Chapter 10 Network Functions and s-Domain Analysis

10.1 Complex frequency and generalized impedance

- Complex frequency: oscillating voltages or currents with exponential amplitudes.

- $$x(t) = X_m e^{\sigma t}\cos(\omega t + \phi_x) = \mathrm{Re}\left[X_m e^{\sigma t} e^{j(\omega t + \phi_x)}\right] = \mathrm{Re}\left[(X_m e^{j\phi_x}) e^{(\sigma + j\omega)t}\right].$$

- Complex frequency: $s \equiv \sigma + j\omega$.

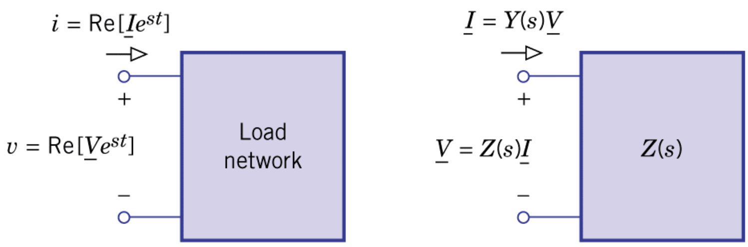
- Phasor: $\underline{X} \equiv X_m \angle \phi_x = X_m e^{j\phi_x}$.

Example 10.1: A Complex-Frequency Waveform

- For a "real" frequency, we have
- $i(t) = I_m \cos(\omega t + \phi_i) = \mathrm{Re}\left[\underline{I} e^{j\omega t}\right]$.
- $v(t) = V_m \cos(\omega t + \phi_v) = \mathrm{Re}\left[\underline{V} e^{j\omega t}\right]$.
- $\underline{V} = V_m \angle \phi_v$, $\underline{I} = I_m \angle \phi_i$.

- For a "complex" frequency, we replace $j\omega$ with s.

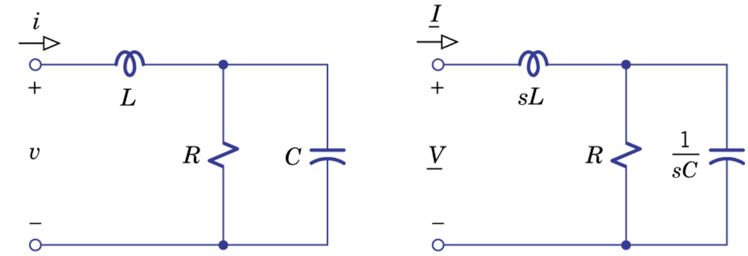
- $i(t) = \mathrm{Re}\left[\underline{I}e^{st}\right] = I_m e^{\sigma t}\cos(\omega t + \phi_i)$.
- $v(t) = \mathrm{Re}\left[\underline{V}e^{st}\right] = V_m e^{\sigma t}\cos(\omega t + \phi_v)$.
- Generalized impedance: $Z(s) \equiv \underline{V}/\underline{I}$, or $\underline{V} = Z(s)\underline{I}$.
- Generalized admittance: $Y(s) \equiv 1/Z(s) = \underline{I}/\underline{V}$, or $\underline{I} = Y(s)\underline{V}$.



(a) Load network (b) s-domain diagram Figure 10.1.

- Generalized impedance $Z(s)$ and admittance $Y(s)$ are directly related to the circuit's behavior given an input signal with a specific complex frequency. This will be covered in more detail in Chapter 13, where we apply Laplace transform to analyze a circuit.

Example 10.2: Calculations with Complex Frequency



(a) Network for Example 10.2 (b) s-domain diagram Figure 10.2.

- Impedance analysis.

Example 10.3: Miller-Effect Capacitance

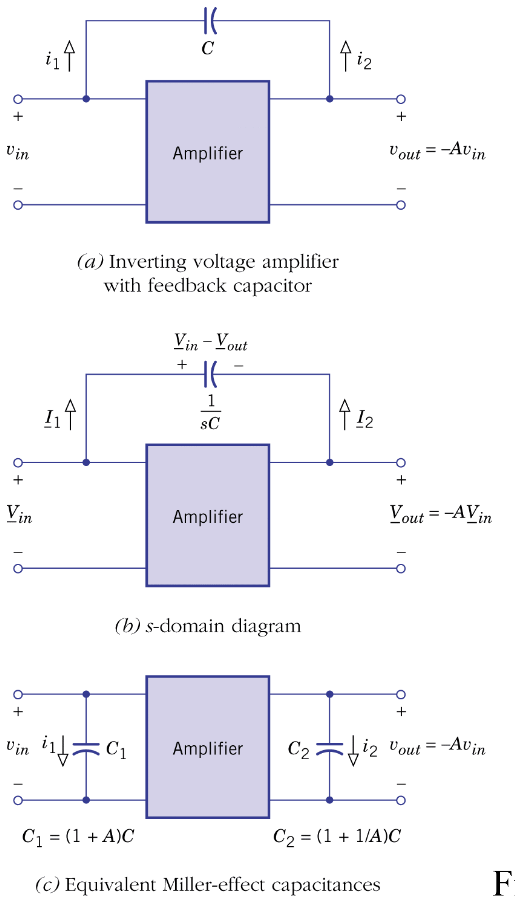


Figure 10.3.

Example 10.4: Generalized Impedance Converter

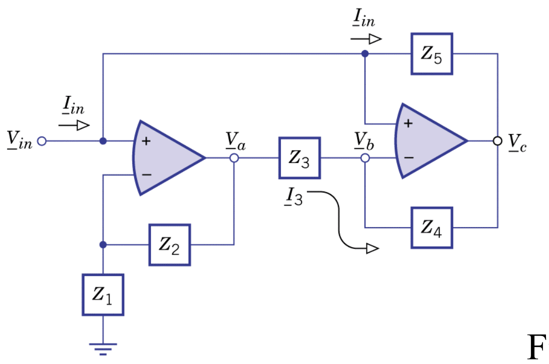


Figure 10.4.

10.2 Network functions

- Any response forced by a complex-frequency excitation.
- Input: $x(t) = X_m e^{\sigma t}\cos(\omega t + \phi_x) = \mathrm{Re}\left[\underline{X} e^{st}\right]$,

  $\underline{X} \equiv X_m \angle \phi_x = X_m e^{j\phi_x}$.
- Response: $y(t) = Y_m e^{\sigma t}\cos(\omega t + \phi_x) = \mathrm{Re}\left[\underline{Y} e^{st}\right]$,

  $\underline{Y} \equiv Y_m \angle \phi_Y = Y_m e^{j\phi_Y}$.
- Network function: $H(s) \equiv \underline{Y} / \underline{X}$.
- A network function is also known as a driving point function if it relates a network's terminal variables. It can also be a transfer function since $y(t)$ can be any voltage or current within the network.

- A network function is also a rational function. Its numerator is a polynomial obtained from the right hand side of the differential equation with derivatives replaced by powers, the denominator is a polynomial obtained from the left-hand side of the differential equation.

Example 10.5: Series LRC Network Functions

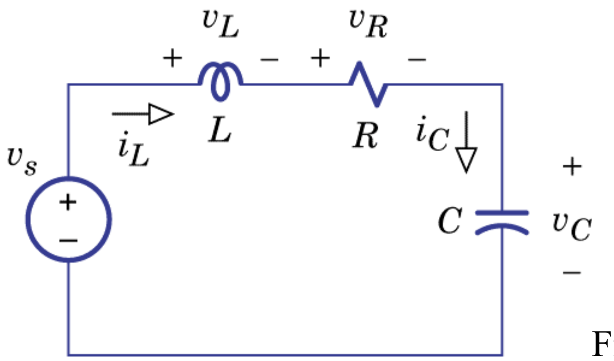
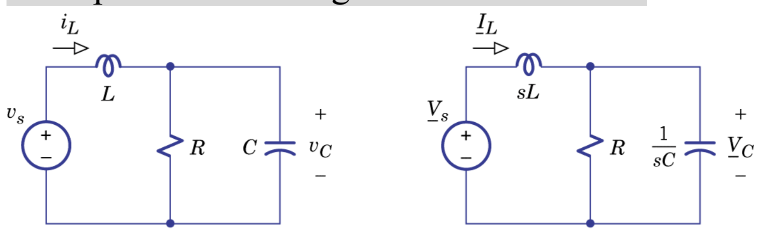


Figure 10.6.

- The network function can also be obtained by using *s*-domain impedances and admittances. Impedance analysis can be done by series-parallel reduction, voltage and current dividers, proportionality, source conversions and node/mesh equations.

Example 10.6: Finding Network Functions

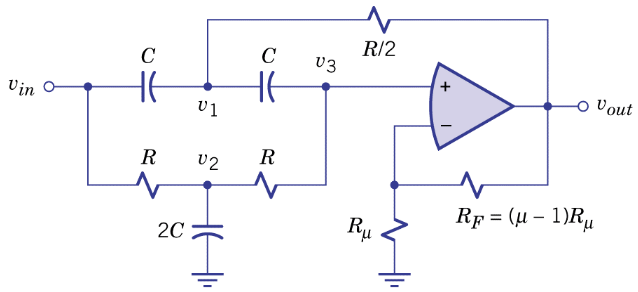


(a) Circuit for Example 10.6

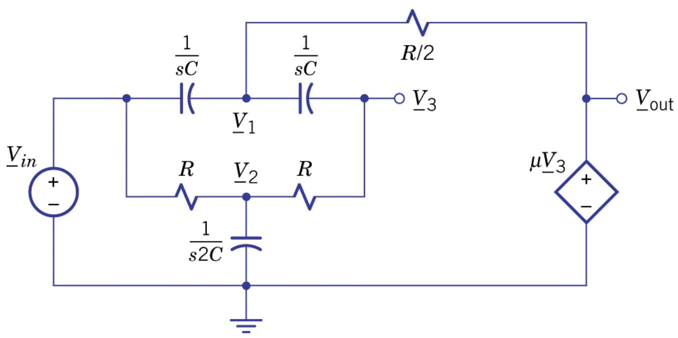
(b) s-domain diagram

Figure 10.7.

Example 10.7: Twin-Tee Network with an Op-Amp



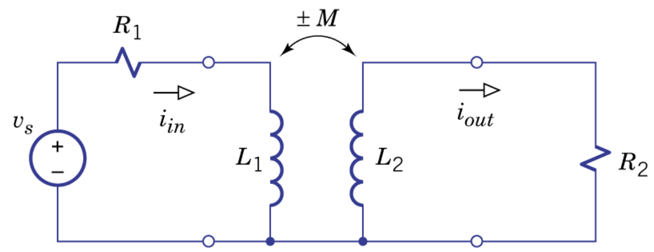
(a) Twin-tee network with an op-amp



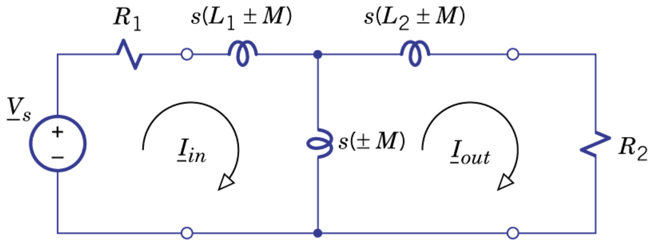
(b) *s*-domain diagram

Figure 10.8.

10.3 Network functions with mutual inductance



(a) Circuit with mutual inductance



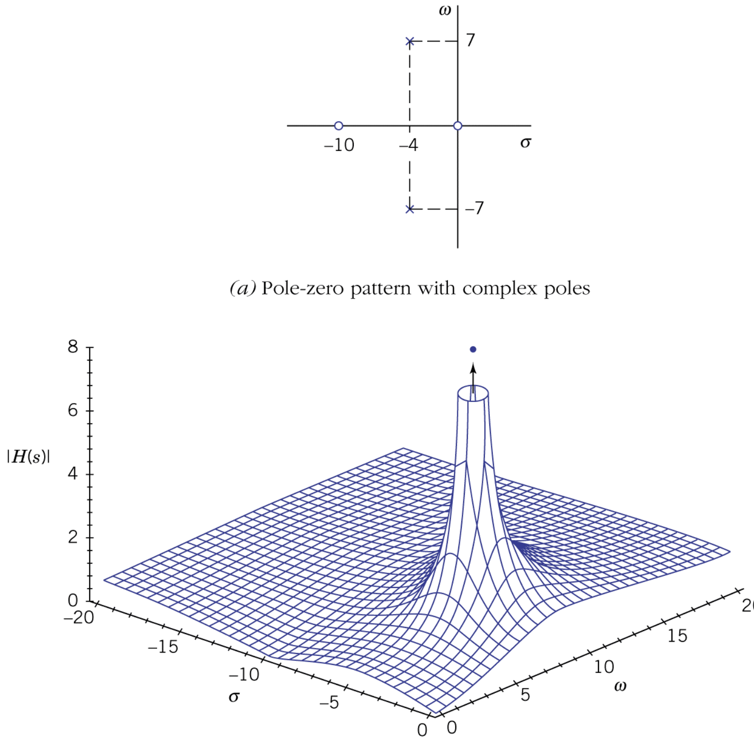
(b) s-domain diagram with tee network

Figure 10.10.

10.4 s-domain analysis

- The network function is more easily obtained from impedance analysis than from differential equations.
- Both forced response and natural response can be determined.
- Poles and zeros: poles are roots of the denominator, zeros are roots of the numerator.
- Gain factor corresponds to the dc gain.

- Gain factor K is real. Poles and zeros are either real or in complex conjugate pairs. The number of poles is the order of the circuit (number of independent energy-storage elements in the circuit).



(a) Pole-zero pattern with complex poles

(b) Surface contour |H(s)|

Figure 10.11.

Example 10.8: Pole-Zero Pattern of a Fifth-Order Network

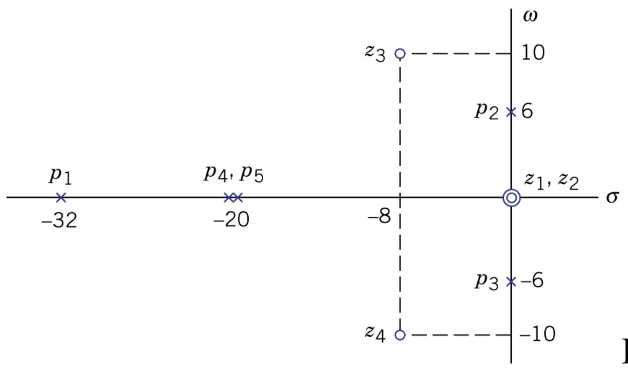


Figure 10.12.

- Forced response and *s*-plane vectors.

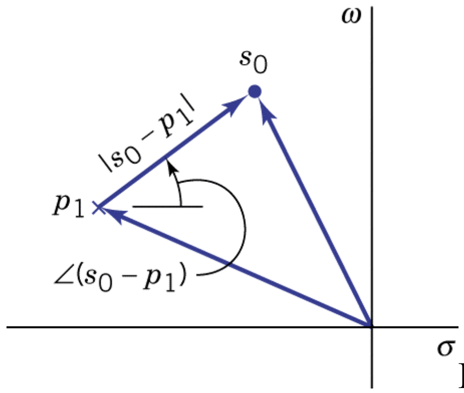


Figure 10.13.

Example 10.9: Calculations with *s*-Plane Vectors

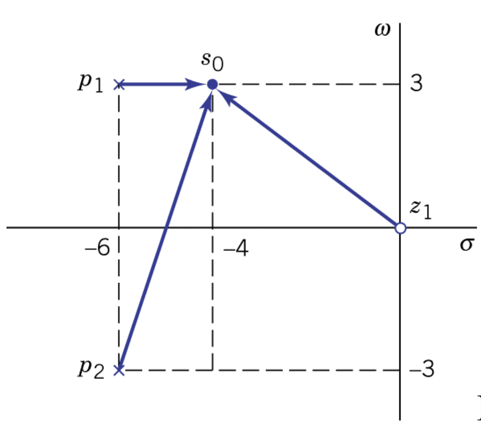


Figure 10.14.

- Natural response and stability: poles of the network function are characteristic values of the circuit natural response, each pole corresponds to a mode.

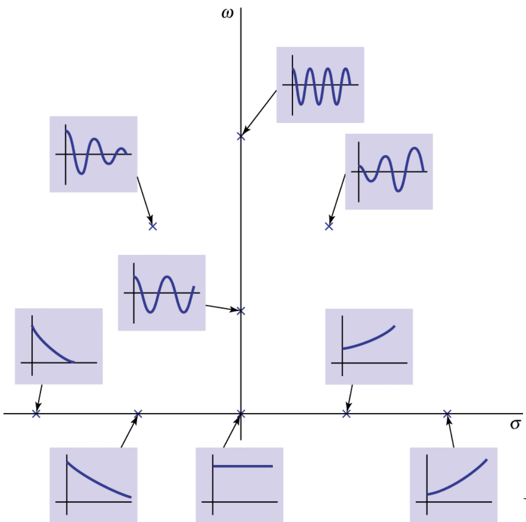


Figure 10.15.

- A circuit is stable if all poles are in the left half of the *s* plane.
- Oscillator and pole-zero cancellation.

Example 10.10: Natural Responses of a Third-Order Circuit