so the current phasors can be easily seen.

Kirchhoff's Laws in Phasor Form

We can apply KVL directly to phasors. The sum of the phasor voltages equals zero for any closed path.

The sum of the phasor currents entering a node must equal the sum of the phasor currents leaving.

Circuit Analysis Using Phasors and Impedances

1. Replace the time descriptions of the voltage and current sources with the corresponding phasors. (All of the sources must have the same frequency.)

2. Replace inductances by their complex impedances $Z_L = j\omega L$. Replace capacitances by their complex impedances $Z_C = 1/(j\omega C)$. Resistances have impedances equal to their resistances.

3. Analyze the circuit using any of the techniques studied earlier in Chapter 2, performing the calculations with complex arithmetic.

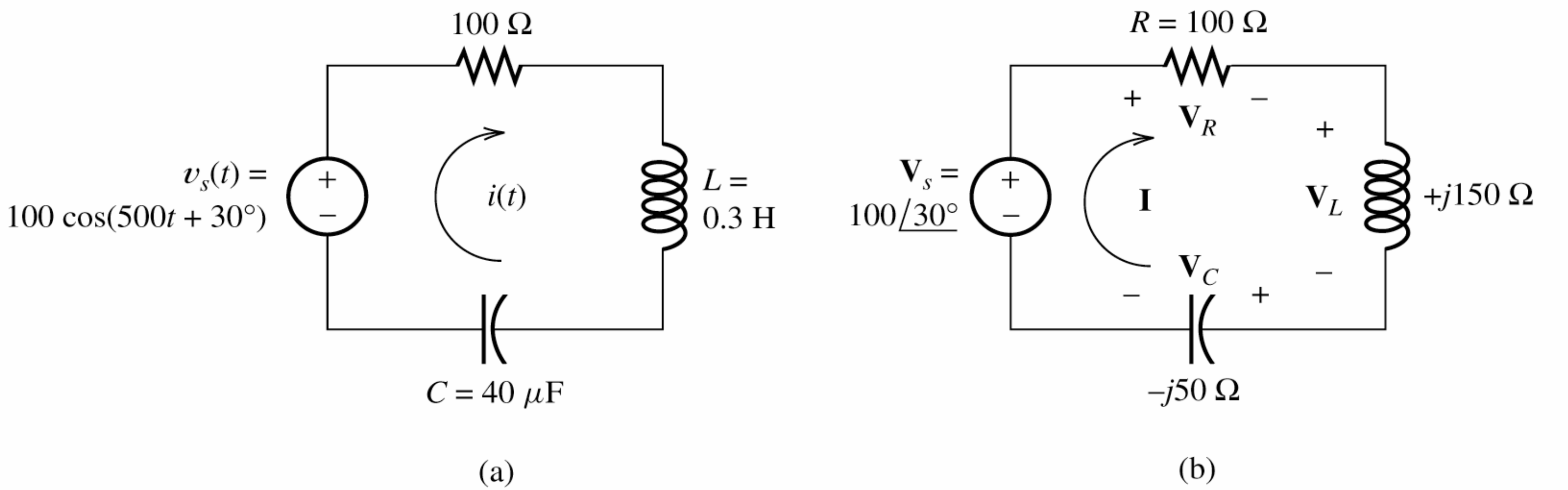


Figure 5.11 Circuit for Example 5.3.

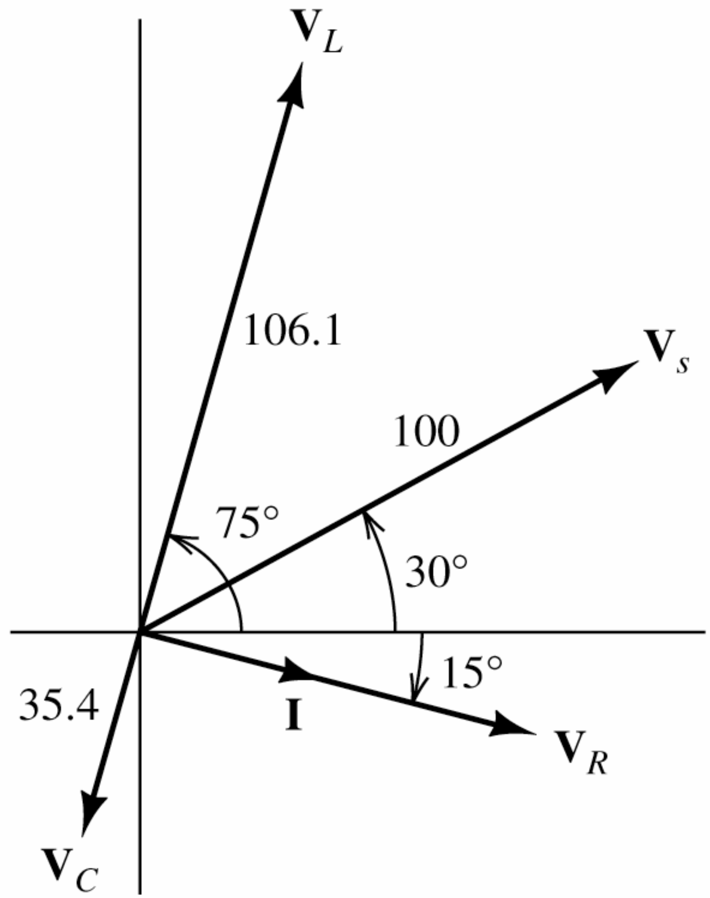


Figure 5.12 Phasor diagram for Example 5.3.

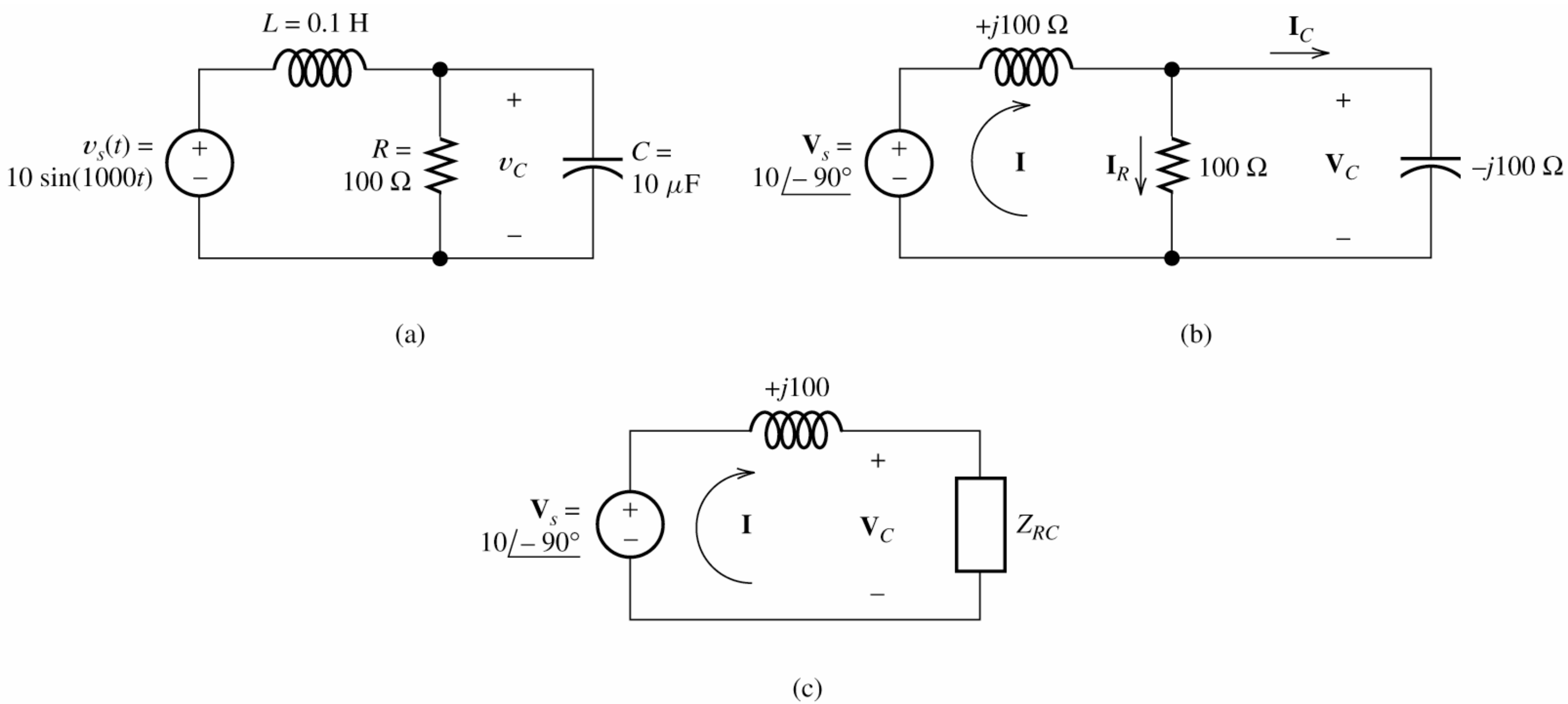


Figure 5.13 Circuit for Example 5.4.

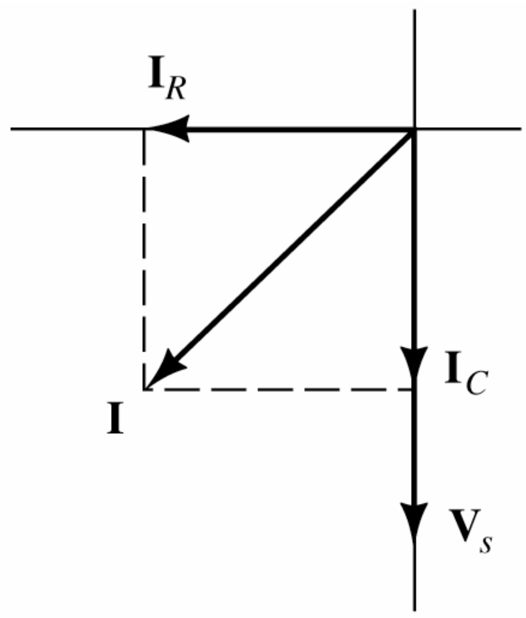


Figure 5.14 Phasor diagram for Example 5.4.

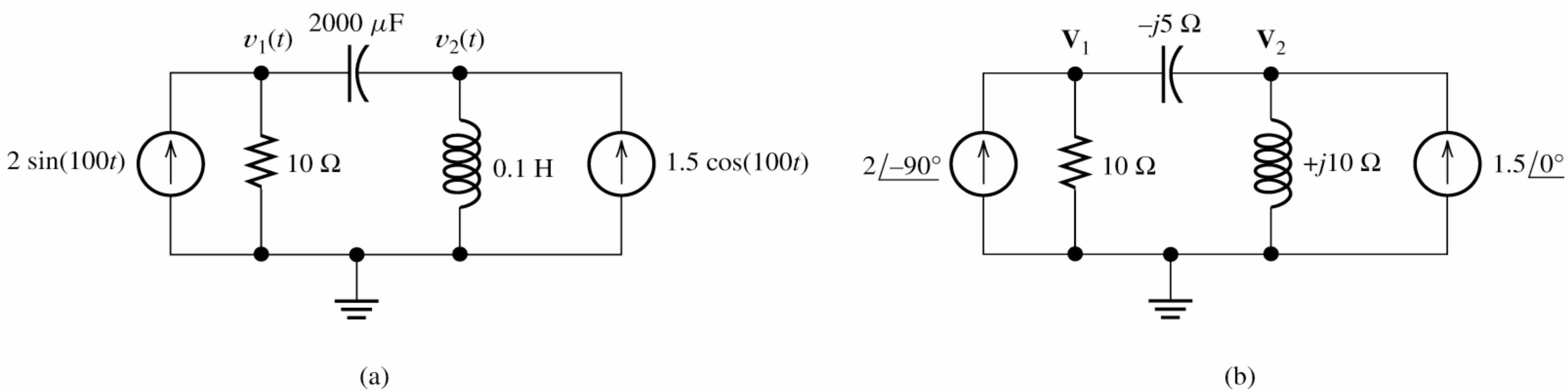


Figure 5.15 Circuit for Example 5.5.

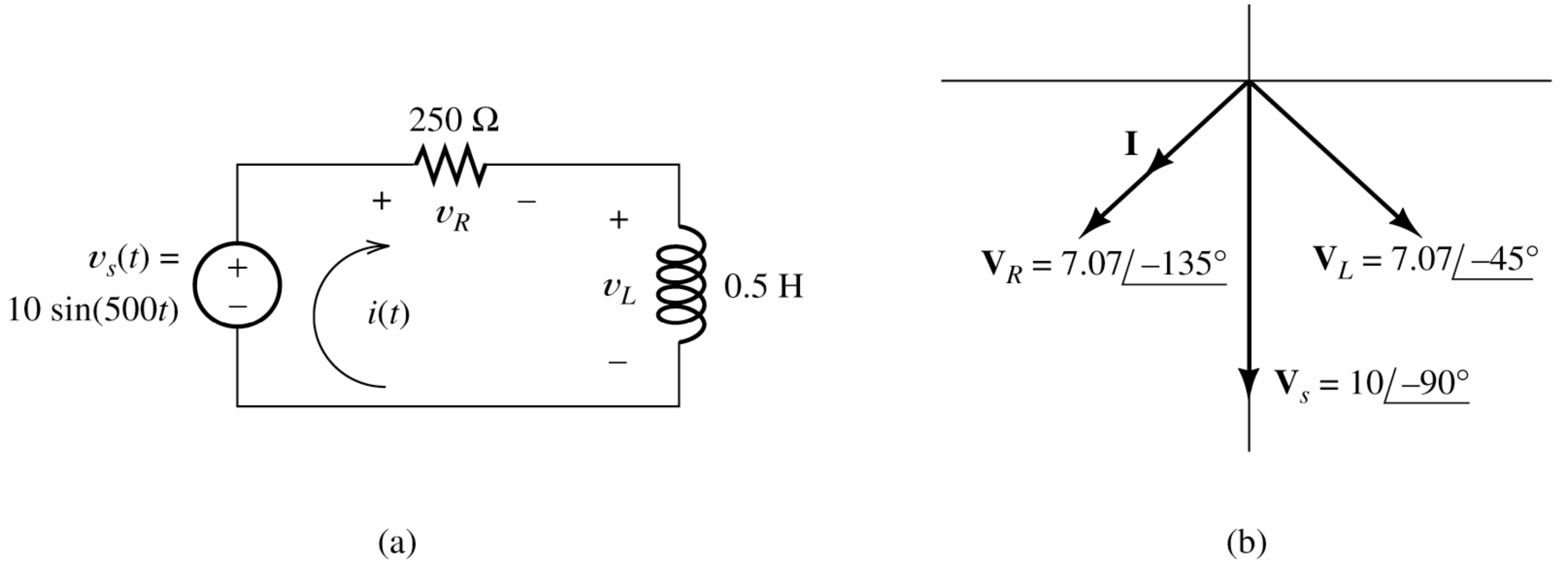


Figure 5.16 Circuit and phasor diagram for Exercise 5.9.

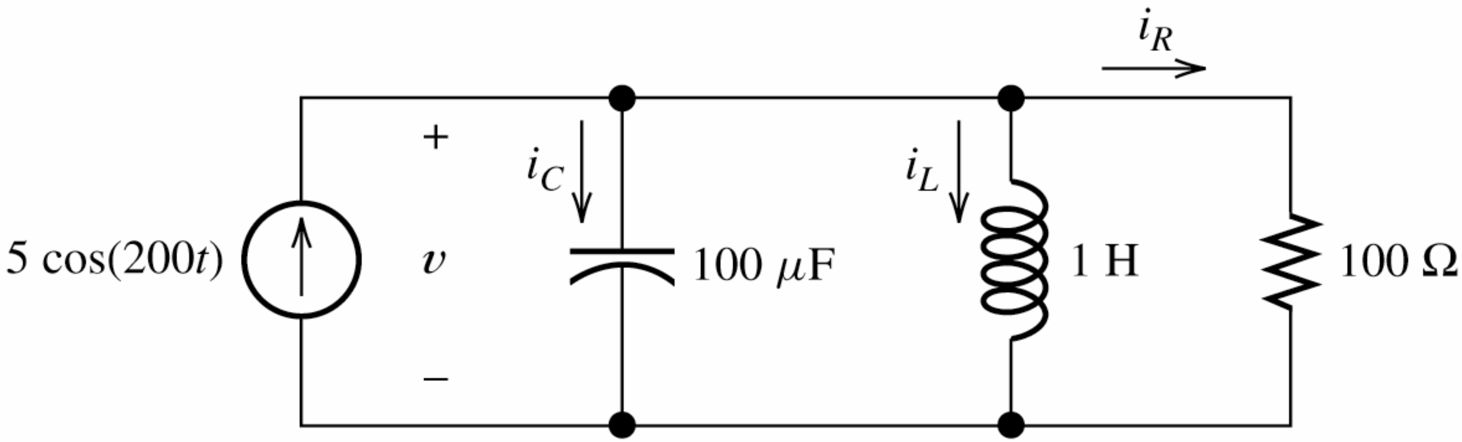


Figure 5.17 Circuit for Exercise 5.10.

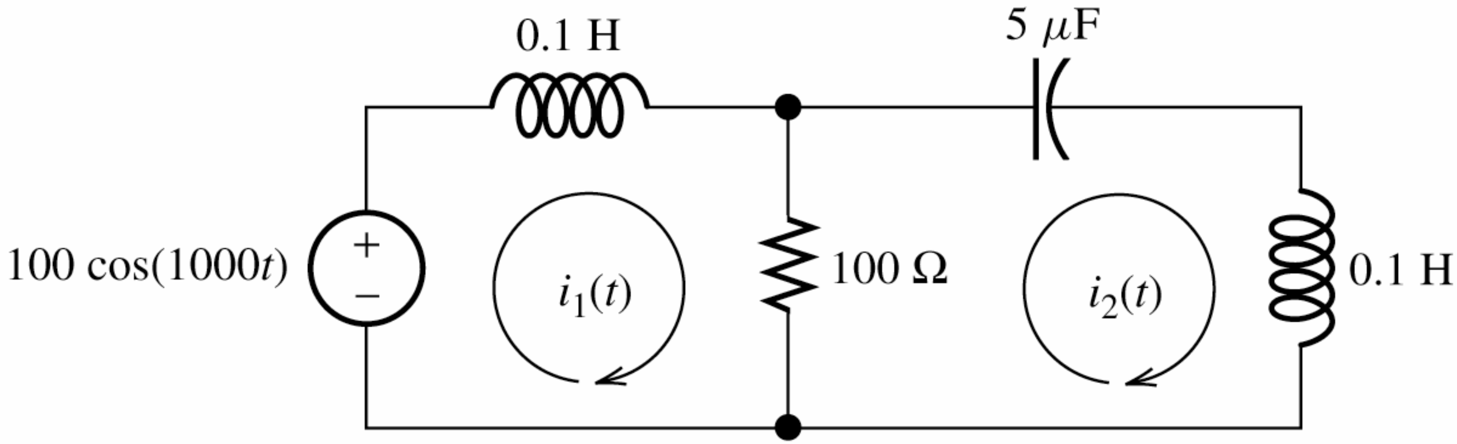


Figure 5.18 Circuit for Exercise 5.11.

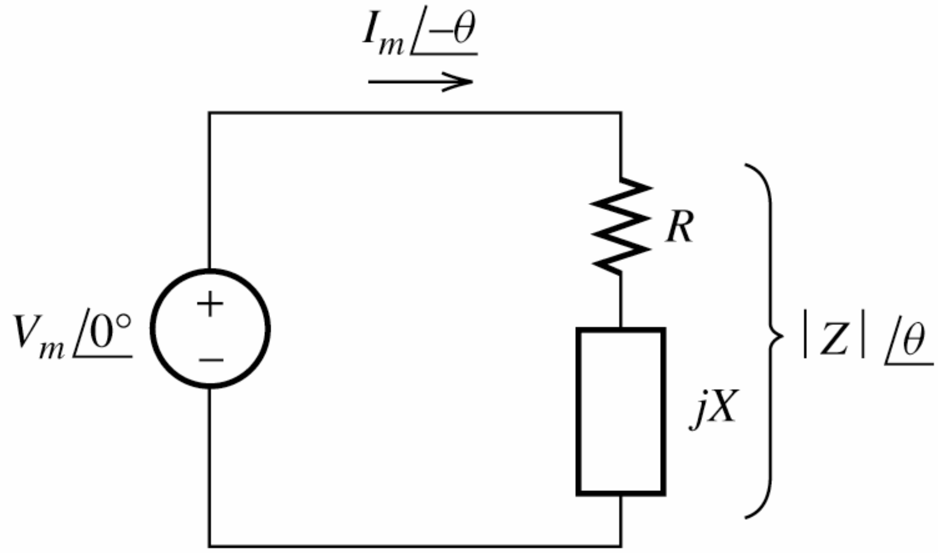


Figure 5.19 A voltage source delivering power to a load impedance $Z = R + jX$.

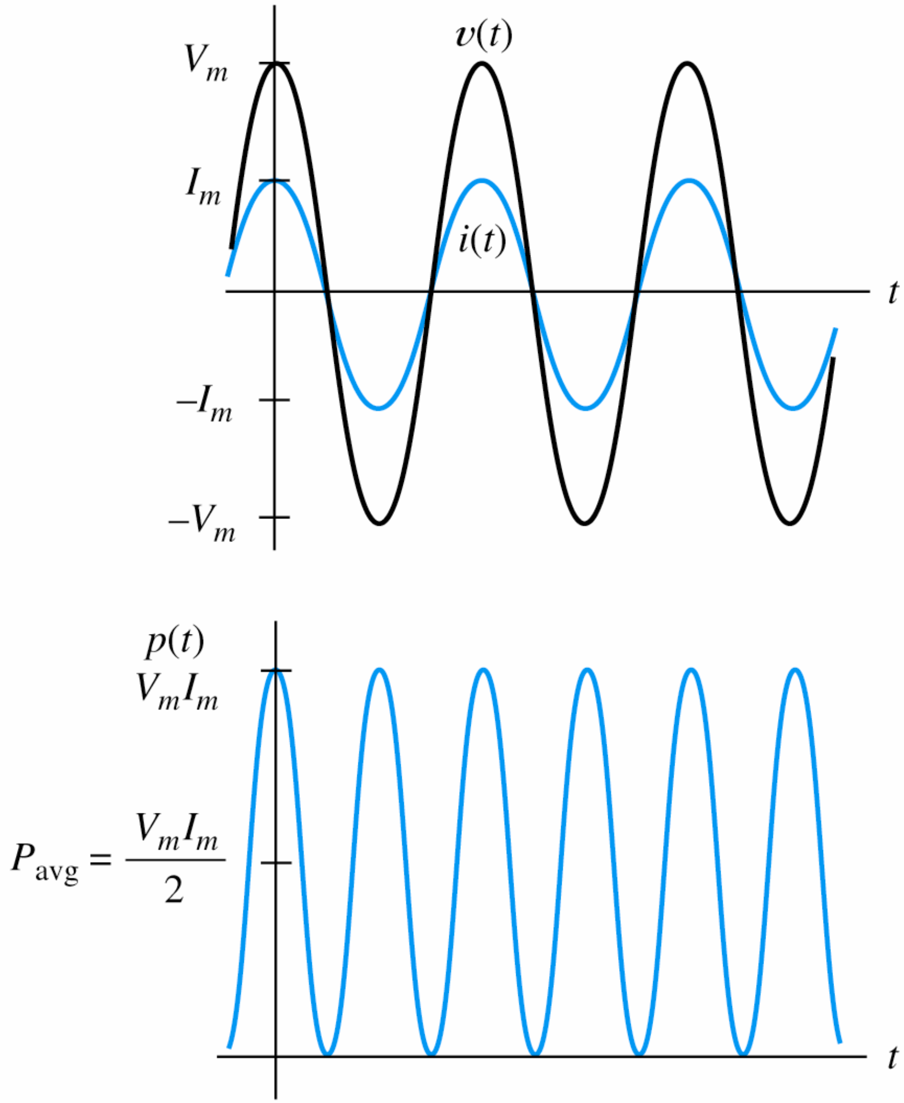
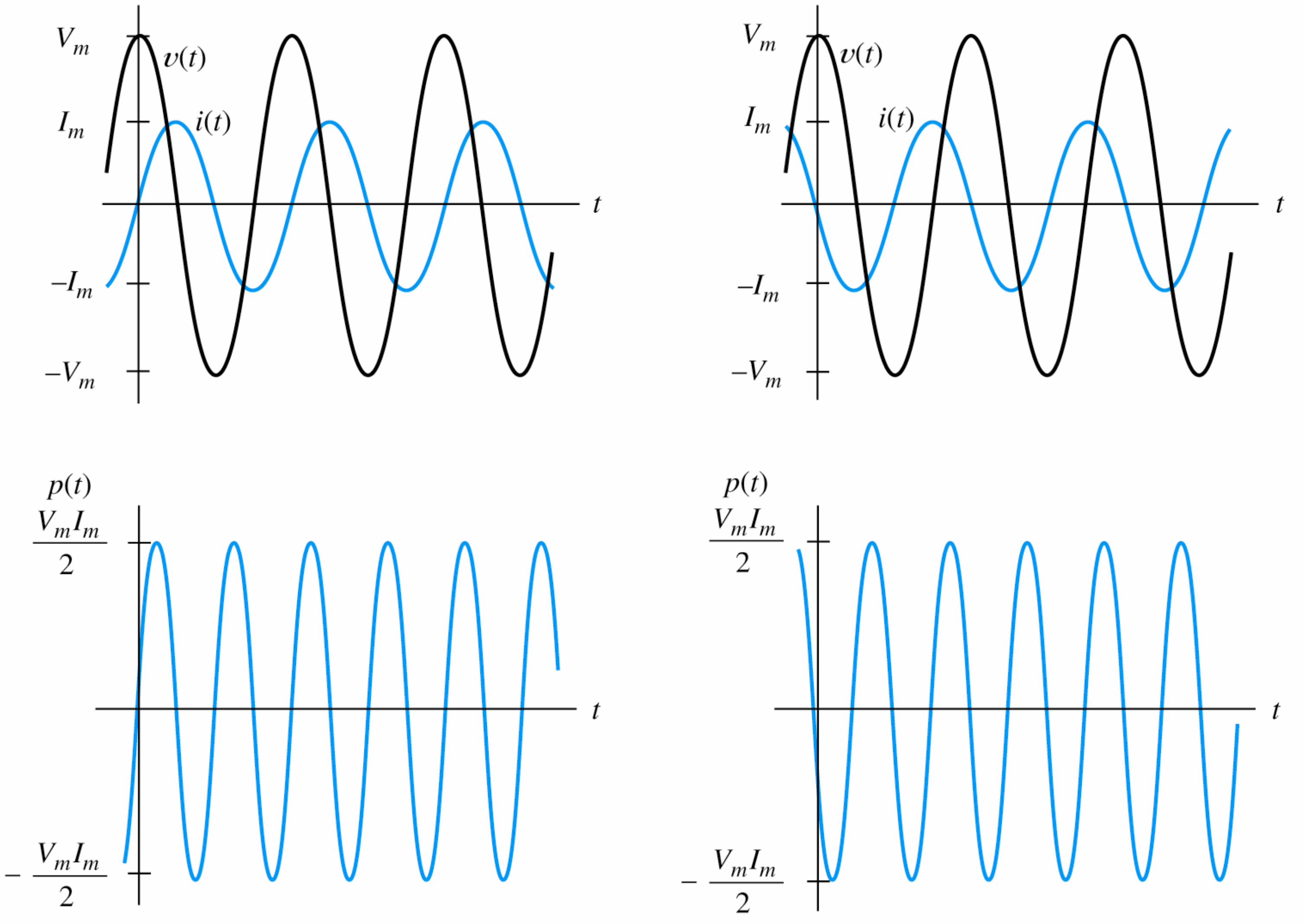


Figure 5.20 Current, voltage, and power versus time for a purely resistive load.



(a) Pure inductive load

(b) Pure capacitive load

Figure 5.21 Current, voltage, and power versus time for pure energy-storage elements.

AC Power Calculations

$$P = V_{\mathrm{rms}} I_{\mathrm{rms}} \cos(\theta)$$

$$\mathrm{PF} = \cos(\theta)$$

$$\theta = \theta_v - \theta_i$$

$$Q = V_{\mathrm{rms}} I_{\mathrm{rms}} \sin(\theta)$$

$$\text{apparent power} = V_{\text{rms}} I_{\text{rms}}$$

$$P^2 + Q^2 = \left(V_{\text{rms}} I_{\text{rms}}\right)^2$$

$$P = I_{\text{rms}}^2 R \qquad P = \frac{V_{R\text{rms}}^2}{R}$$

$$Q = I_{\text{rms}}^2 X \qquad Q = \frac{V_{X\text{rms}}^2}{X}$$

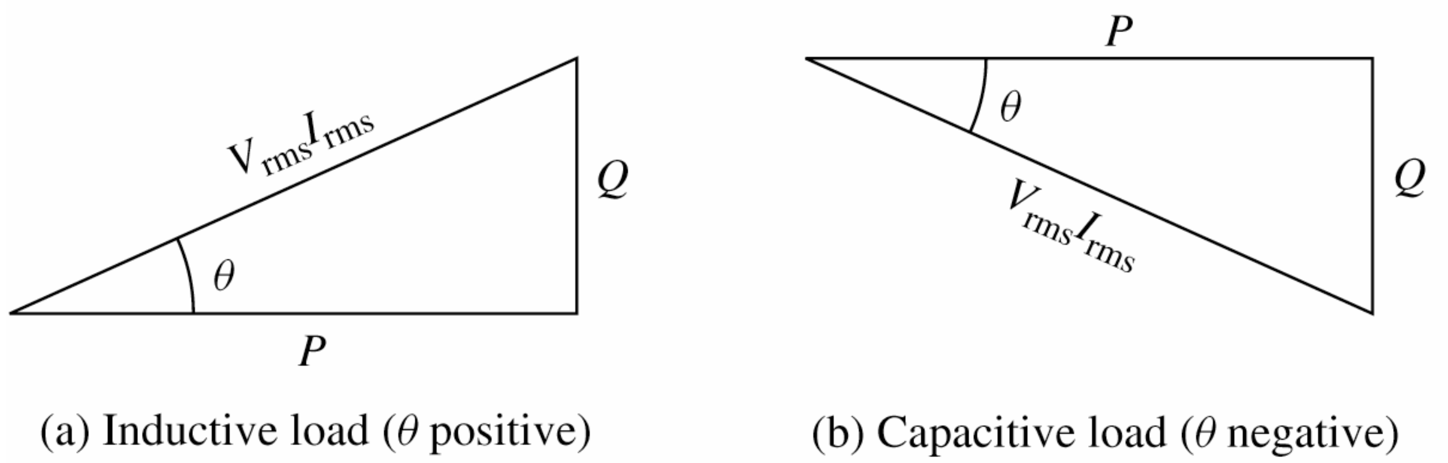


Figure 5.22 Power triangles for inductive and capacitive loads.

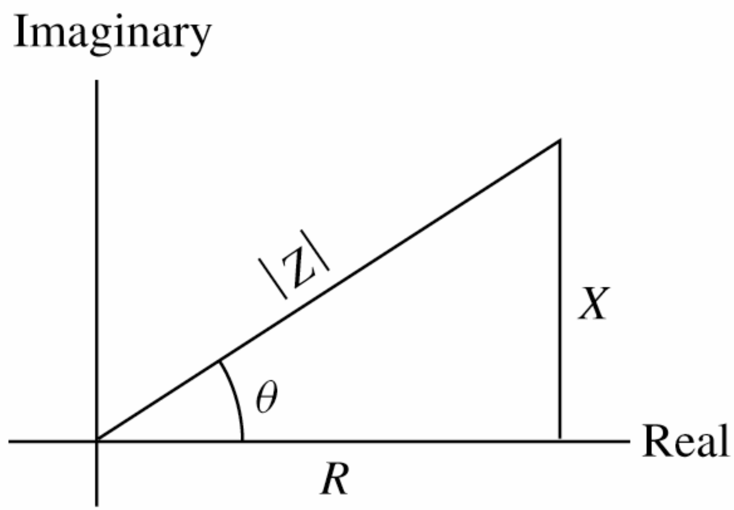


Figure 5.23 The load impedance in the complex plane.

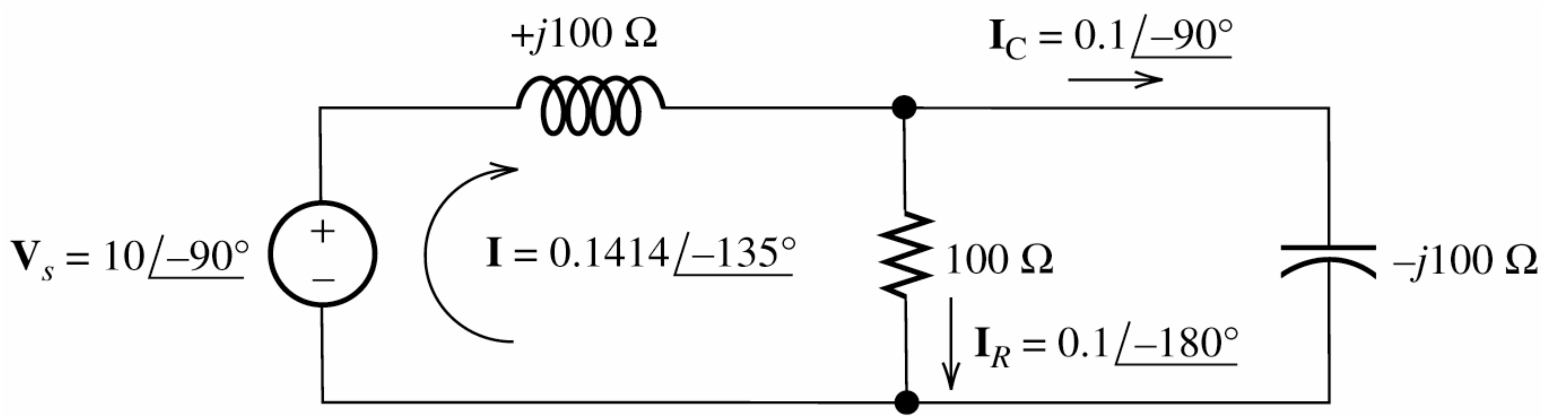


Figure 5.24 Circuit and currents for Example 5.6.

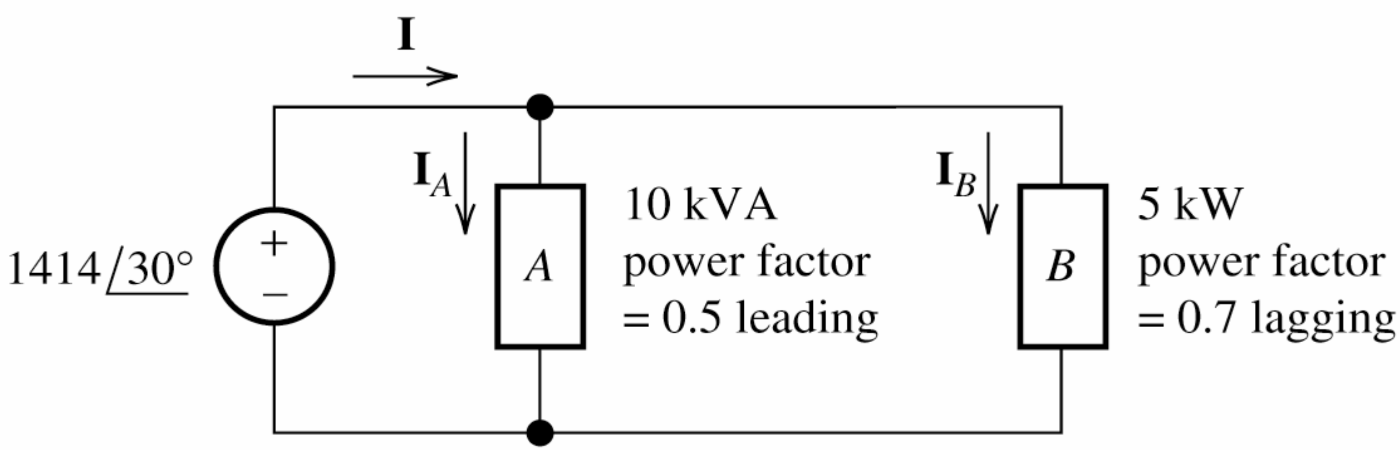
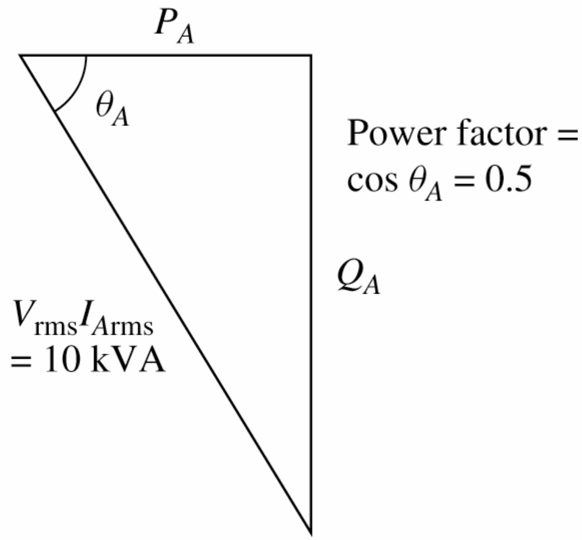
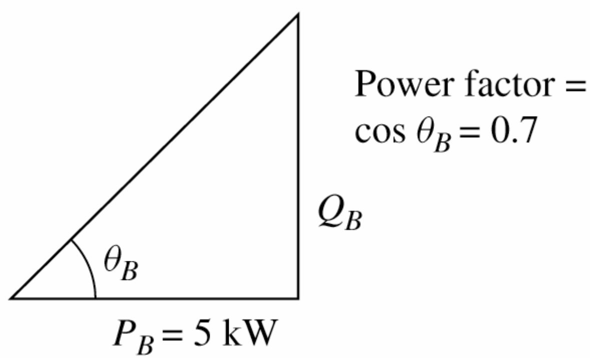


Figure 5.25 Circuit for Example 5.7.



(a)



(b)

Figure 5.26 Power triangles for loads A and B of Example 5.7.

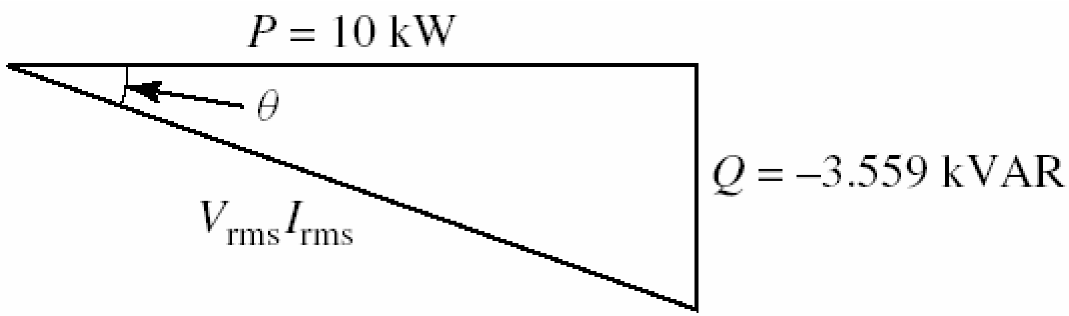


Figure 5.27 Power triangle for the source of Example 5.7.

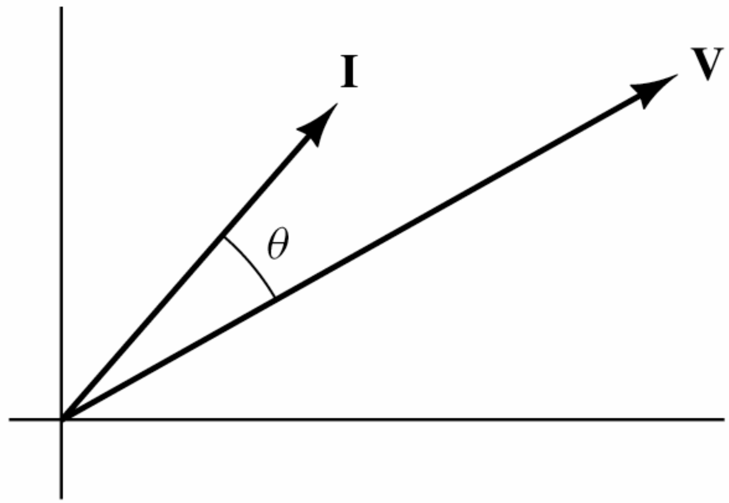


Figure 5.28 Phasor diagram for Example 5.7.

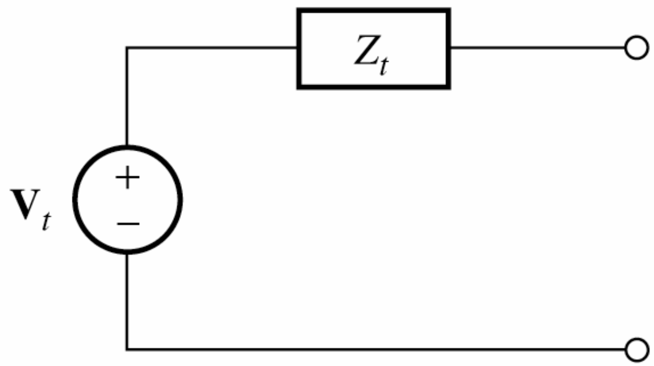


Figure 5.29 The Thévenin equivalent for an ac circuit consists of a phasor voltage source $\mathbf{V}_t$ in series with a complex impedance Z_t.

THÉVENIN EQUIVALENT CIRCUITS

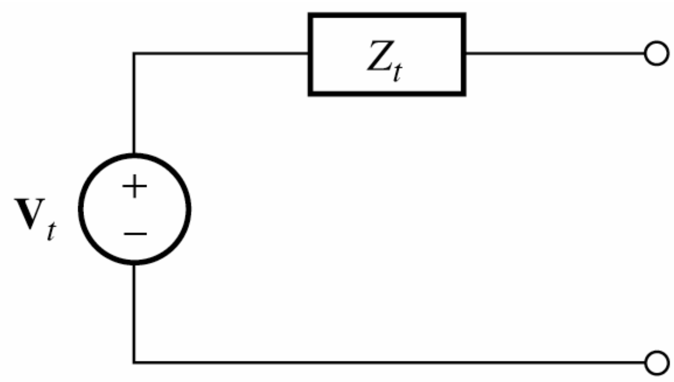


Figure 5.29 The Thévenin equivalent for an ac circuit consists of a phasor voltage source $\mathbf{V}_t$ in series with a complex impedance Z_t.

The Thévenin voltage is equal to the open-circuit phasor voltage of the original circuit.

$$\mathbf{V}_t = \mathbf{V}_{\mathrm{oc}}$$

We can find the Thévenin impedance by zeroing the independent sources and determining the impedance looking into the circuit terminals.

The Thévenin impedance equals the open-circuit voltage divided by the short-circuit current.

$$Z_t = \frac{\mathbf{V}_{\mathrm{oc}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{\mathbf{V}_t}{\mathbf{I}_{\mathrm{sc}}}$$

$$\mathbf{I}_n = \mathbf{I}_{\mathrm{sc}}$$

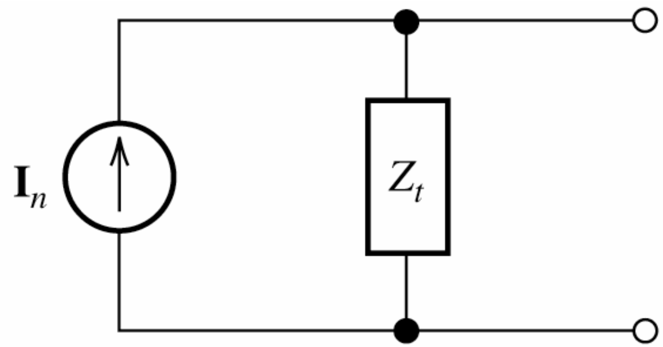
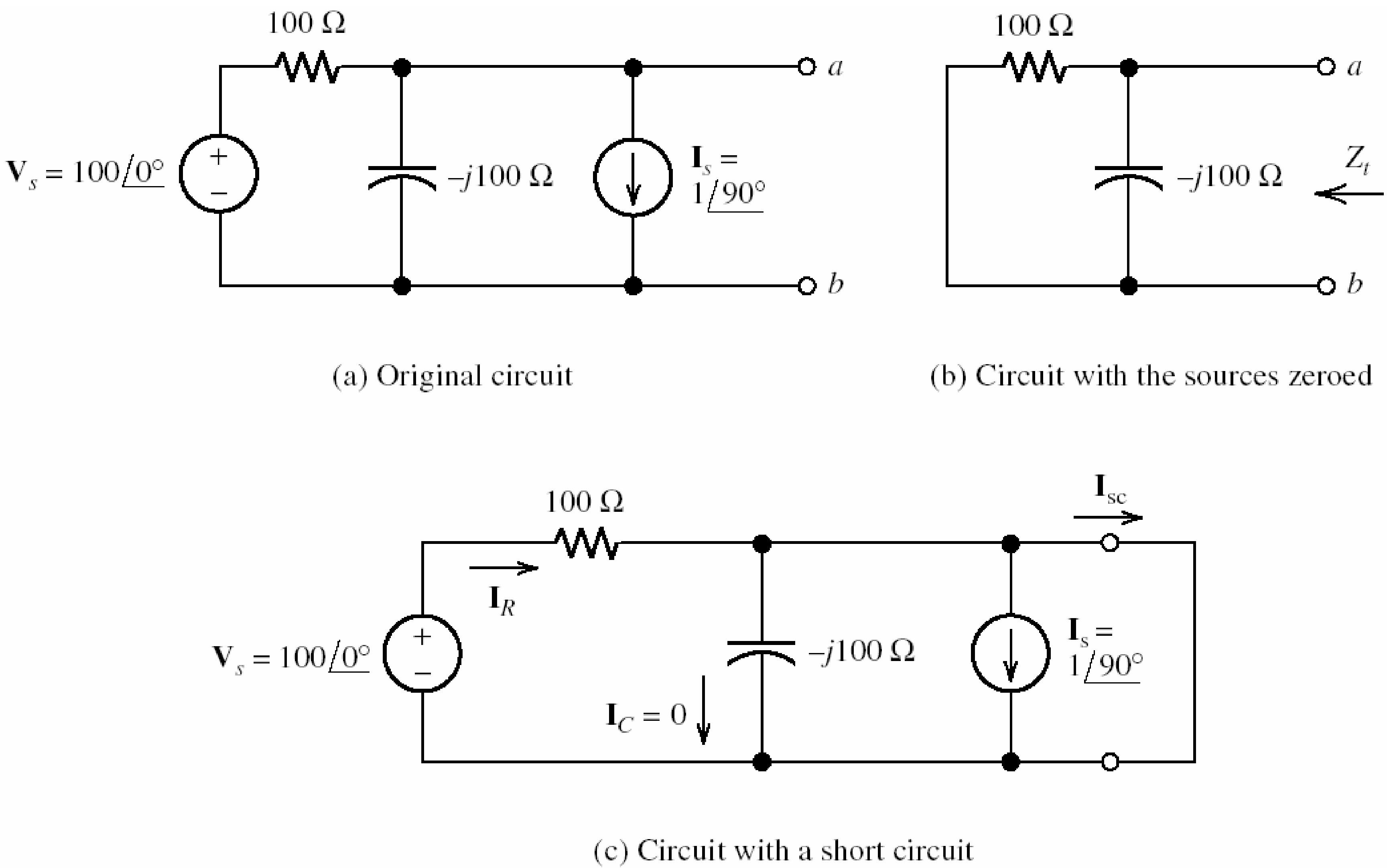


Figure 5.30 The Norton equivalent circuit consists of a phasor current source $\mathbf{I}_n$ in parallel with the complex impedance Z_t.



(a) Original circuit

(b) Circuit with the sources zeroed

(c) Circuit with a short circuit

Figure 5.31 Circuit of Example 5.9.

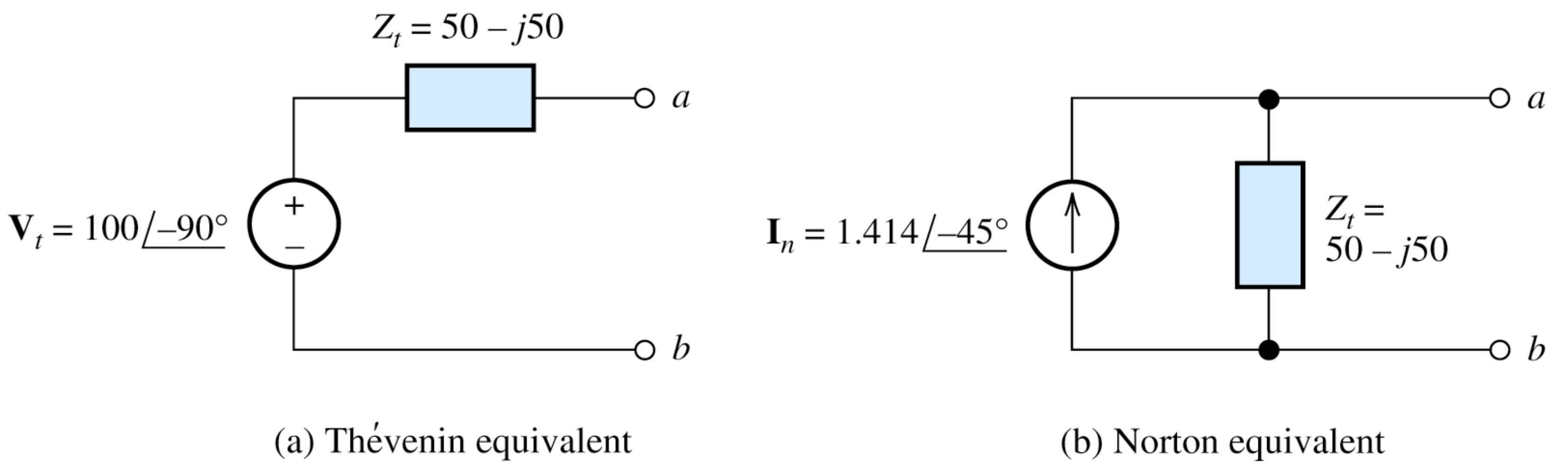


Figure 5.32 Thévenin and Norton equivalents for the circuit of Figure 5.31a.

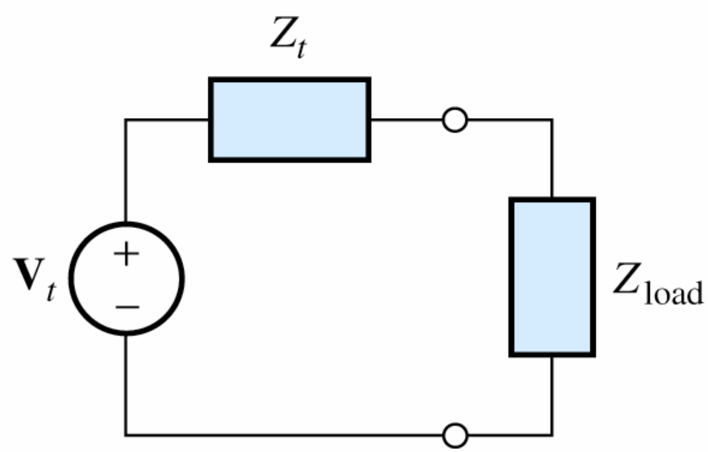


Figure 5.33 The Thévenin equivalent of a two-terminal circuit delivering power to a load impedance.

Maximum Average Power Transfer

If the load can take on any complex value, maximum power transfer is attained for a load impedance equal to the complex conjugate of the Thévenin impedance.

If the load is required to be a pure resistance, maximum power transfer is attained for a load resistance equal to the magnitude of the Thévenin impedance.

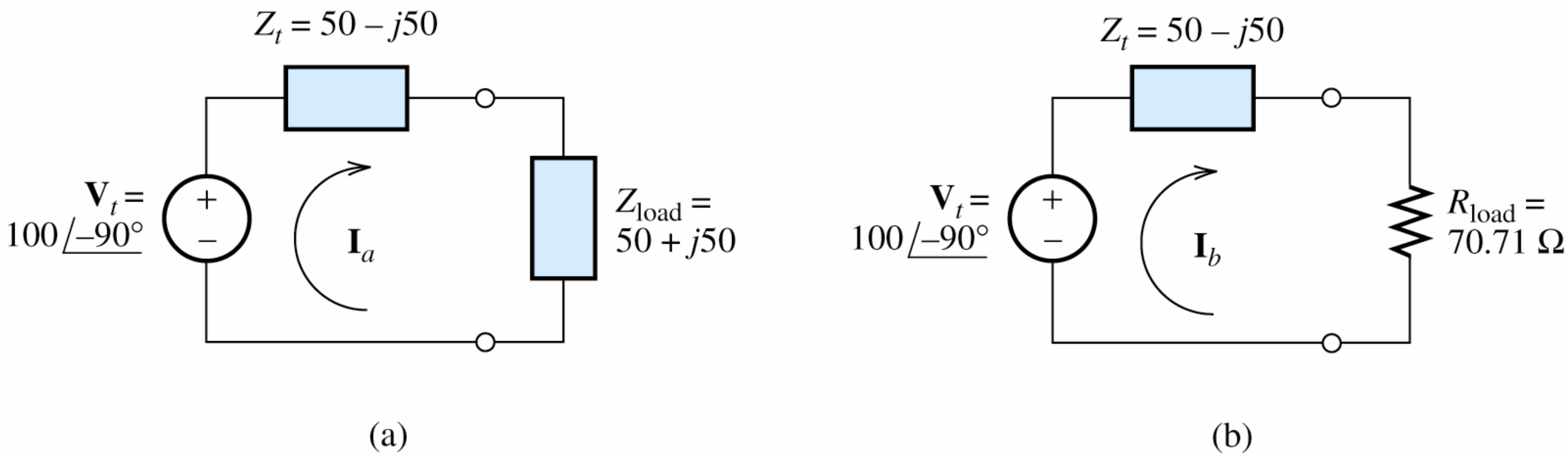


Figure 5.34 Thévenin equivalent circuit and loads of Example 5.10.

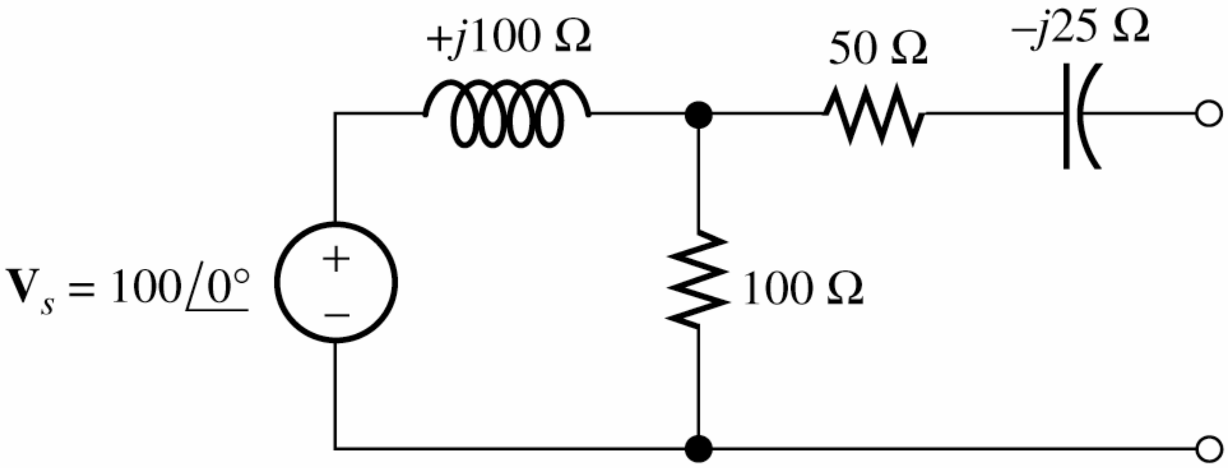
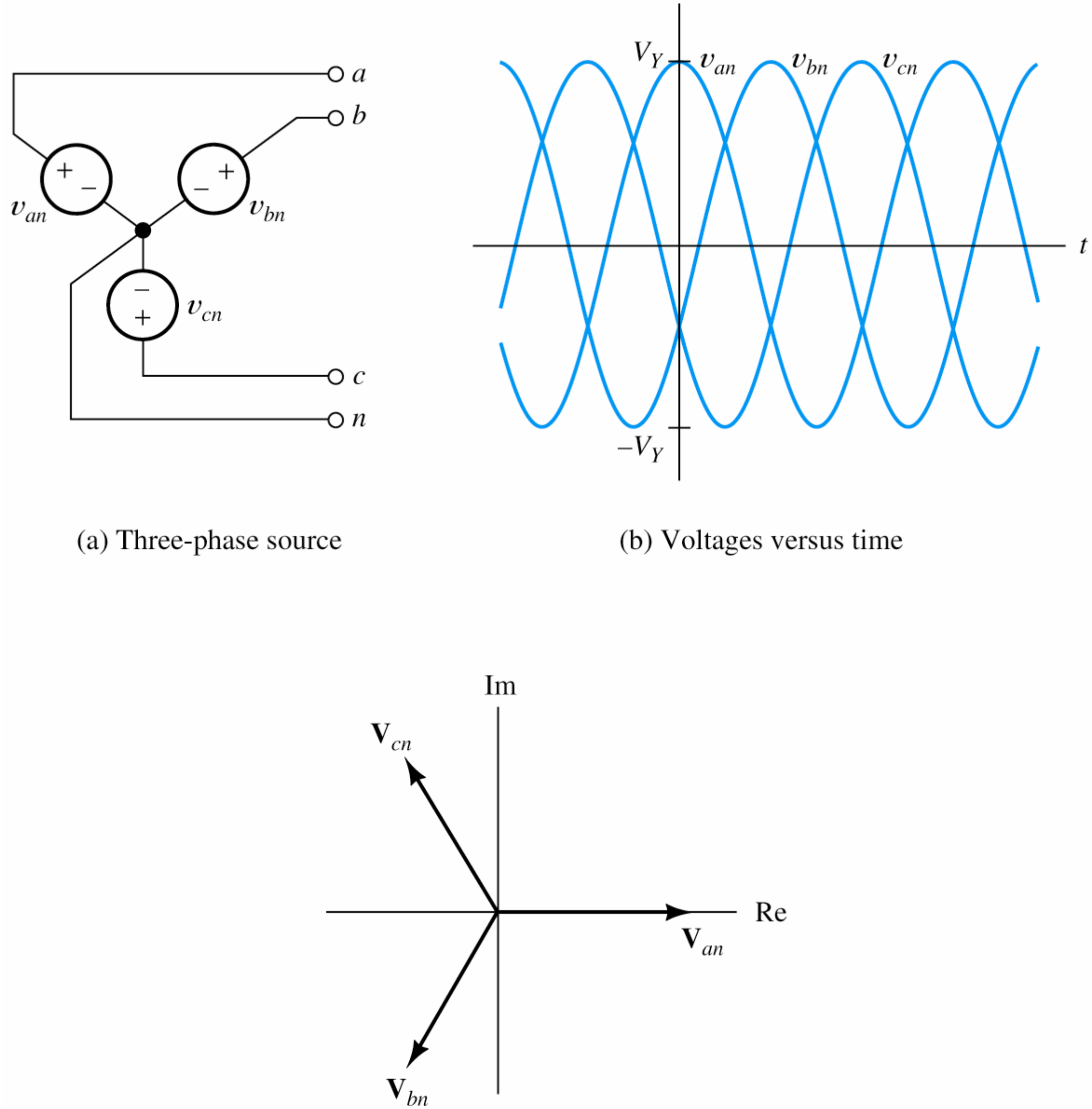


Figure 5.35 Circuit of Exercises 5.14 and 5.15.

BALANCED THREE-PHASE CIRCUITS

Much of the power used by business and industry is supplied by three-phase distribution systems. Plant engineers need to be familiar with three-phase power.



(a) Three-phase source

(b) Voltages versus time

(c) Phasor diagram

Figure 5.36 A balanced three-phase voltage source.

Phase Sequence

Three-phase sources can have either a positive or negative phase sequence.

The direction of rotation of certain three-phase motors can be reversed by changing the phase sequence.

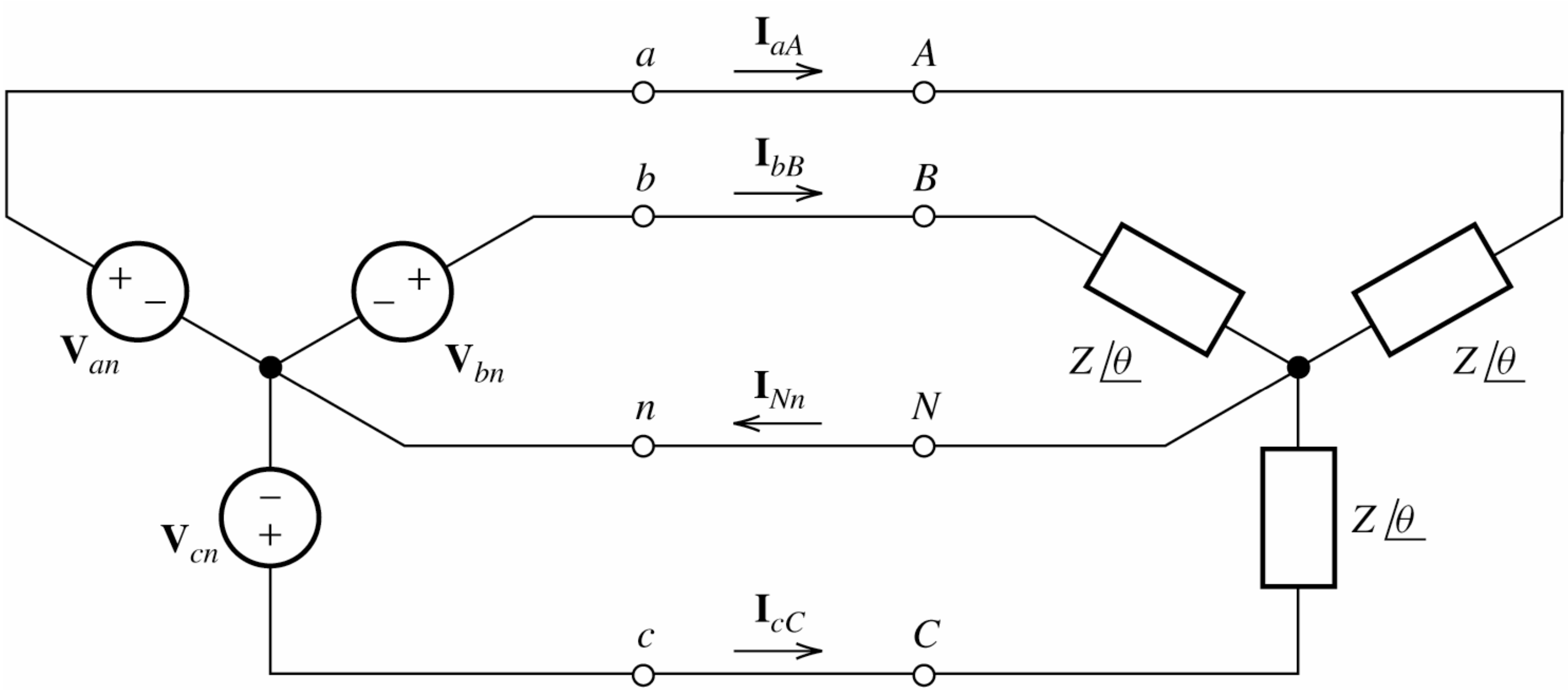


Figure 5.37 A three-phase wye–wye connection with neutral.

Wye–Wye Connection

Three-phase sources and loads can be connected either in a wye configuration or in a delta configuration.

The key to understanding the various three-phase configurations is a careful examination of the wye–wye circuit.

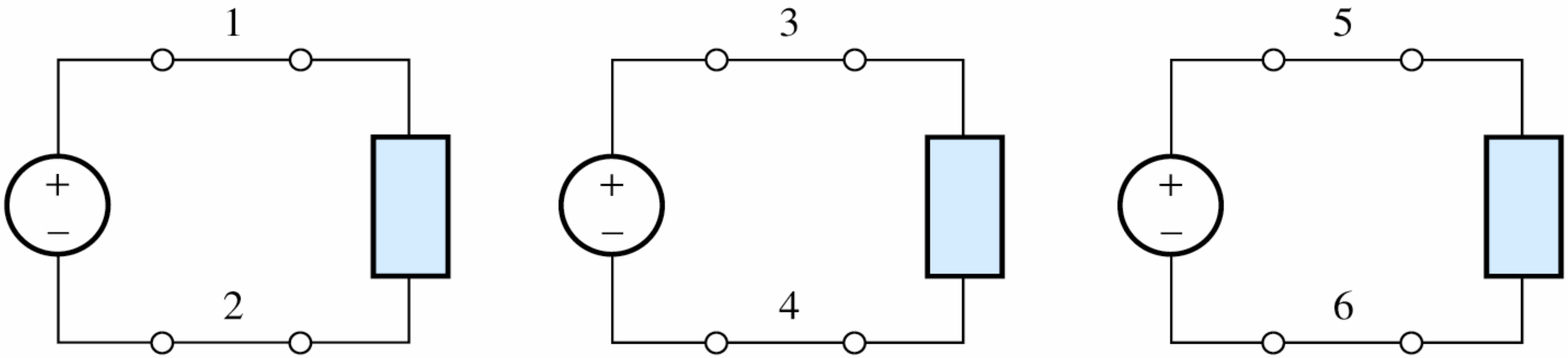


Figure 5.38 Six wires are needed to connect three single-phase sources to three loads. In a three-phase system, the same power transfer can be accomplished with three wires.

$$P_{\mathrm{avg}} = p(t) = 3V_{Y\mathrm{rms}} I_{L\mathrm{rms}} \cos(\theta)$$

$$Q = 3\frac{V_Y I_L}{2}\sin(\theta) = 3V_{Y\mathrm{rms}} I_{L\mathrm{rms}} \sin(\theta)$$

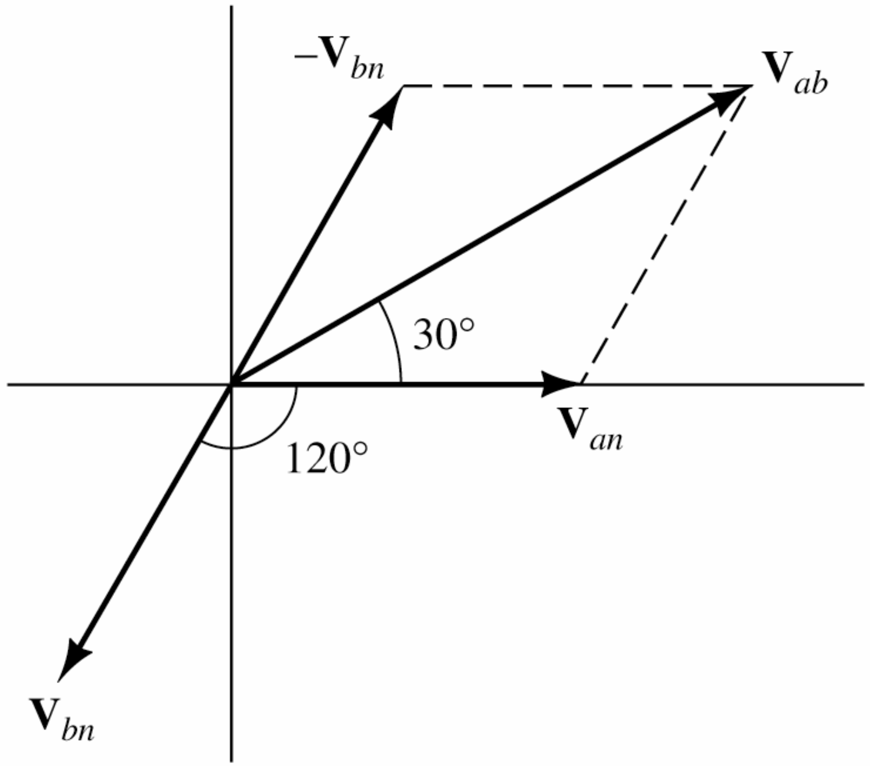
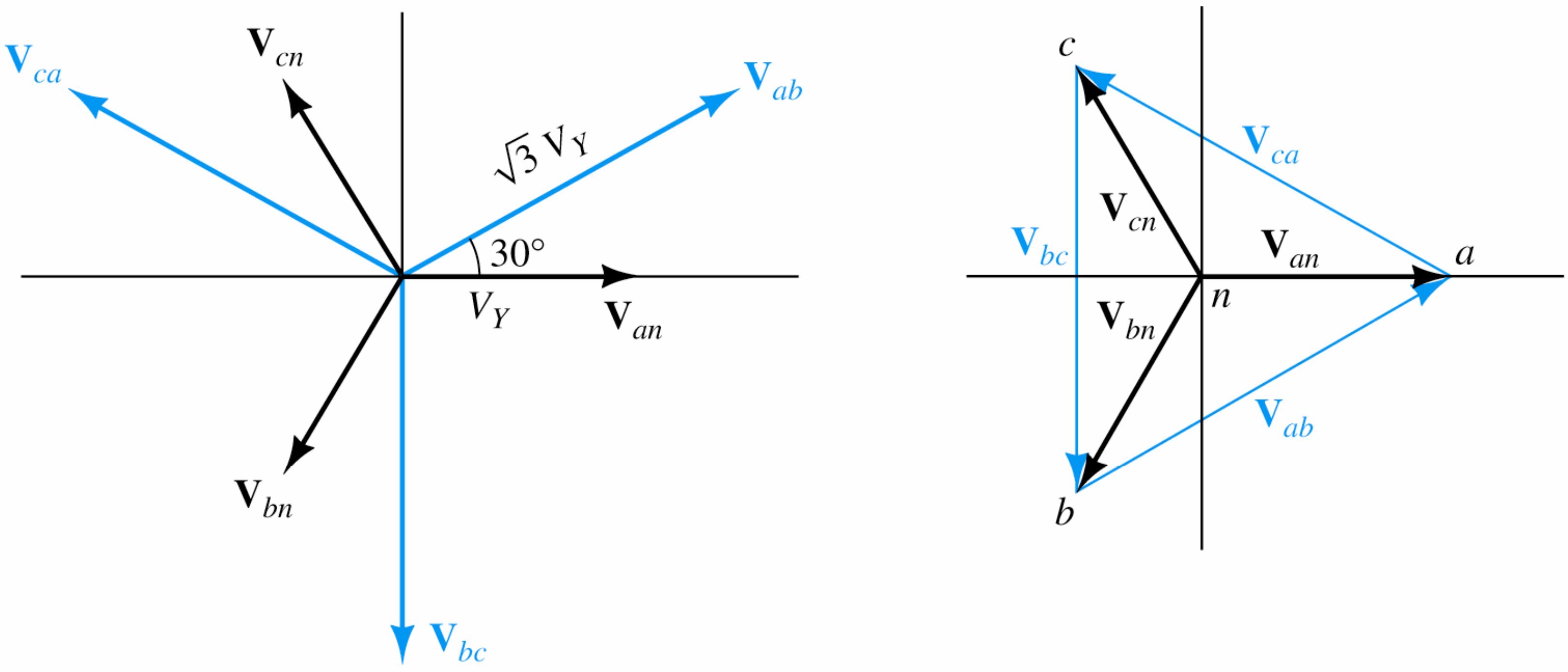


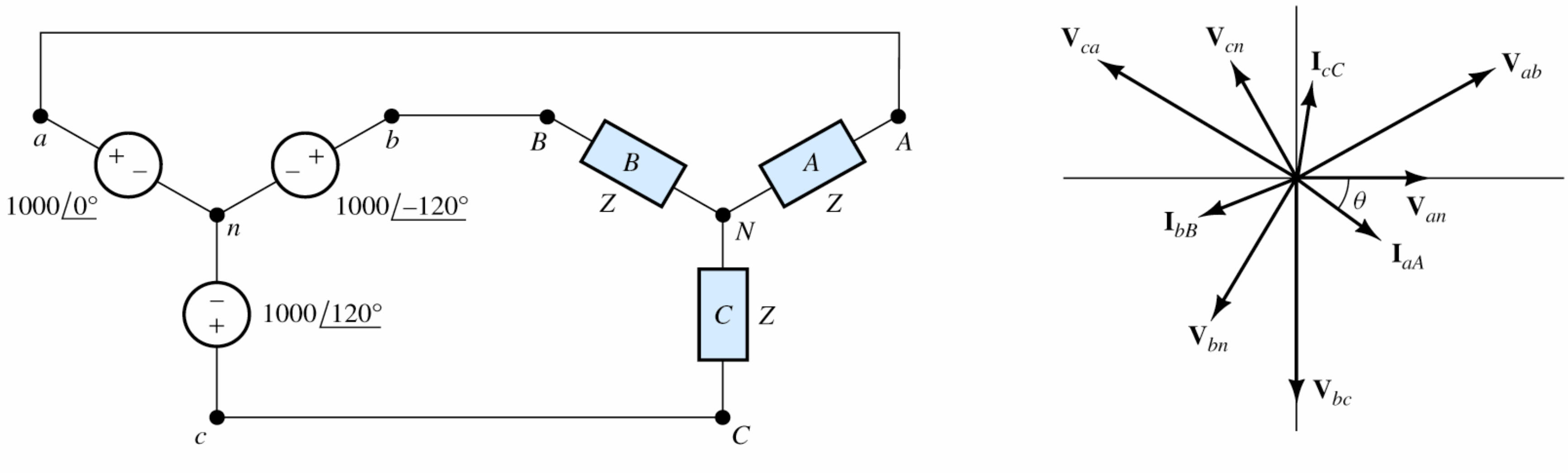
Figure 5.39 Phasor diagram showing the relationship between the line-to-line voltage $\mathbf{v}_{ab}$ and the line-to-neutral voltages $\mathbf{v}_{an}$ and $\mathbf{v}_{bn}$.



(a) All phasors starting from the origin

(b) A more intuitive way to draw the phasor diagram

Figure 5.40 Phasor diagram showing line-to-line voltages and line-to-neutral voltages.



(a) Circuit diagram

(b) Phasor diagram

Figure 5.41 Circuit and phasor diagram for Example 5.11.

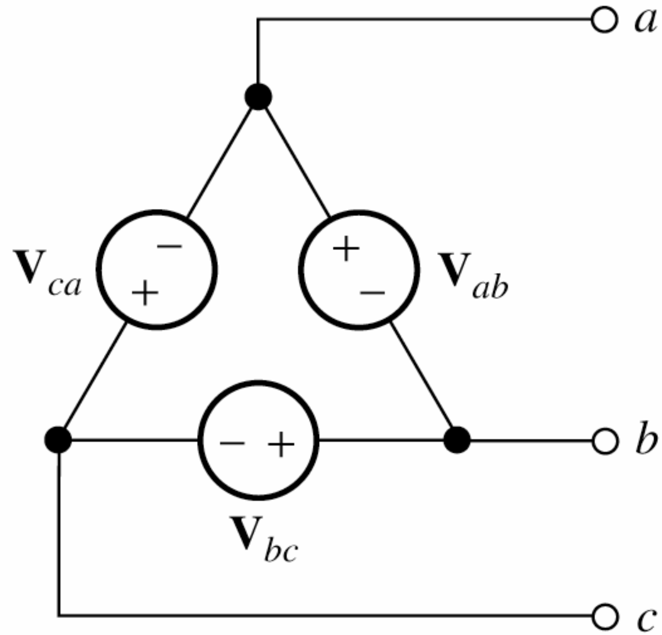


Figure 5.42 Delta-connected three-phase source.

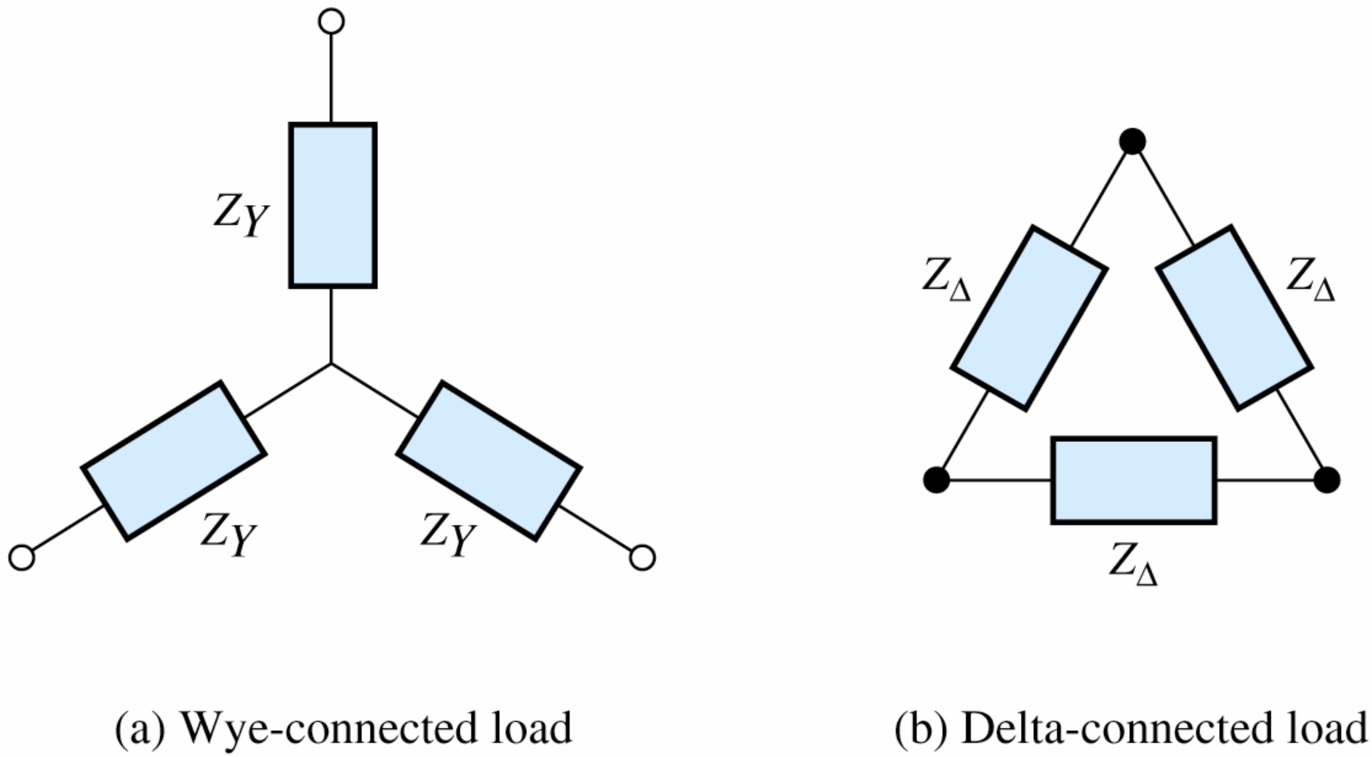
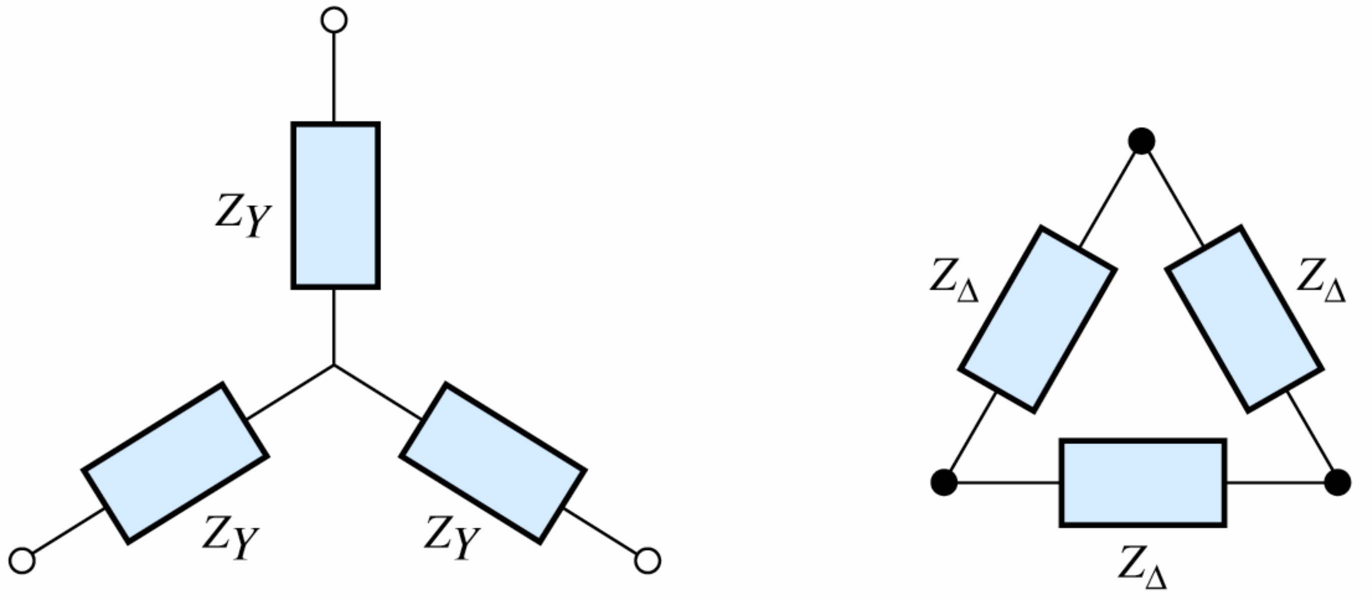


Figure 5.43 Loads can be either wye-connected or delta-connected.

$$Z_\Delta = 3Z_Y$$



(a) Wye-connected load

(b) Delta-connected load

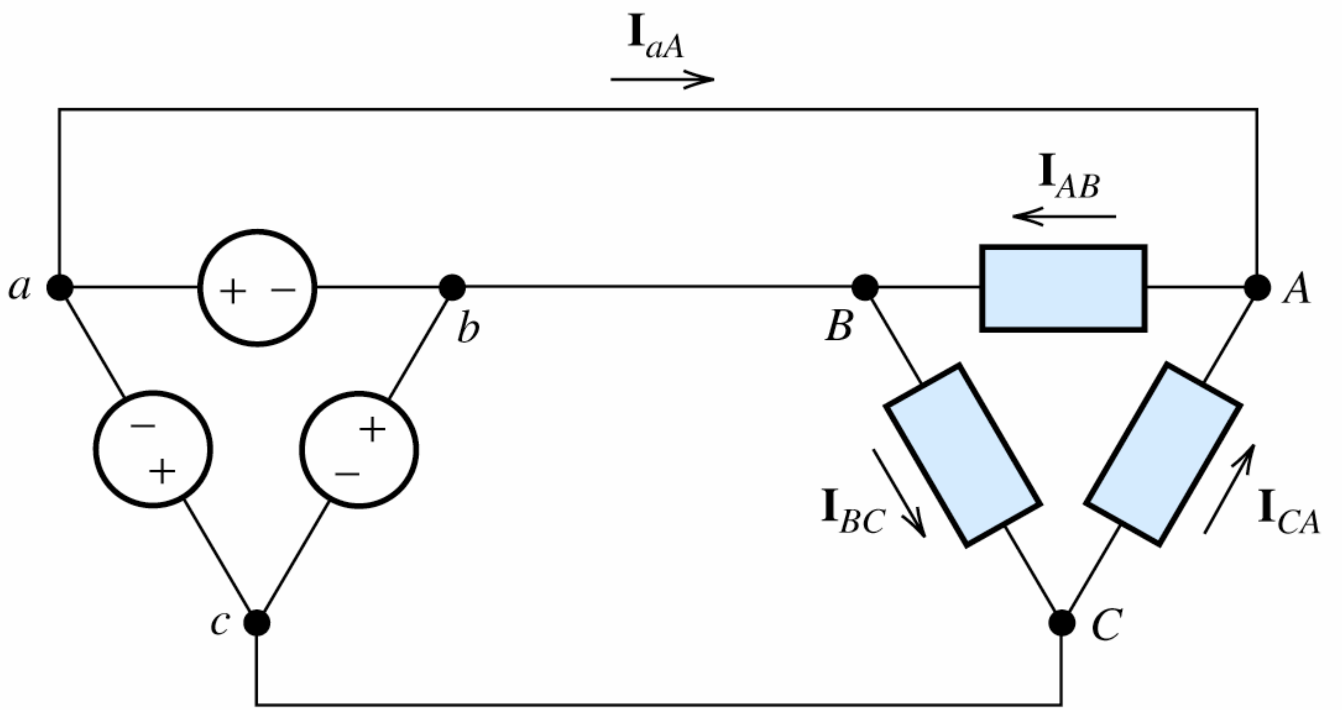
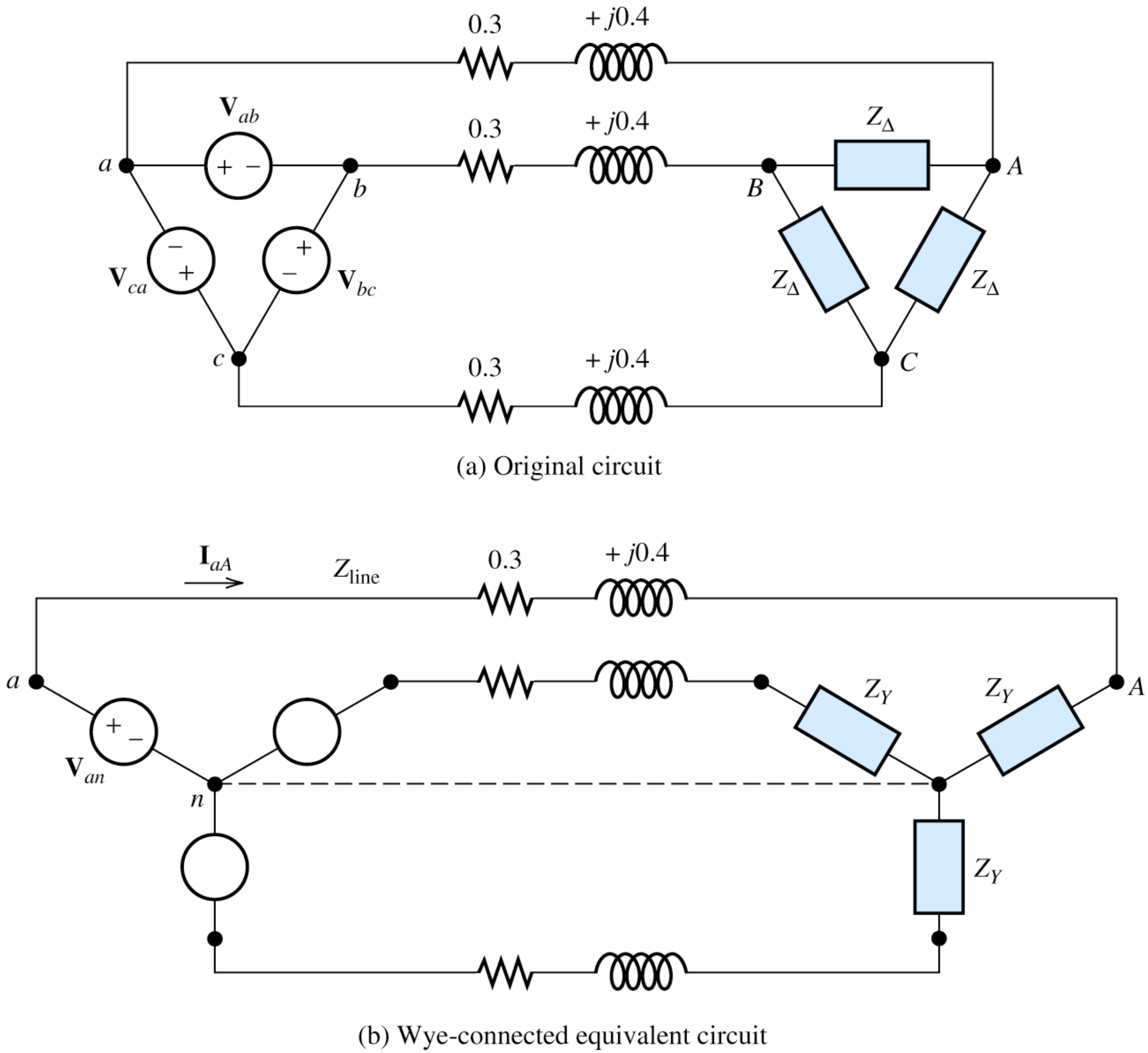


Figure 5.44 A delta-connected source delivering power to a delta-connected load.



(a) Original circuit

(b) Wye-connected equivalent circuit

Figure 5.45 Circuit of Example 5.12.