

Chapter 3

Inductance and Capacitance

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1. Find the current (voltage) for a capacitance or inductance given the voltage (current) as a function of time.
2. Compute the capacitances of parallel-plate capacitors.

3. Compute the stored energies in capacitances or inductances.

4. Describe typical physical construction of capacitors and inductors and identify parasitic effects.

5. Find the voltages across mutually coupled inductances in terms of the currents.

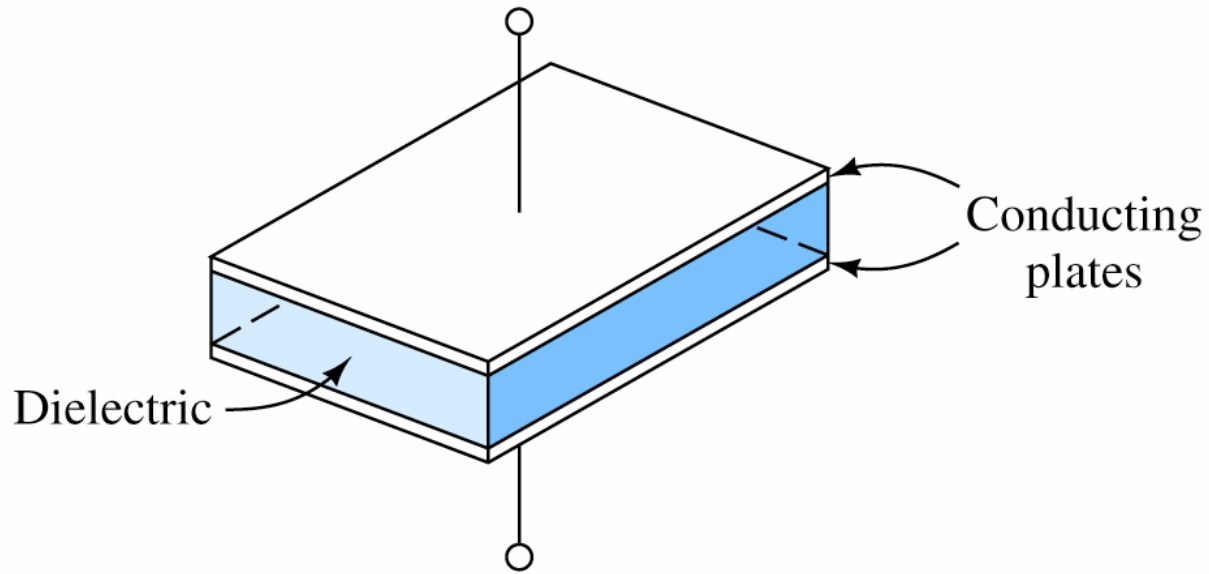


Figure 3.1 A parallel-plate capacitor consists of two conductive plates separated by a dielectric layer.

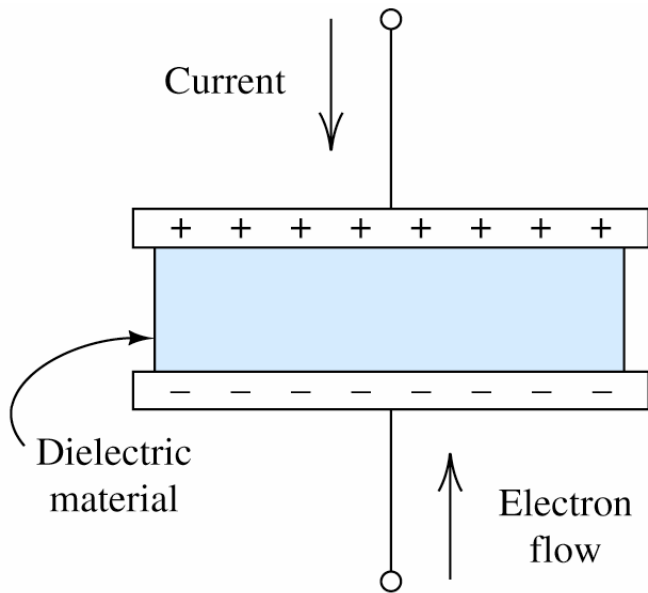
CAPACITANCE

$$q = Cv$$

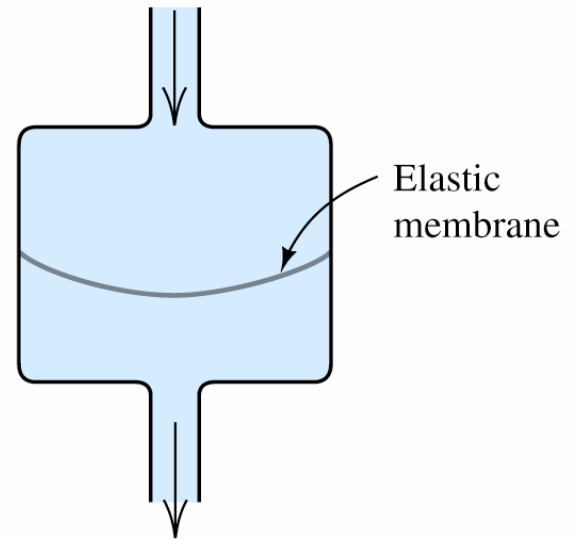
$$q(t) = \int_{t_0}^t i(t)dt + q(t_0)$$

$$i = C \frac{dv}{dt}$$

$$v(t) = \frac{1}{C} \int_{t_0}^t i(t)dt + v(t_0)$$

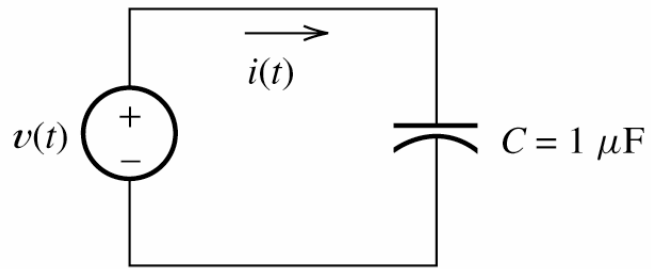


(a) As current flows through a capacitor, charges of opposite sign collect on the respective plates

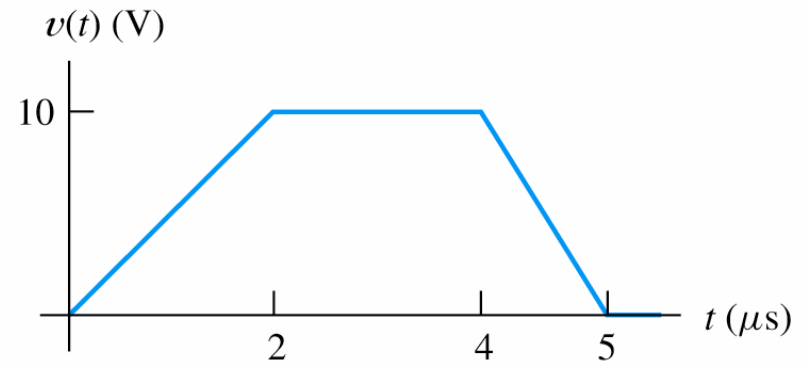


(b) Fluid-flow analogy for capacitance

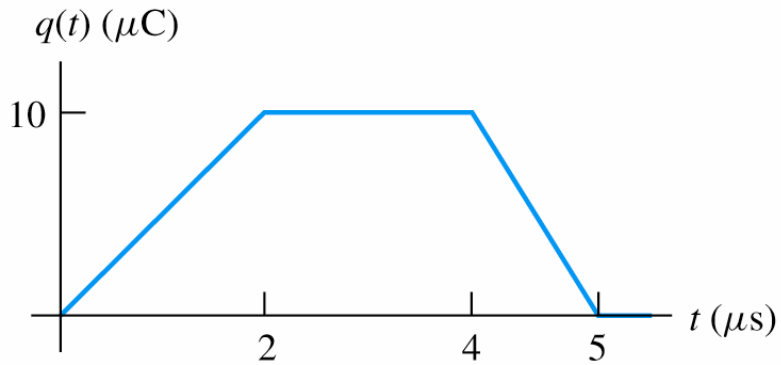
Figure 3.2 A capacitor and its fluid-flow analogy.



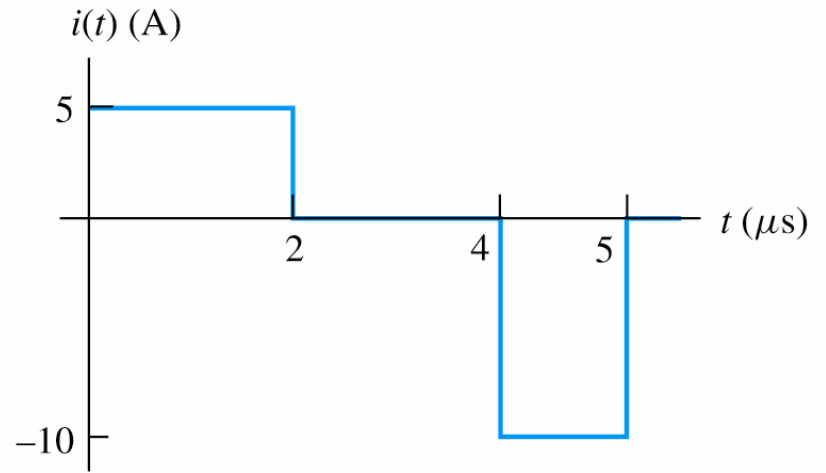
(a)



(b)

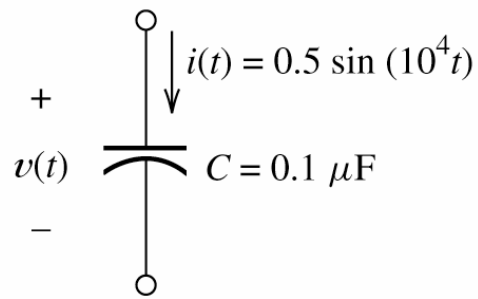


(c)

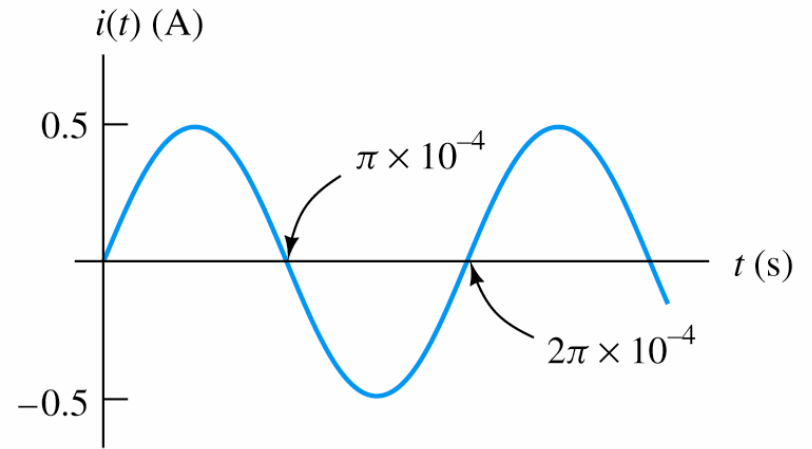


(d)

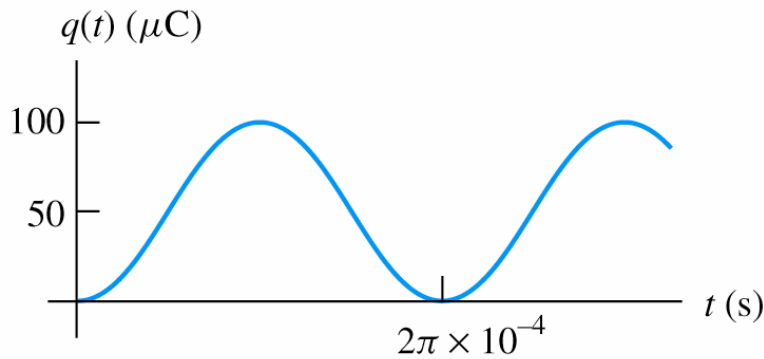
Figure 3.4 Circuit and waveforms for Example 3.1.



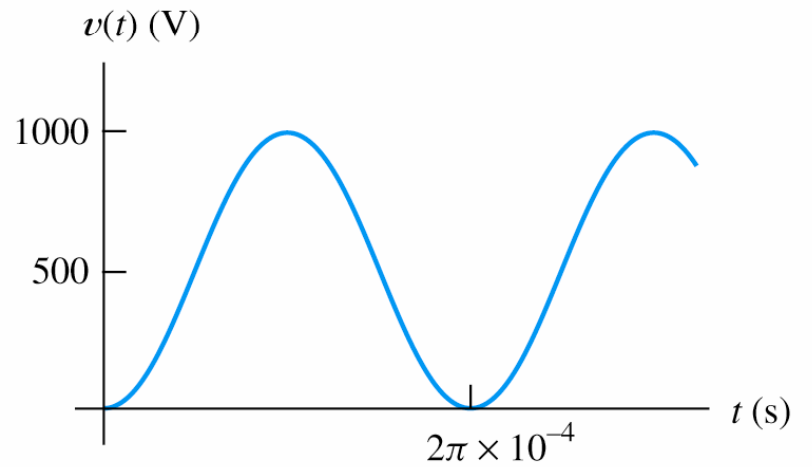
(a)



(b)

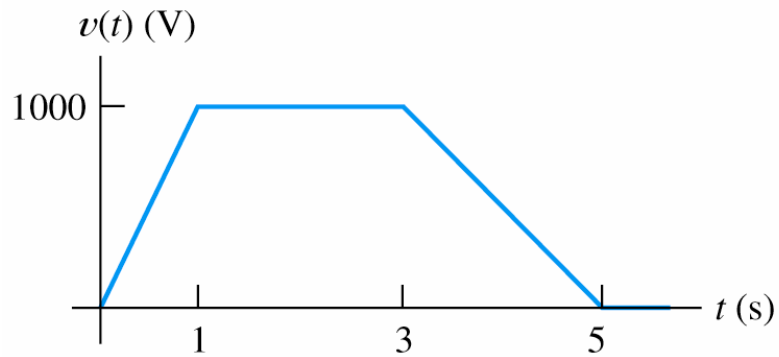


(c)

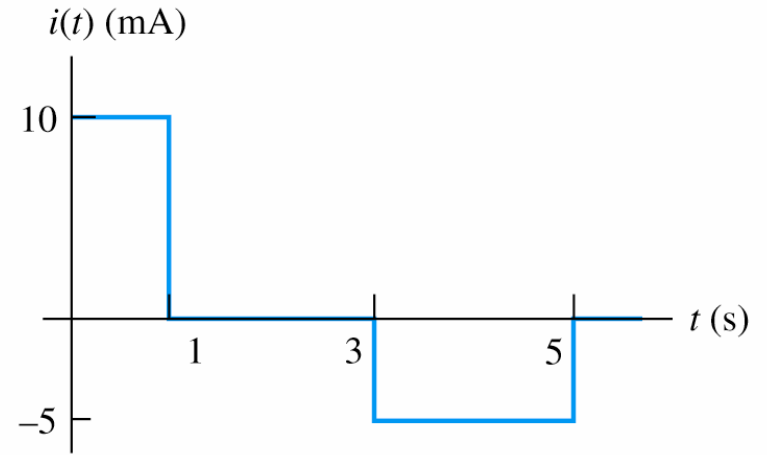


(d)

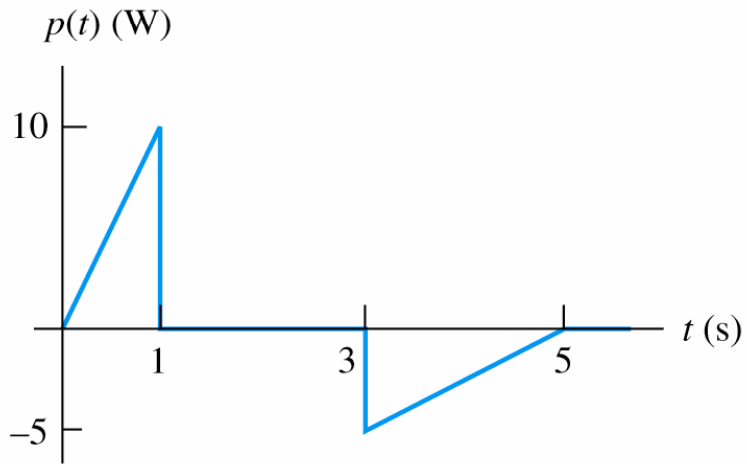
Figure 3.5 Waveforms for Example 3.2.



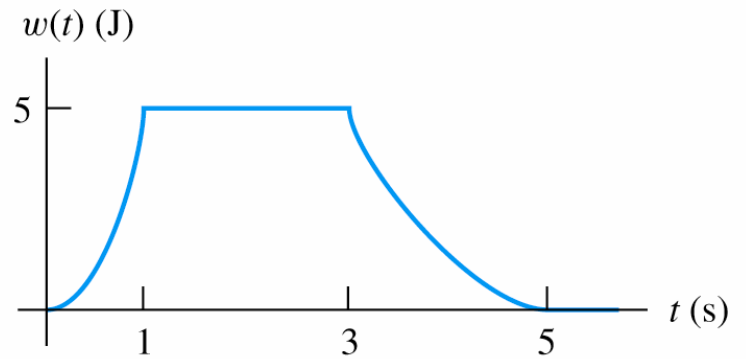
(a)



(b)



(c)



(d)

Figure 3.6 Waveforms for Example 3.3.

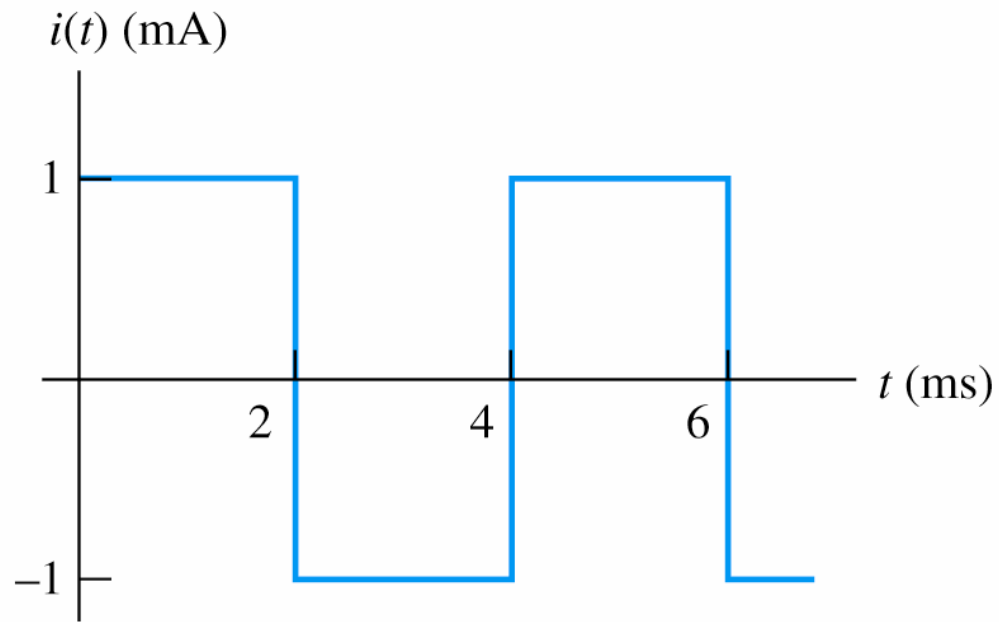
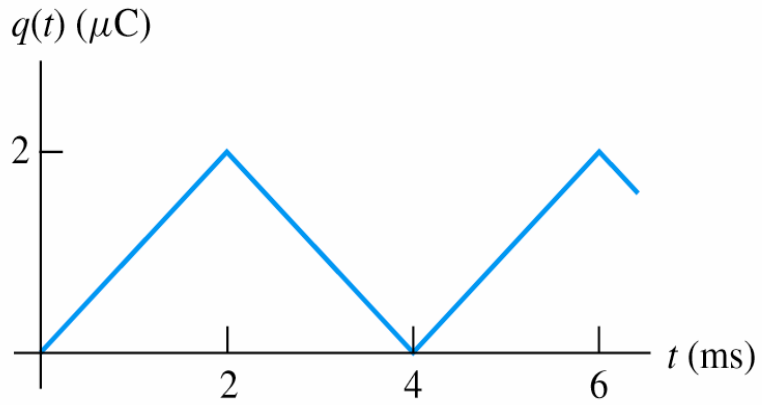
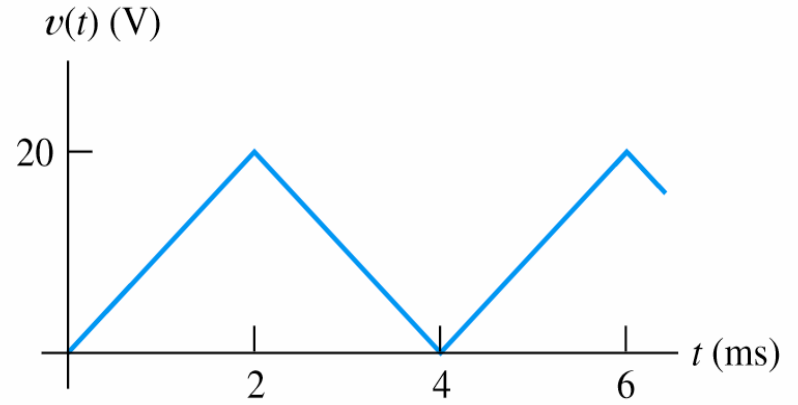


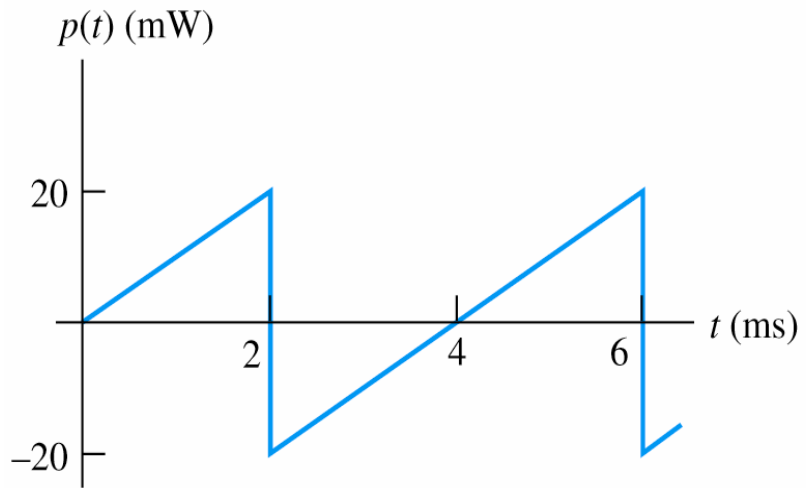
Figure 3.7 Square-wave current for Exercise 3.2.



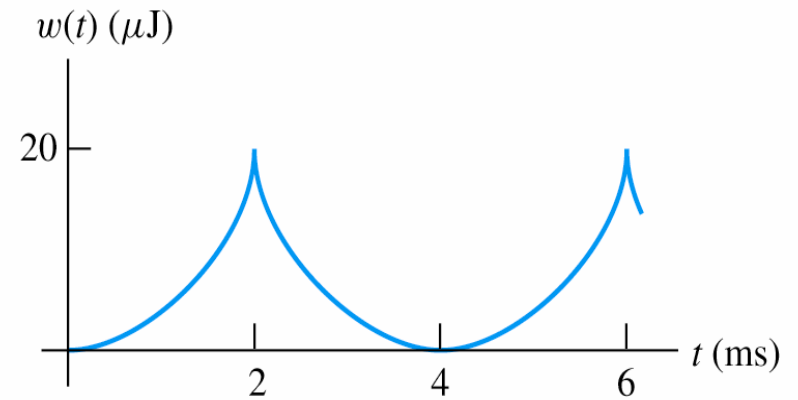
(a)



(b)

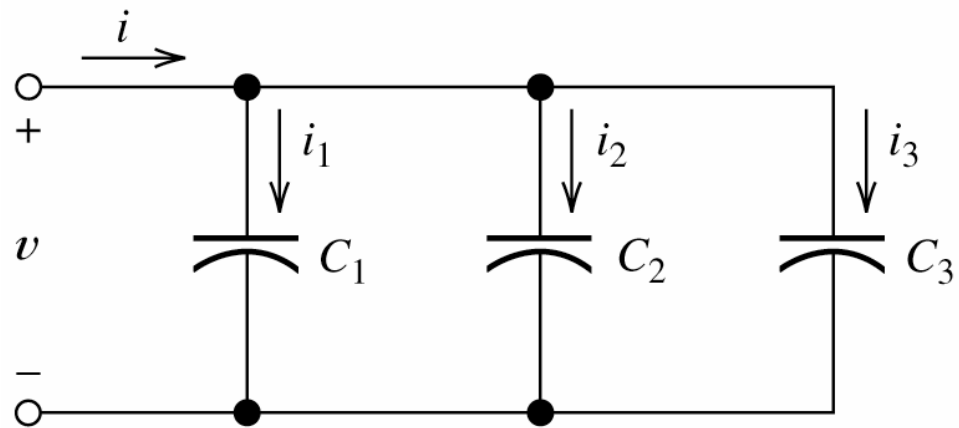


(c)



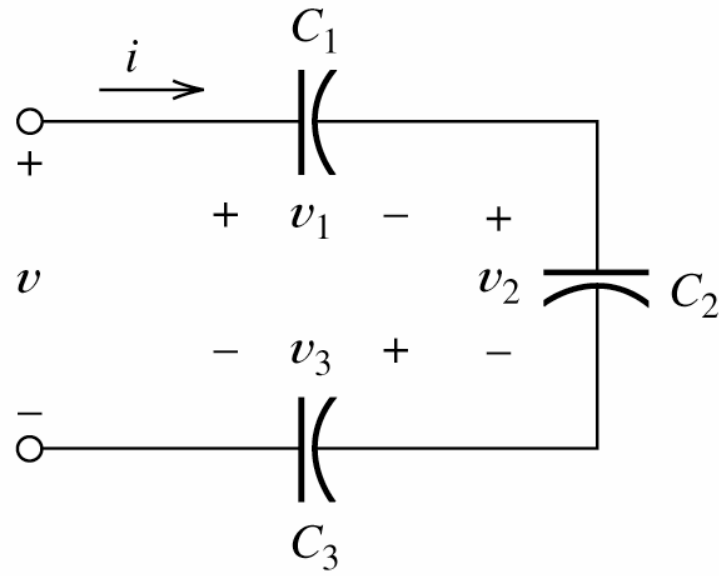
(d)

Figure 3.8 Answers for Exercise 3.2.



$$C_{\text{eq}} = C_1 + C_2 + C_3$$

Figure 3.9 Three capacitances in parallel.



$$C_{\text{eq}} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

Figure 3.10 Three capacitances in series.

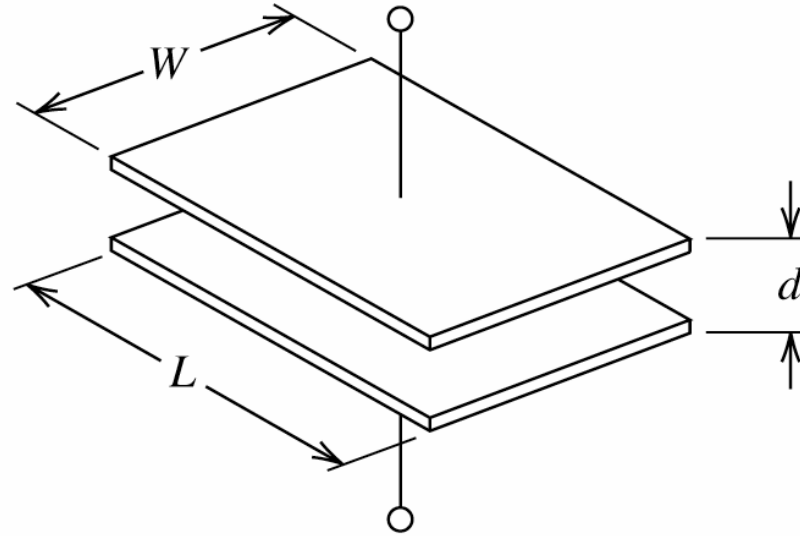
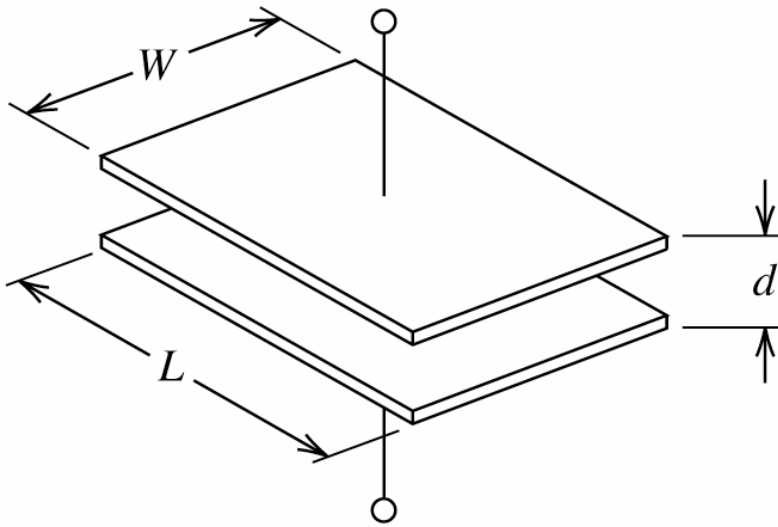


Figure 3.11 A parallel-plate capacitor including dimensions.

Capacitance of the Parallel-Plate Capacitor



$$C = \frac{\epsilon A}{d} \quad A = WL$$

$$\epsilon_0 \cong 8.85 \times 10^{-12} \text{ F/m}$$

$$\epsilon = \epsilon_r \epsilon_0$$

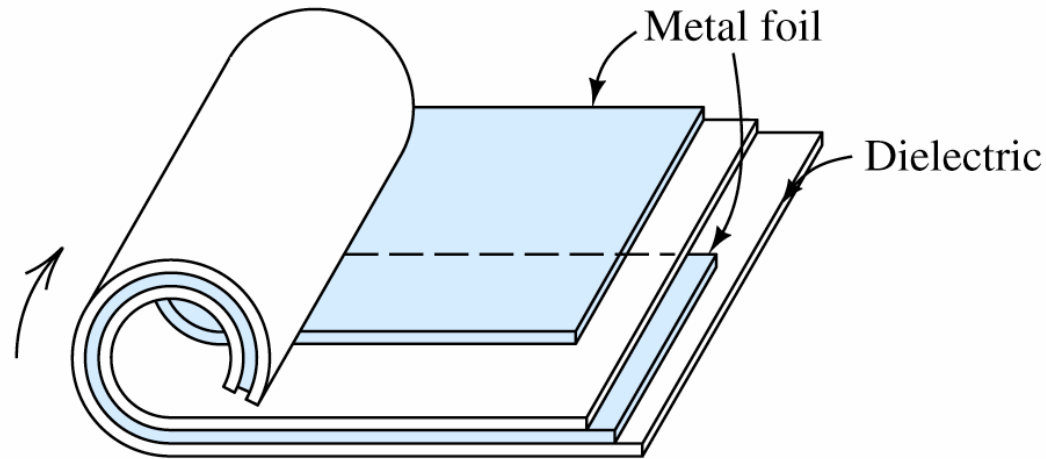


Figure 3.12 Practical capacitors can be constructed by interleaving the plates with two dielectric layers and rolling them up. By staggering the plates, connection can be made to one plate at each end of the roll.

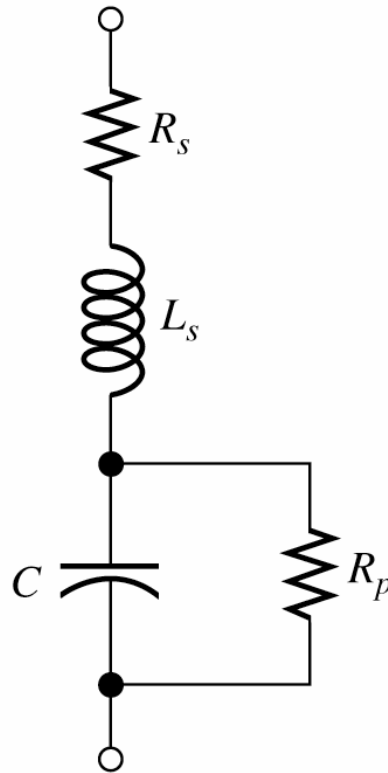


Figure 3.13 The circuit model for a capacitor including the parasitic elements R_s , L_s , and R_p .

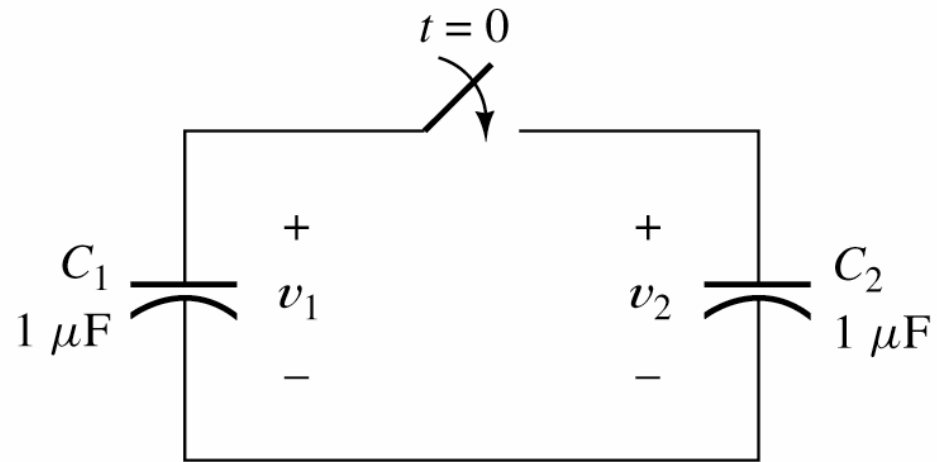
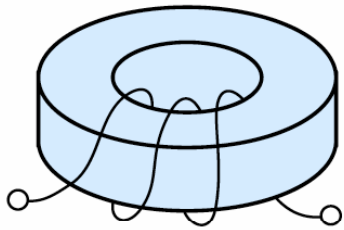
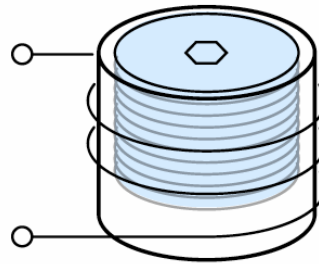


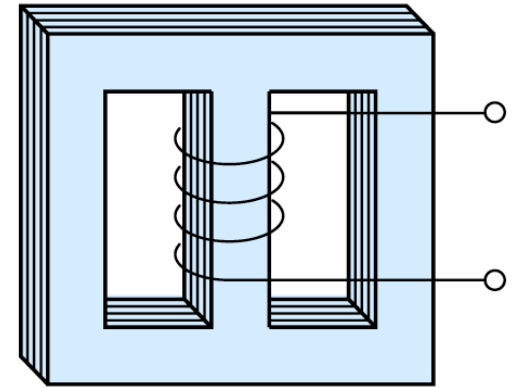
Figure 3.14 See Example 3.5.



(a) Toroidal inductor

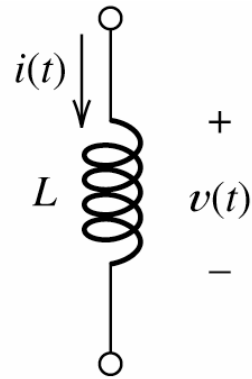


(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance



(c) Inductor with a laminated iron core

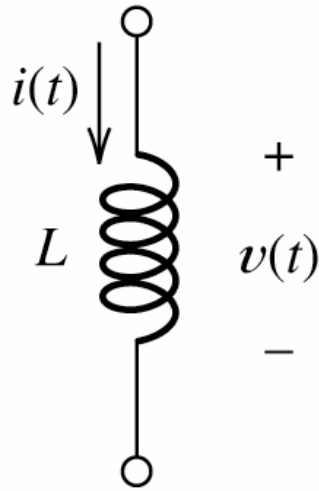
Figure 3.15 An inductor is constructed by coiling a wire around some type of form.



$$v(t) = L \frac{di}{dt}$$

Figure 3.16 Circuit symbol and the $v - i$ relationship for inductance.

INDUCTANCE

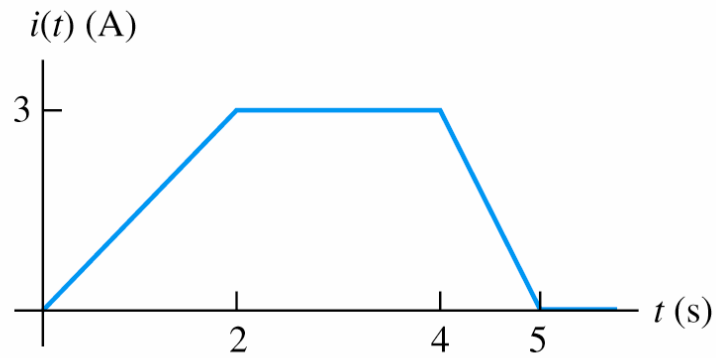


$$v(t) = L \frac{di}{dt}$$

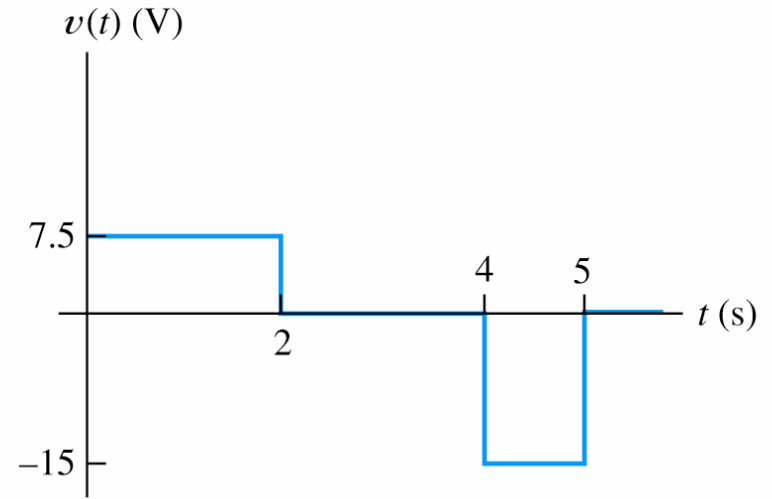
$$v(t) = L \frac{di}{dt}$$

$$i(t) = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$

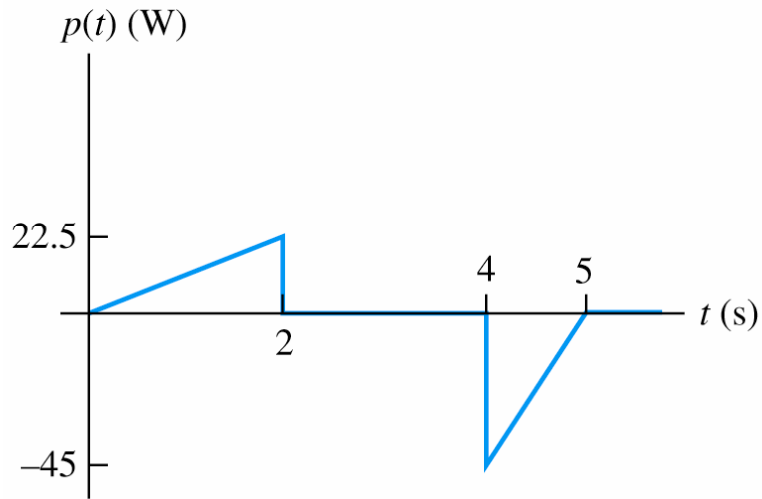
$$w(t) = \frac{1}{2} Li^2(t)$$



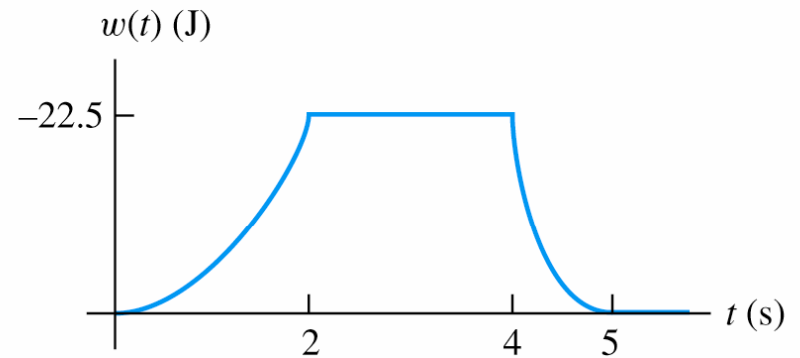
(a)



(b)

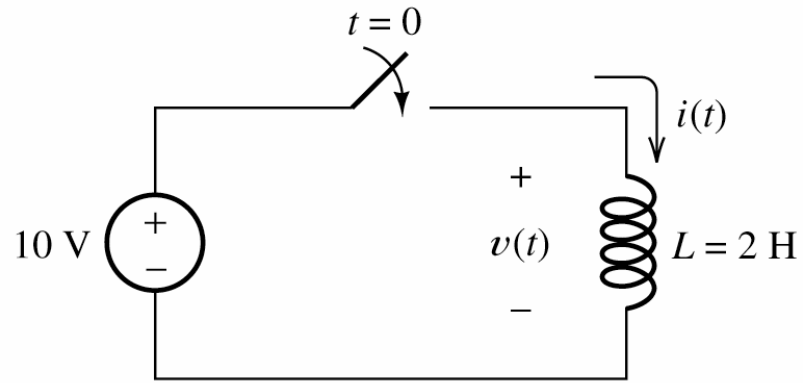


(c)

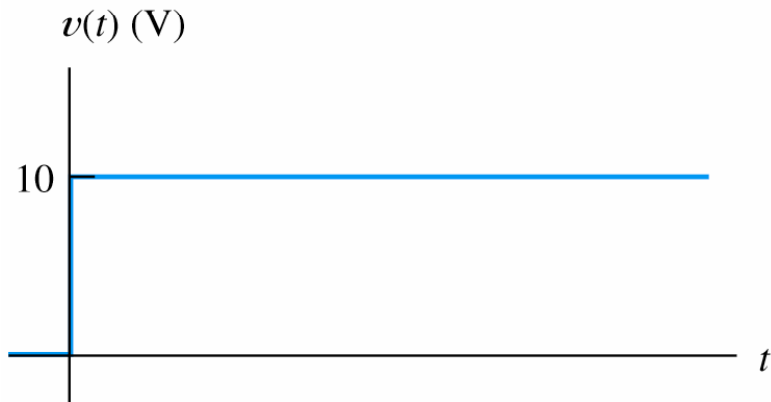


(d)

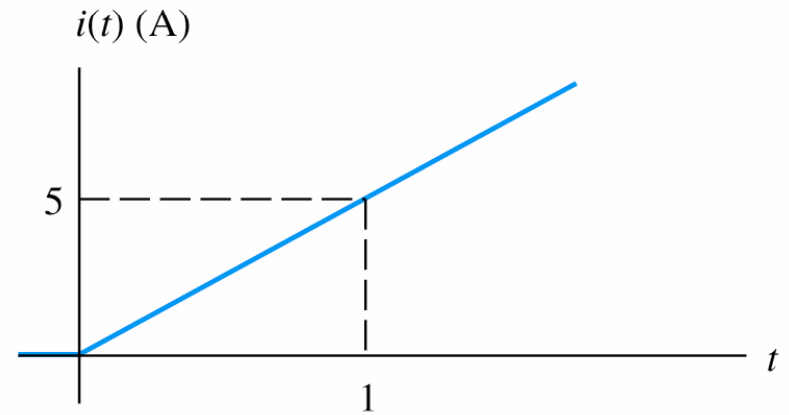
Figure 3.17 Waveforms for Example 3.6.



(a)

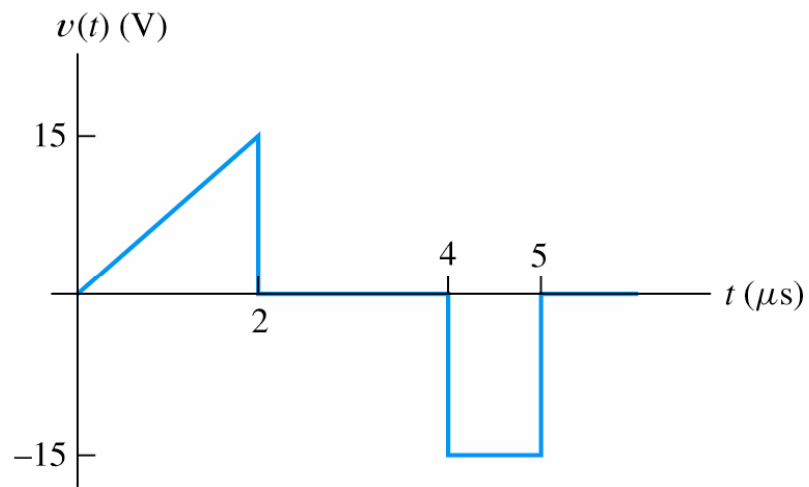


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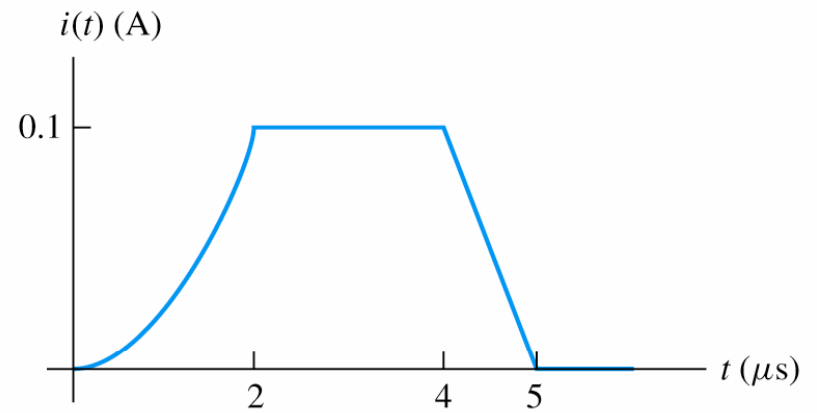


(c)

Figure 3.18 Circuit and waveforms for Example 3.7.



(a)



(b)

Figure 3.19 See Exercise 3.7.

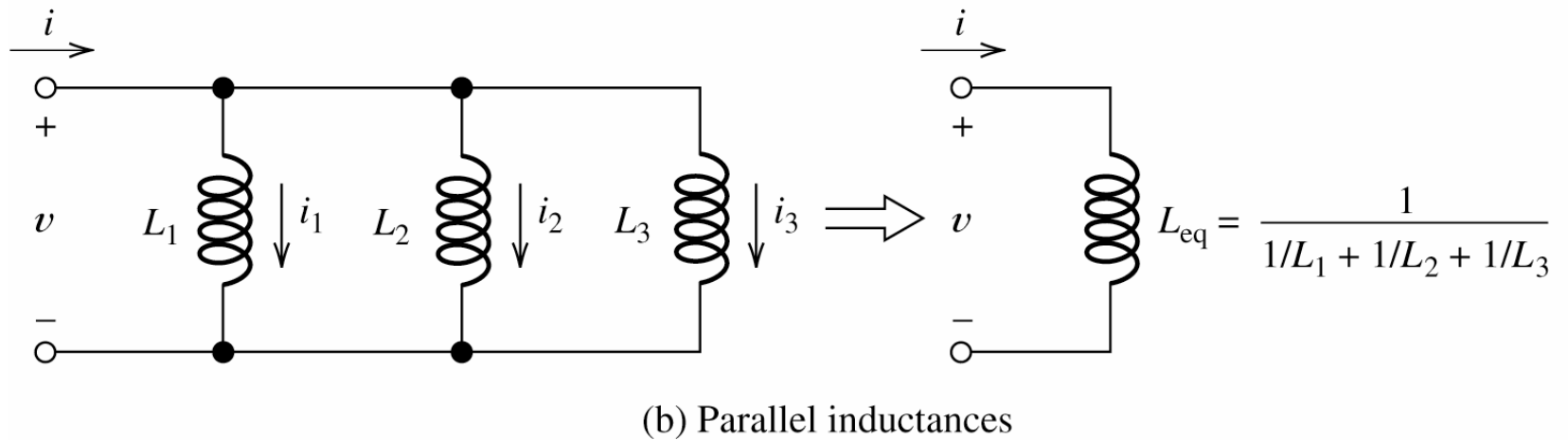
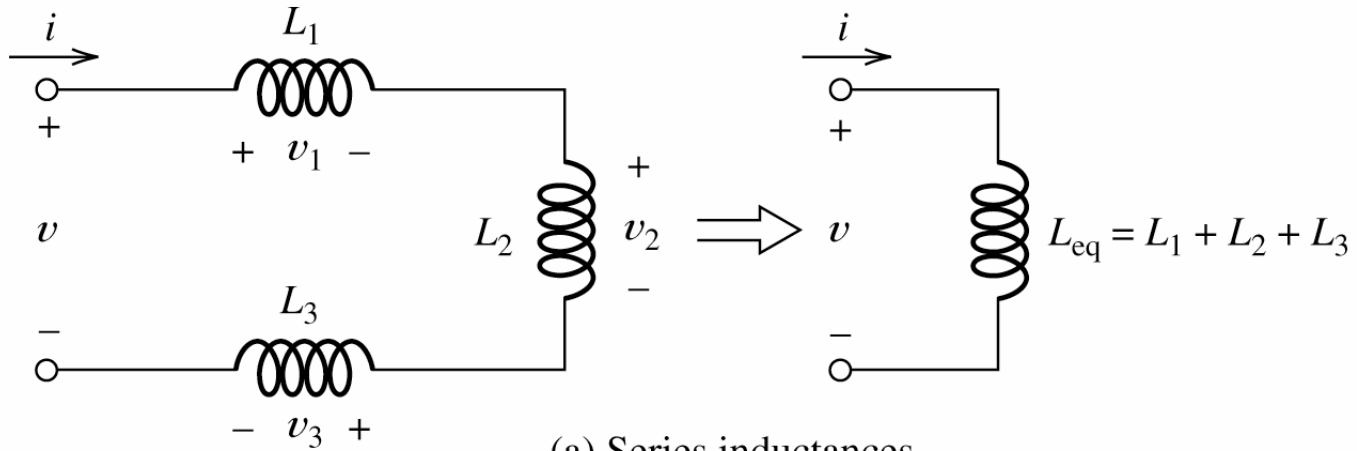
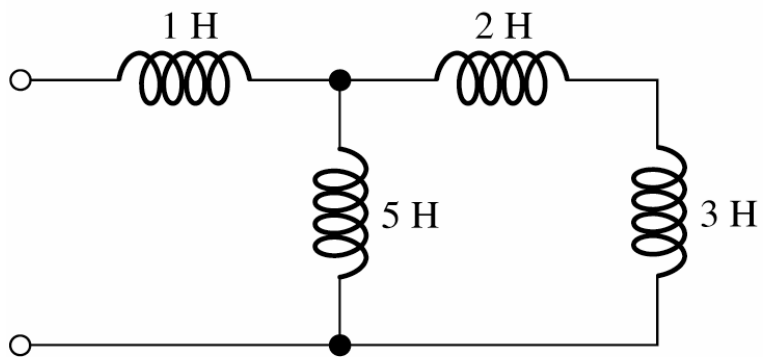
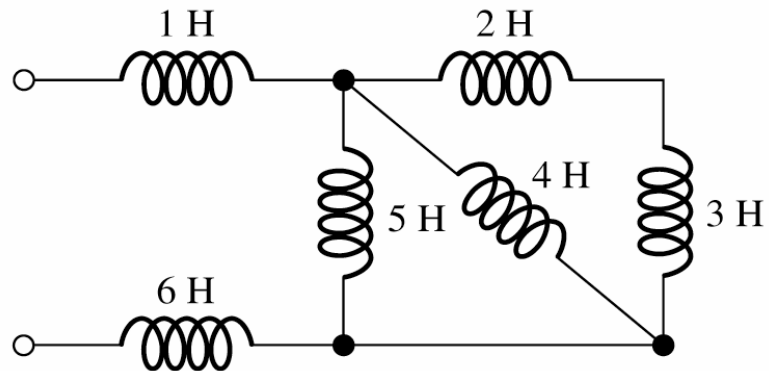


Figure 3.20 Inductances in series and parallel are combined in the same manner as resistances.



(a)



(b)

Figure 3.21 See Exercise 3.10.

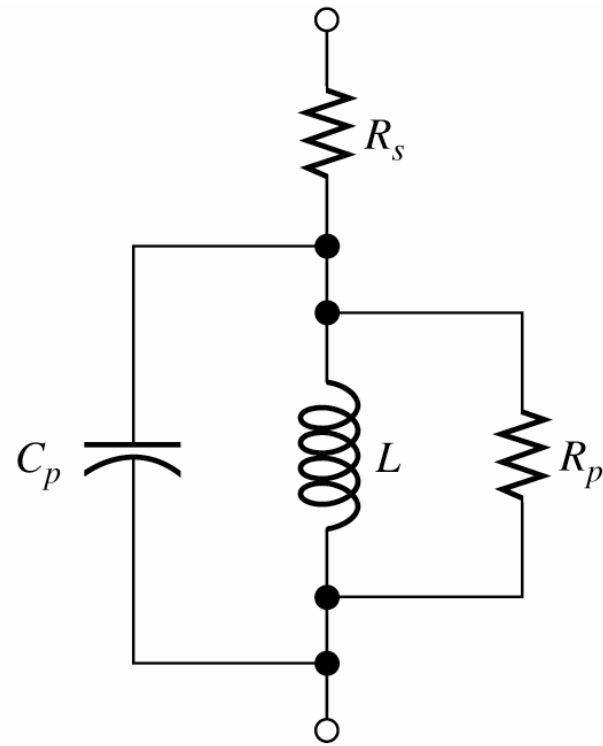
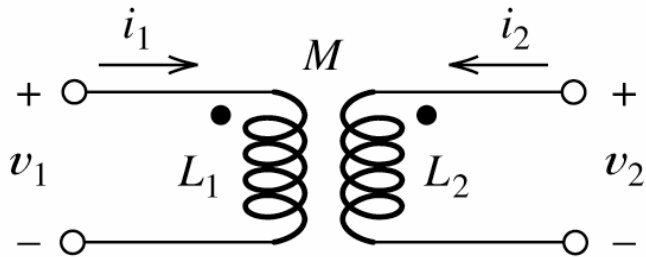


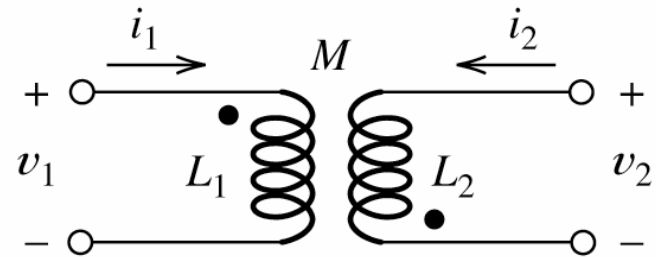
Figure 3.22 Circuit model for real inductors including several parasitic elements.



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

(a)



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

(b)

Figure 3.23 Circuit symbols and $v - i$ relationships for mutually coupled inductances.

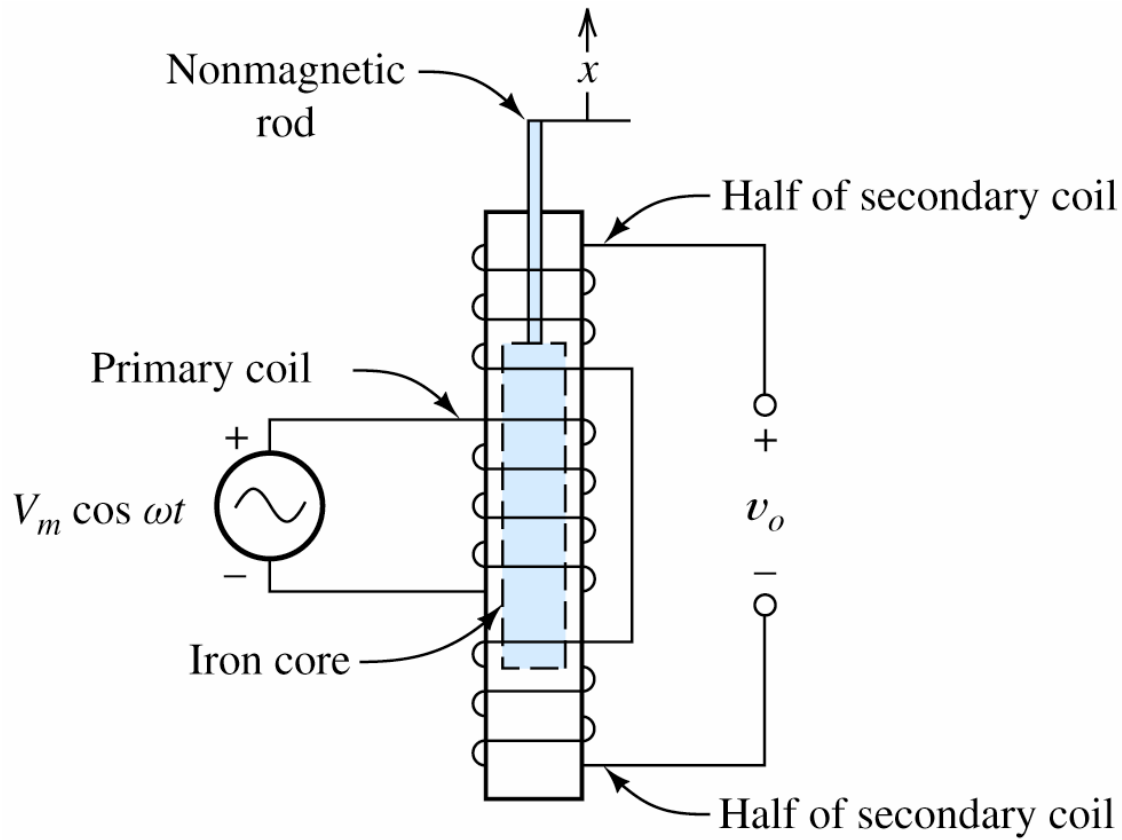


Figure 3.24 A linear variable differential transformer used as a position transducer.