Chapter 11 Frequency Response and Filters

11.1 Frequency response

- Frequency response is the forced response of a circuit to a sinusoid ac waveform of a particular frequency. Amplitude ratio and phase shift are typically used to characterize frequency response.
- Transfer function $H(s) \rightarrow H(j\omega)$ (phasor analysis).

- $\underline{Y} = H(j\omega)\underline{X}$.

- Amplitude ratio: $a(\omega) \equiv |H(j\omega)|$.

- Phase shift: $\theta(\omega) \equiv \angle H(j\omega)$.

- Superposition for waveforms at different frequencies):

Example 11.1: A frequency-Selective Network

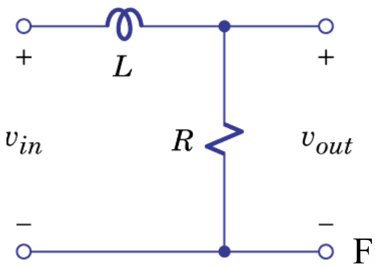


Figure 11.1.

- Frequency response curves: plots of amplitude ratio and phase shift vs. frequency. They can be obtained by analytical method or graphical method.

Example 11.2: An All-Pass Network

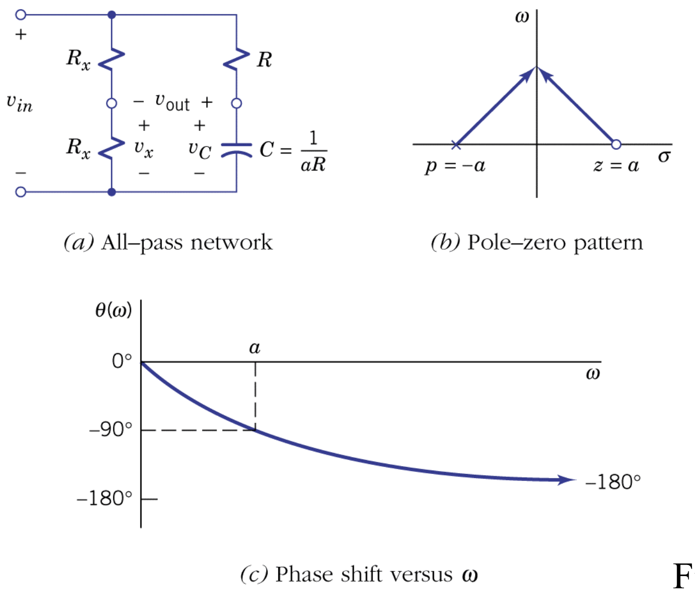


Figure 11.2.

Example 11.3: Frequency-Response Calculations

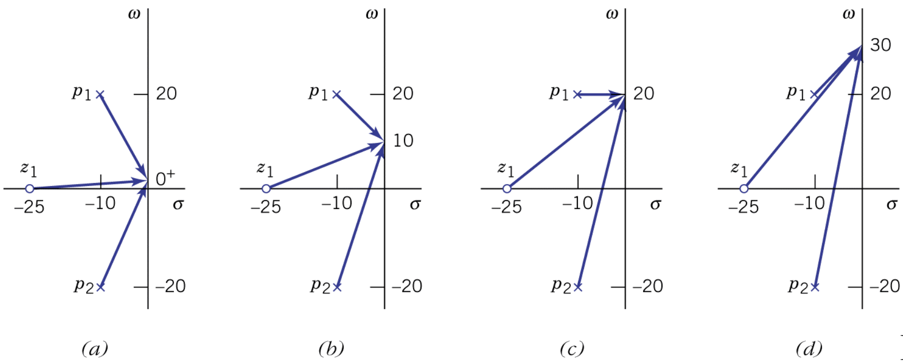


Figure 11.3.

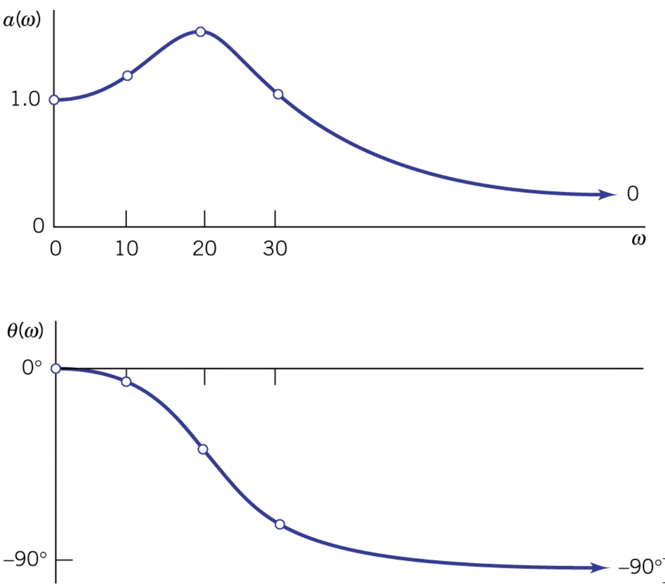


Figure 11.4.

11.2 Filters

- Filters are frequency-selective networks that pass certain frequencies but suppress/reject the others.
- Four common categories: lowpass, highpass, bandpass and notch.
- A positive gain constant K is assumed.
- Ideal lowpass filter, ideal highpass filter, cutoff frequency, passband and stop band.

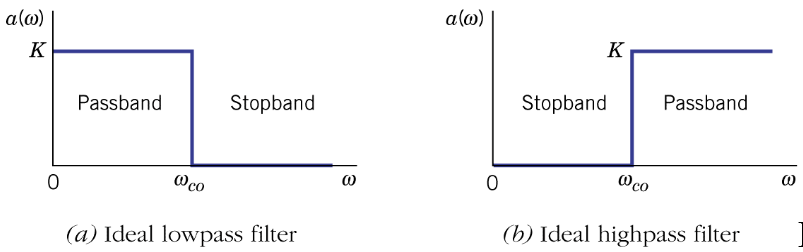


Figure 11.5.

- First-order lowpass filter:

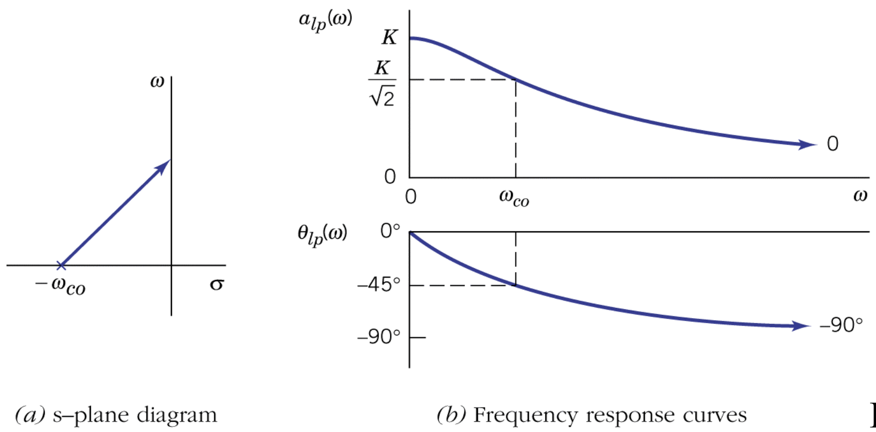
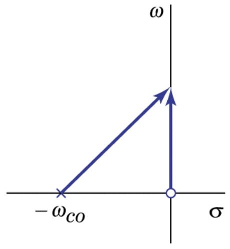
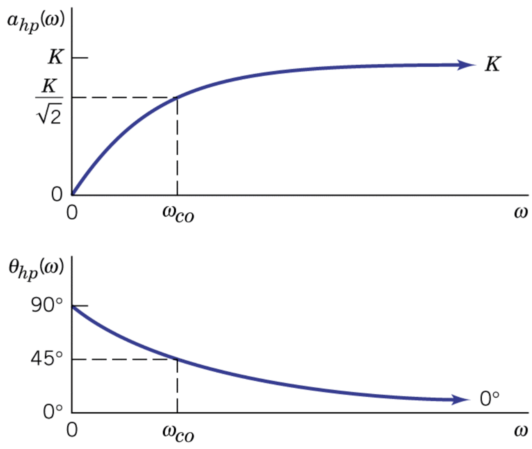


Figure 11.6.

- First-order highpass filter:



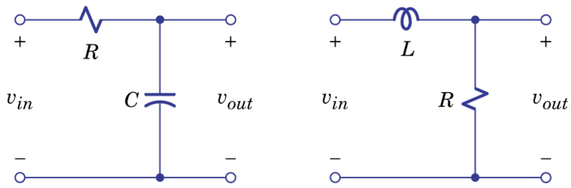
(a) s–plane diagram



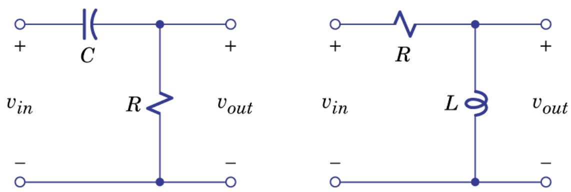
(b) Frequency response curves

Figure 11.7.

- First-order filter networks:



(a) Lowpass filters



(b) Highpass filters

Figure 11.8.

Example 11.4: Parallel Filter Network

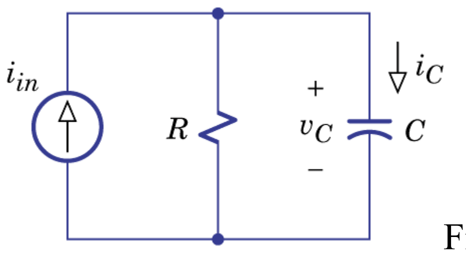


Figure 11.9.

Example 11.5: Design of a Lowpass Filter

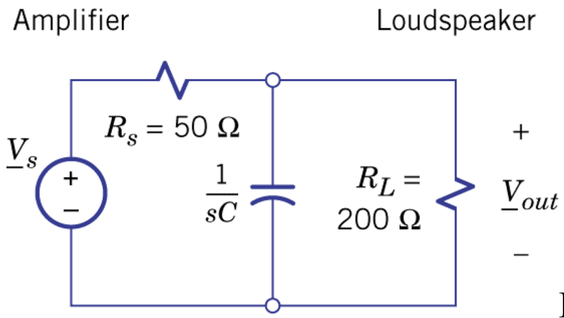


Figure 11.10.

- Ideal bandpass filter, ideal notch filter (band-reject filter), lower cutoff frequency, upper cutoff frequency and bandwidth.

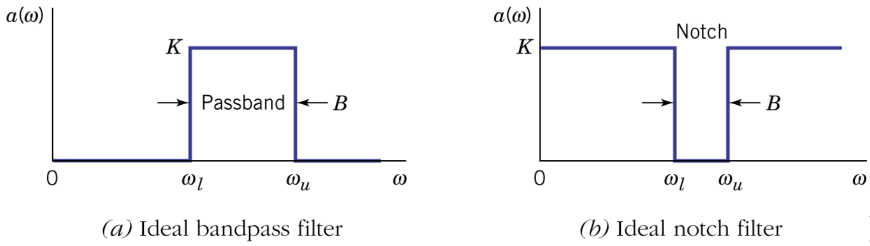
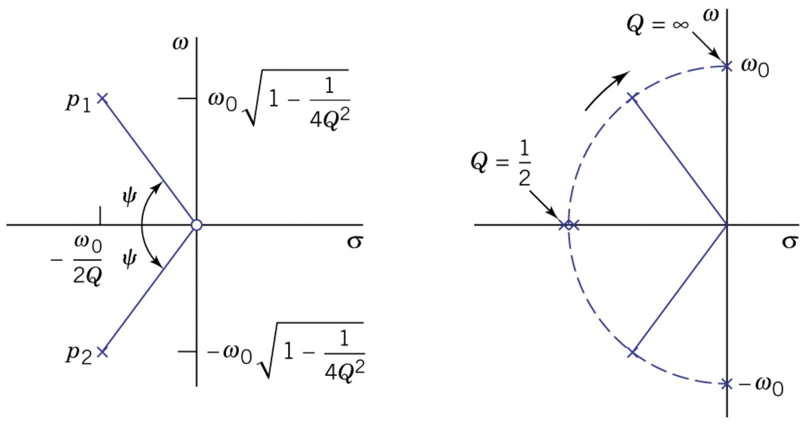


Figure 11.11.

- Second order bandpass filter and quality factor.



(a) Pole–zero pattern for second–order bandpass filter

(b) Pole movement as *Q* changes

Figure 11.12.

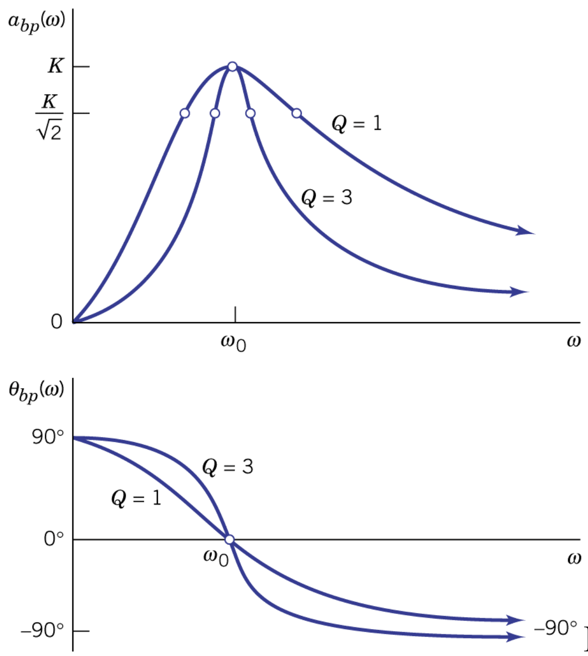


Figure 11.13.

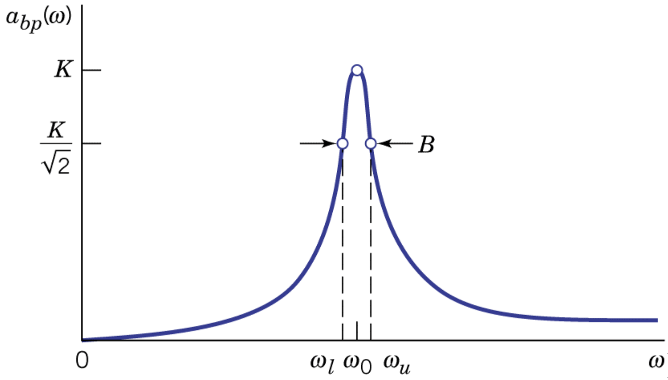


Figure 11.14.

- Second-order notch filter.

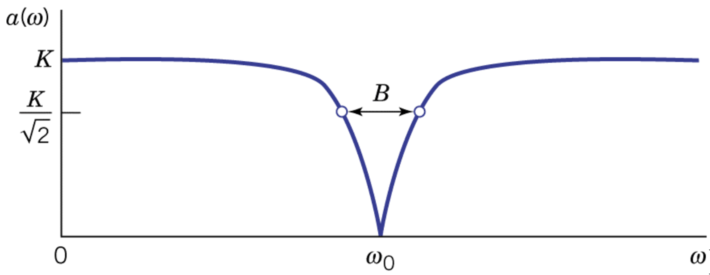


Figure 11.16.

- Resonant circuits for bandpass and notch filters.

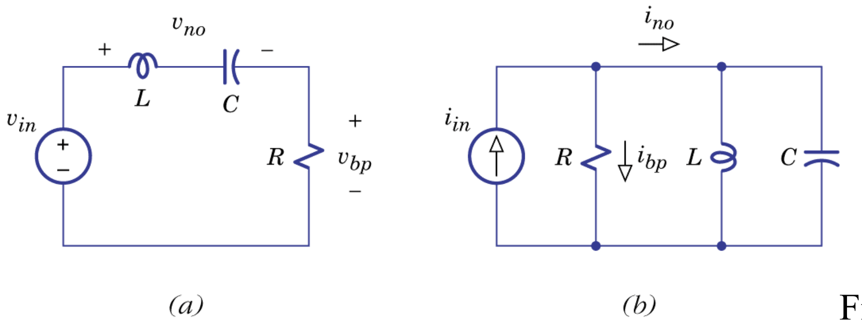


Figure 11.17.

- Winding resistance.

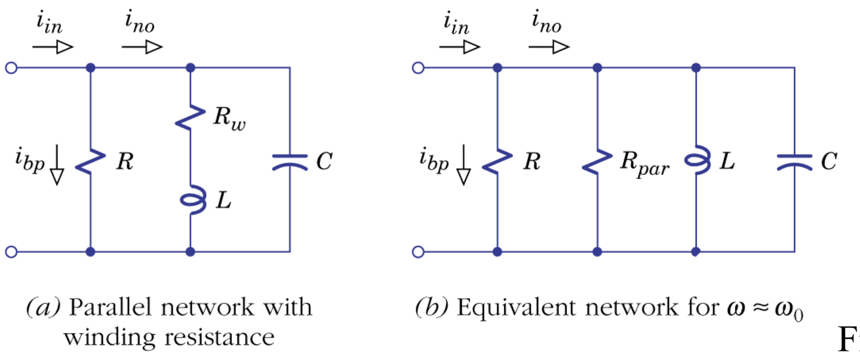
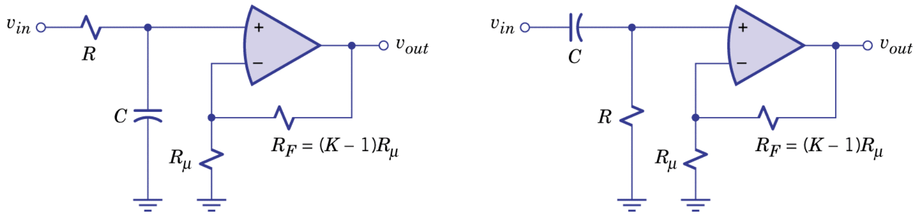


Figure 11.18.

Example 11.6: Design of a Bandpass Filter

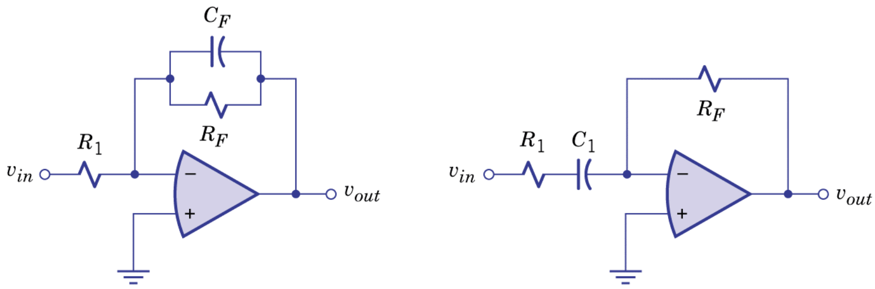
11.3 Op-Amp filter circuits

- Op-amps are included in filter circuit design to avoid loading effects and to eliminate the need for inductors in bandpass and notch filters.
- Noninverting lowpass and highpass filters:



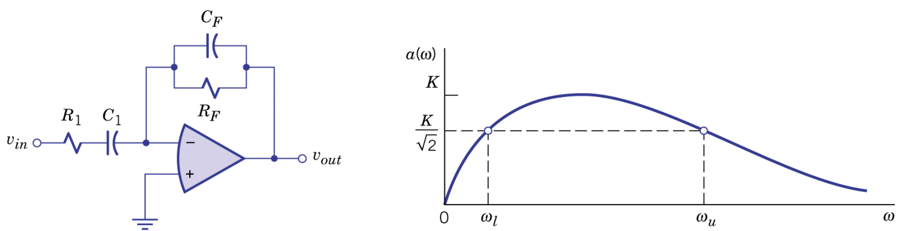
(a) Lowpass (b) Highpass Figure 11.19.

- Inverting lowpass and highpass filters:



(a) Lowpass (b) Highpass Figure 11.20.

- Wideband bandpass filters:



(a) Circuit (b) Amplitude ratio Figure 11.21.

- Narrowband bandpass filters:

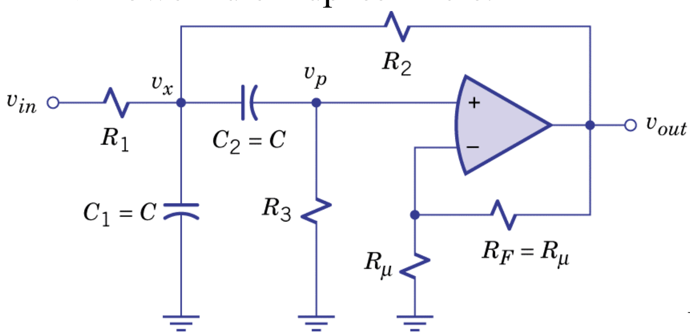
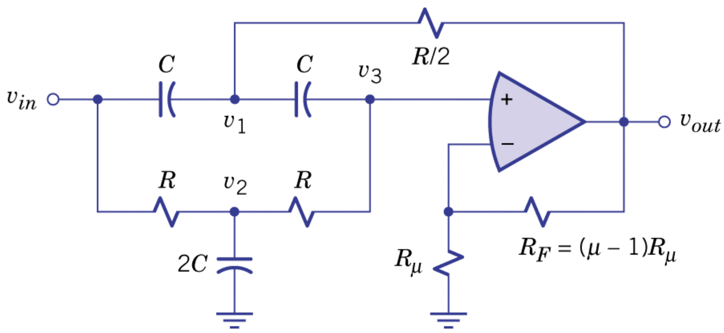
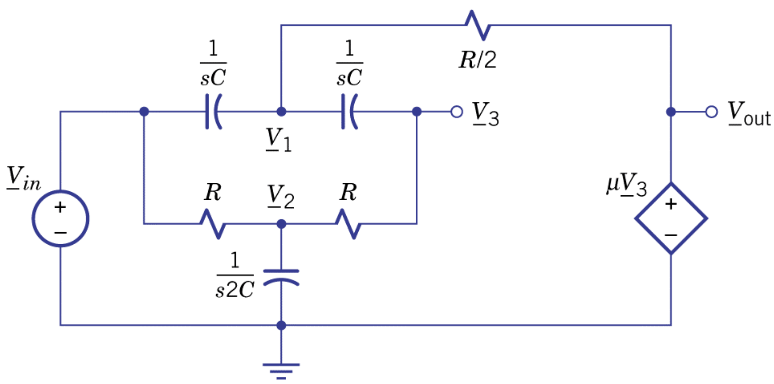


Figure 11.22.

- Notch filters:



(a) Twin-tee network with an op-amp



(b) *s*-domain diagram

(Figure 10.8.)

Example 11.7: Design of an Active Filter

11.4 Bode plots

- Amplitude ratio and frequency are converted to a logarithmic scale.
- Factored functions and decibels:

$$H(s) = KH_1(s)H_2(s)\cdots$$

$$a(\omega) = |H(j\omega)| = |K|a_1(\omega)a_2(\omega)\cdots$$

$$g(\omega) \equiv 20\log a(\omega) = 20\log|K| + 20\log a_1(\omega) + 20\log a_2(\omega) + \cdots$$
$$= K_{dB} + g_1(\omega) + g_1(\omega) + \cdots$$

$$\theta(\omega) = \angle H(j\omega) = \angle K + \theta_1(\omega) + \theta_2(\omega) + \cdots$$

- First-order factors (ramp function, highpass function and lowpass function).
- Ramp function:

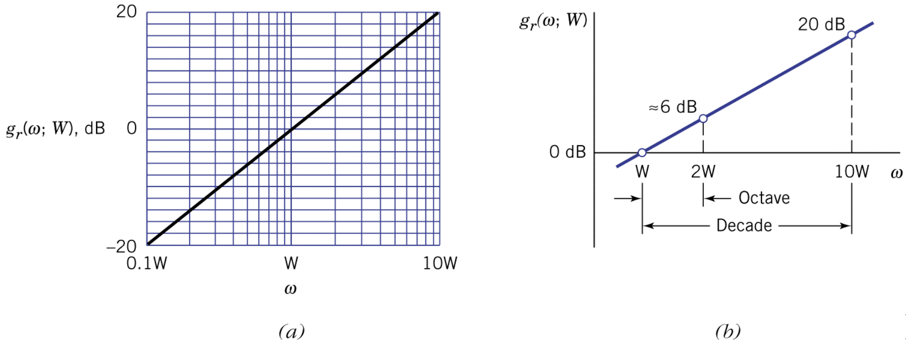


Figure 11.23.

- Highpass function:

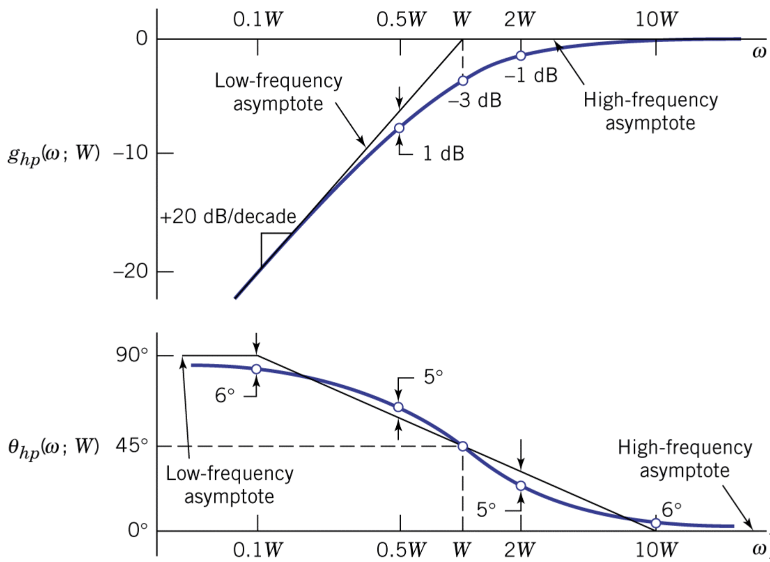


Figure 11.24.

- Lowpass function:

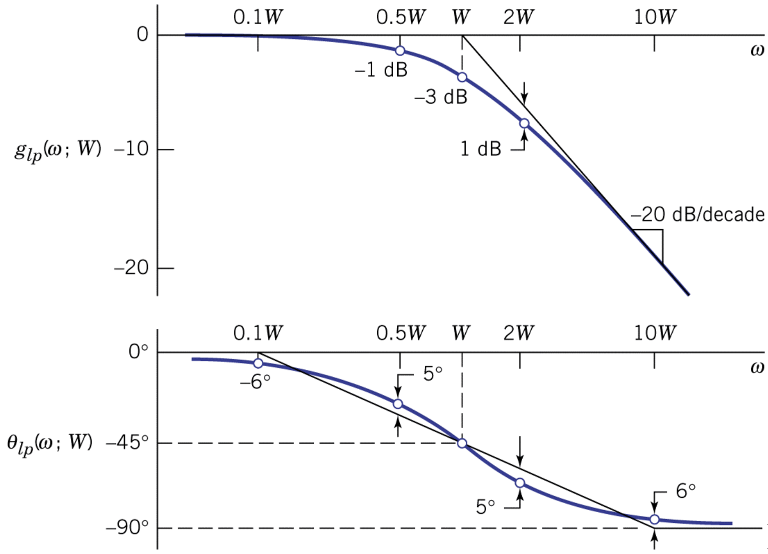


Figure 11.25.

Example An Illustrative Bode Plot

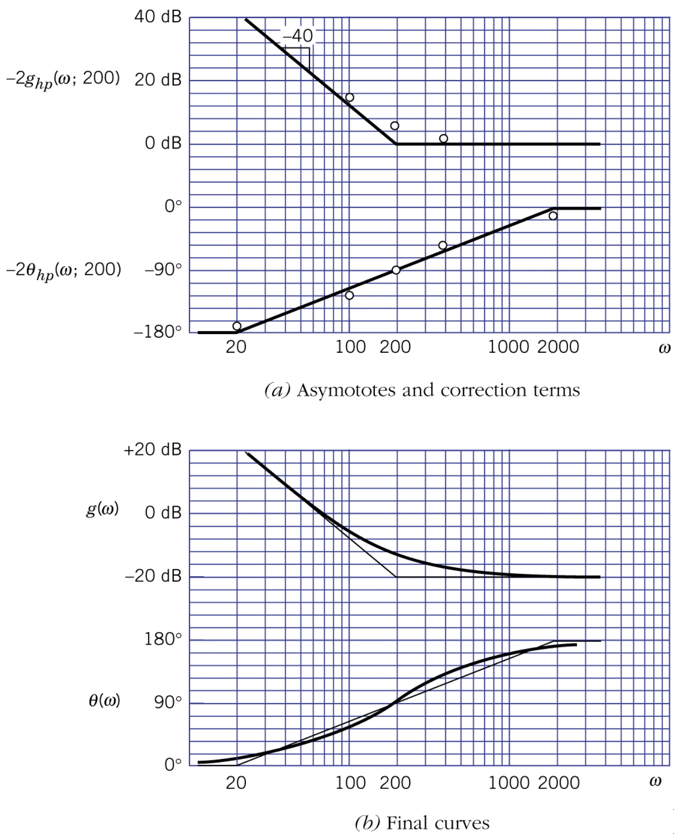


Figure 11.26.

- Products of first-order factors: Bode plots of any transfer functions consisting entirely of first-order factors and powers of first-order factors can be constructed using the additive property of gain and phase. The important elements include: break frequencies, asymptotic gain and phase using straight line approximations and constants K_{dB} and $\angle K$.

Example 11.9: Frequency Response of a Bandpass Amplifier

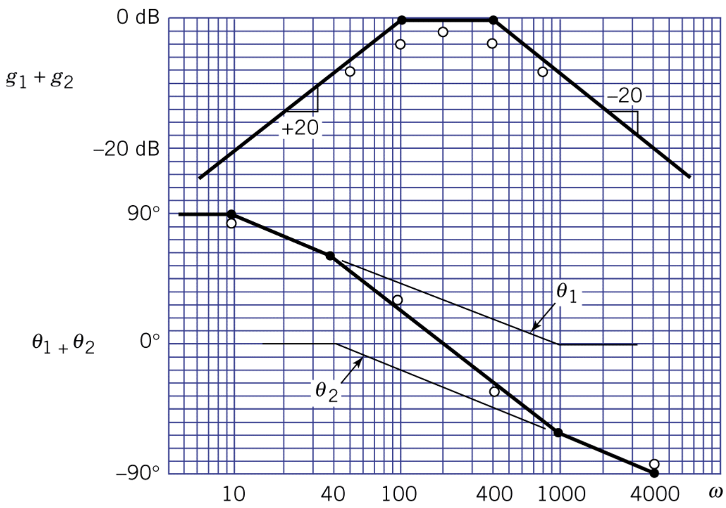


Figure 11.27.

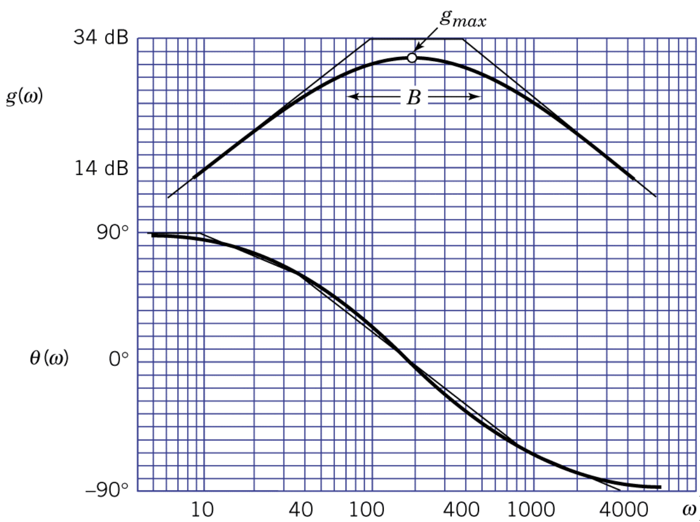


Figure 11.28.

- Quadratic factors for complex-conjugate poles:

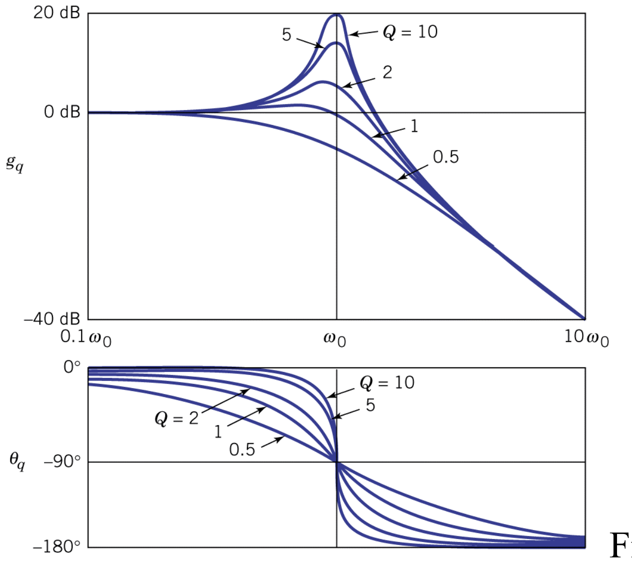


Figure 11.29.

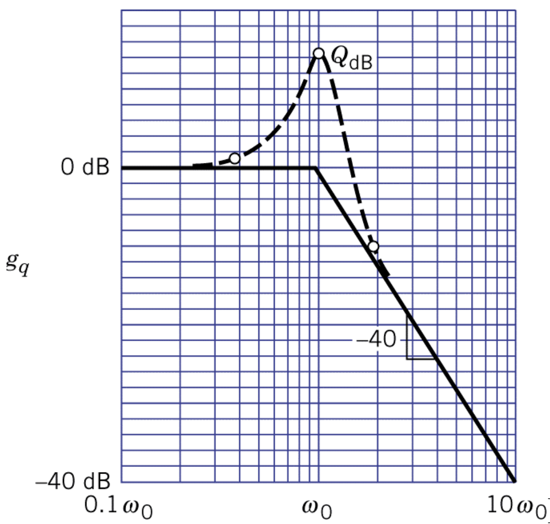


Figure 11.30.

Example 11.10: Bode Plot of a Narrowband Filter

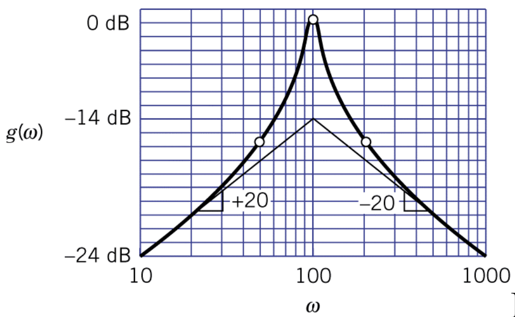


Figure 11.31.

11.5 Frequency response design

- Given a required frequency response, the transfer function can be found by starting from the Bode plot. First, a straight line approximation needs to be obtained. Second, the straight line approximation can be decomposed to a constant term and a set of first-order functions (assuming no resonant peaks or peaks are present). Finally, we can apply the basic op-amp networks to realize the transfer function.

Example 11.11: Design of an FM Pre-emphasis Network

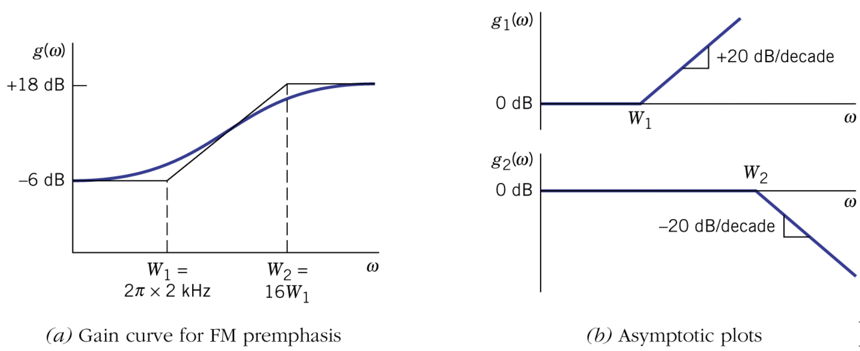


Figure 11.33.

11.6 Butterworth filters

- Two trade-offs in filter design: performance vs. complexity and rejection vs. ripple. Will only cover Butterworth lowpass and highpass filters.
- Butterworth lowpass filters: maximally flat, poles are uniformly spaced by angle of *$180^0/n$* (*n* is the order), rolloff at 20*n* dB per decade.

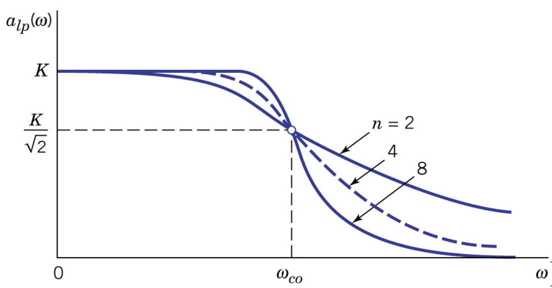


Figure 11.36.

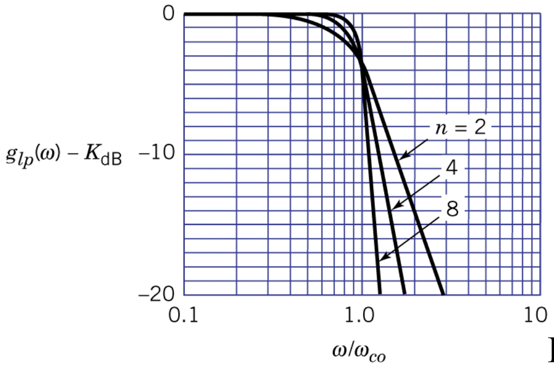


Figure 11.37.

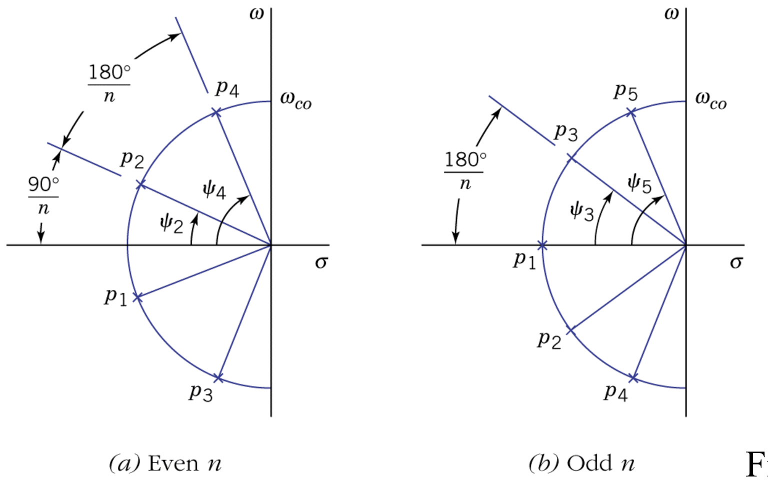
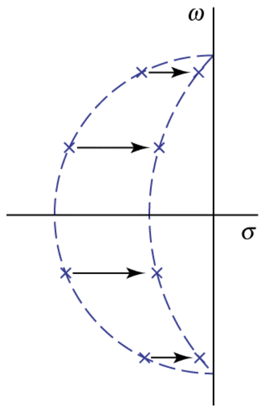


Figure 11.38.



Example 11.12: FM Stereo Separation Filter

- Butterworth highpass filters can be derived from existing lowpass designs via the lowpass-to-highpass transformation: $s \to \omega_{co}^{2} / s$.

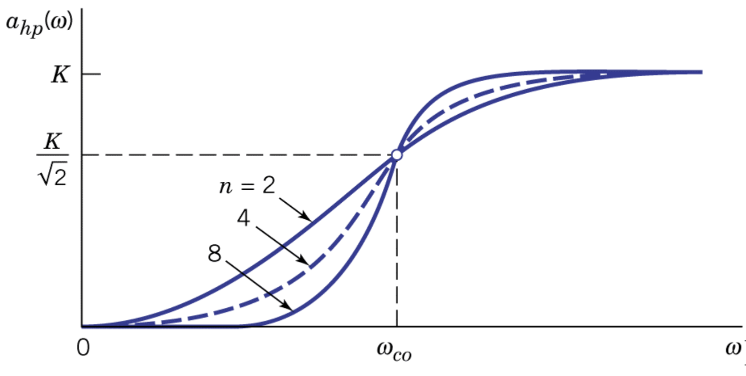


Figure 11.40.

- Op-amps can be used to realize Butterworth filters such that inductors and loading effects can be eliminated.
- Op-amp circuits for second-order transfer functions are shown in Figure 11.41.

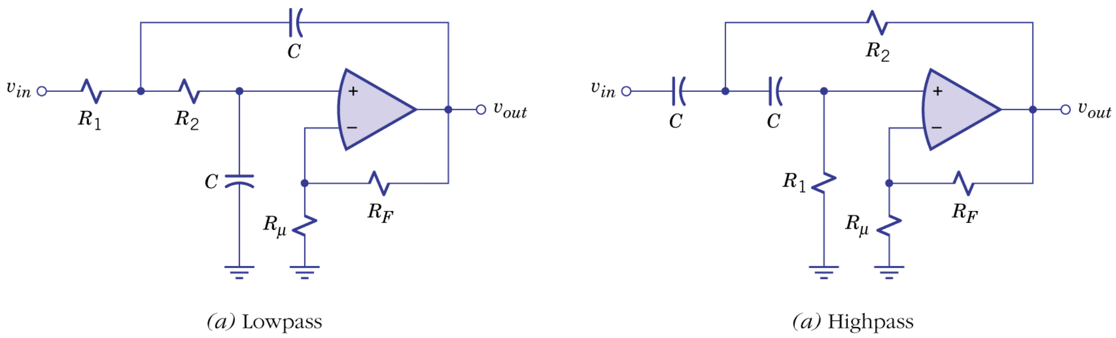


Figure 11.41.

Example 11.13: Op-Amp Circuit for a Lowpass Filter

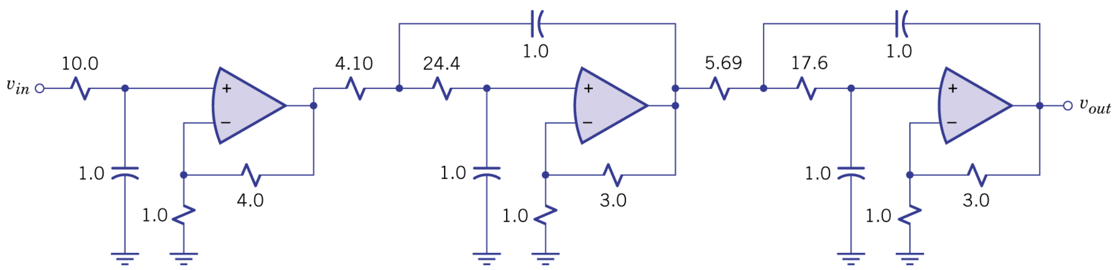


Figure 11.42.