Chapter 4: Systematic Analysis Methods

Chapter 4: Outline

Systematic Methods (still KCL, KVL)

Node Analysis

KCL

Node voltages

Branch voltages

Branch current

Voltage controlled, non-voltage controlled

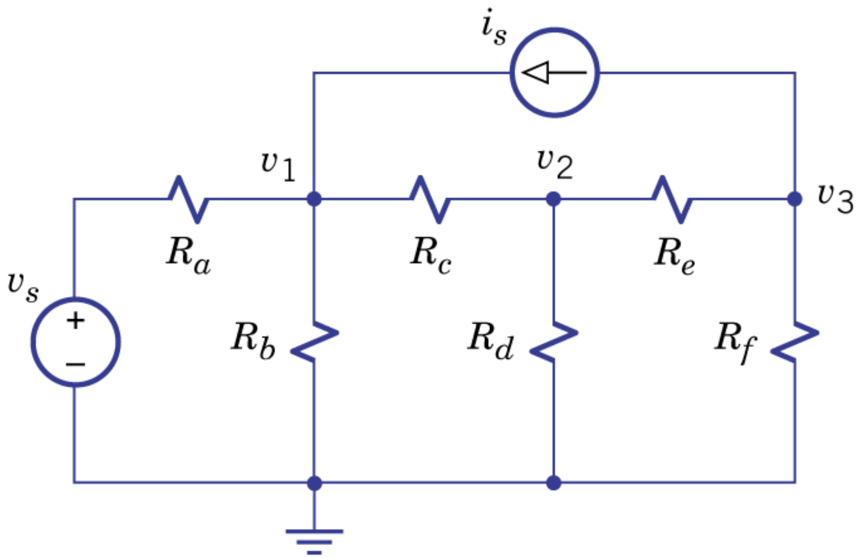
Mesh Analysis

(Dual)

Matrix form (observation)

With Controlled sources

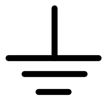
How to Analyze a Circuit?



- Find all voltages and currents.
- A possible solution:
  - Start with node voltages
  - Then find all branch voltages
  - Then find all currents
- Potential issues:
  - Use node KCL to find node voltages
  - How many equations?
  - Voltage controlled?
- Duality: node → mesh

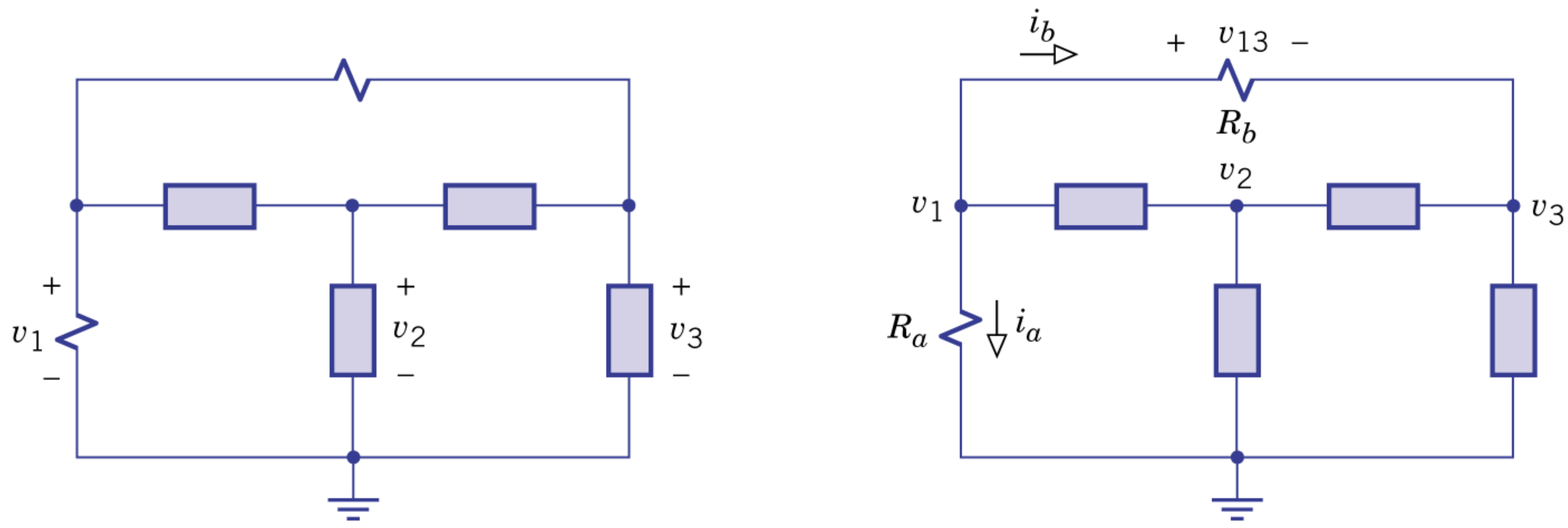
Node Analysis

Node Analysis

- A systematic circuit analysis method using node voltages.
- Reference node: a specific node in a circuit for measuring electric potentials. Denoted by the ground symbol. 
- Node voltage: the potential at a nonreference node with respect to the reference node.
- Branch voltage: the potential difference between the two nodes of a branch.

$$v_{nm} = v_n - v_m$$

Node Analysis



(a) Circuit with four nodes and three node voltages

(b) Relating other branch variables to node voltages

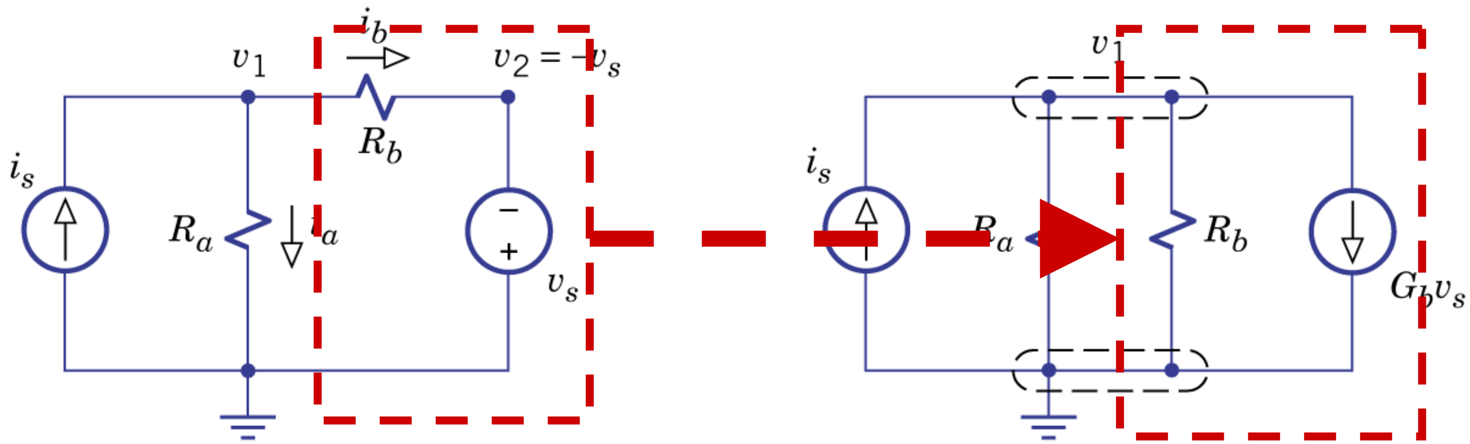
$$v_{nm} = v_n - v_m$$

- Circuit analysis procedures:
  node voltages -> branch voltages -> branch currents.

Voltage controlled assumption

Node Equations

- KCL equations at the specified nodes (excluding the reference node).



(a) Circuit with one unknown node voltage

(b) Equivalent parallel circuit

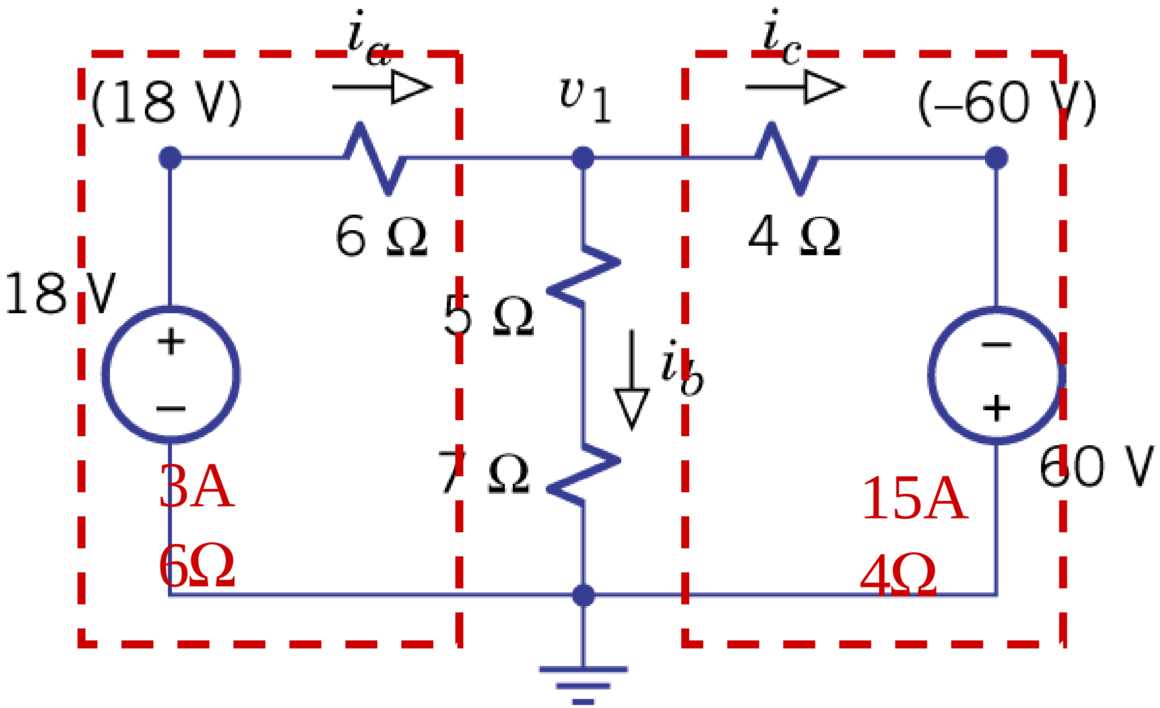
$$G_{11} \cdot v_1 = i_{s1}$$

$$G_{11} = (G_a + G_b)$$ SUM

$$i_{s1} = i_s - G_b \cdot v_s$$ NET

Special treatment required for non-voltage controlled devices.

Example 4.1: Analysis with One Unknown



$$G_{11} \cdot v_1 = i_{s1}$$

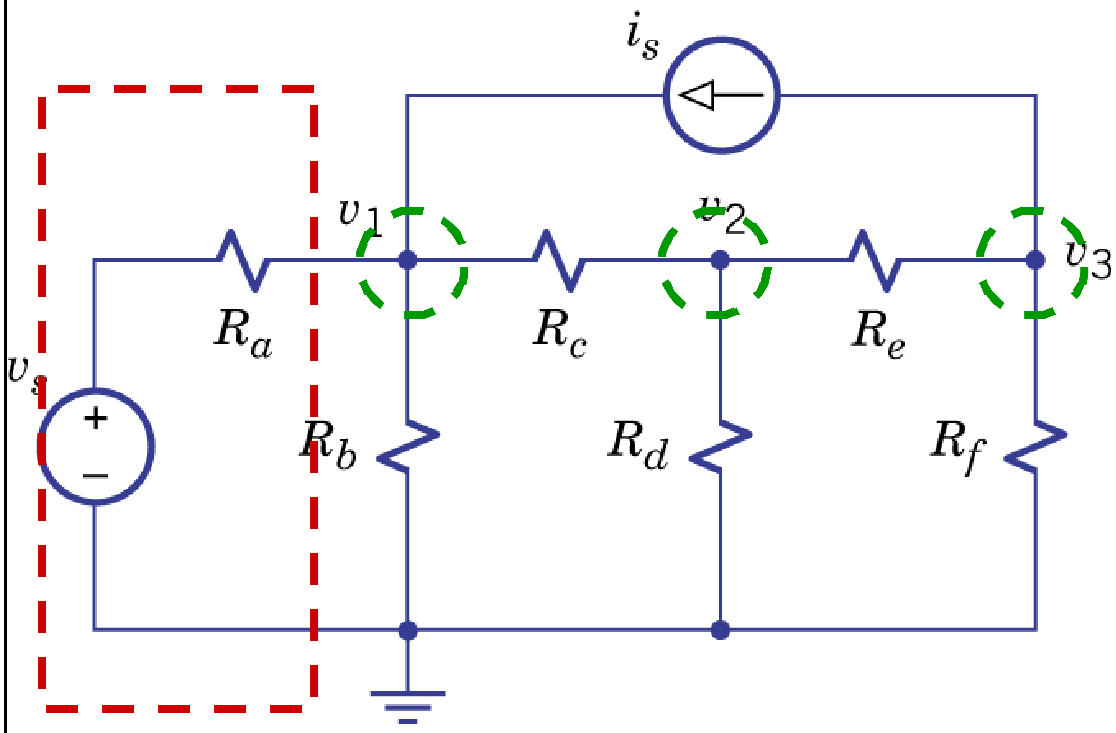
$$G_{11} = \frac{1}{6} + \frac{1}{12} + \frac{1}{4} = \frac{1}{2} S$$

$$i_{s1} = 3 - 15 = -12A$$

$$v_1 = -12 \times 2 = -24V$$

$$i_a = 7A, i_b = -2A, i_c = 9A$$

Matrix Node Equations



$$\text{node } v_1 : \frac{v_1 - v_s}{R_a} + \frac{v_1}{R_b} + \frac{v_1 - v_2}{R_c} = i_s$$

$$\text{node } v_2 : \frac{v_2 - v_1}{R_c} + \frac{v_2}{R_d} + \frac{v_2 - v_3}{R_e} = 0$$

$$\text{node } v_3 : \frac{v_3 - v_2}{R_e} + \frac{v_3}{R_f} = -i_s$$

$$\Downarrow$$

$$\left(\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}\right)v_1 - \frac{1}{R_c}v_2 = i_s + \frac{v_s}{R_a}$$

$$-\frac{1}{R_c}v_1 + \left(\frac{1}{R_c} + \frac{1}{R_d} + \frac{1}{R_e}\right)v_2 - R_e v_3 = 0$$

$$-\frac{1}{R_e}v_2 + \left(\frac{1}{R_e} + \frac{1}{R_f}\right)v_3 = -i_s$$

Matrix Node Equations

$$\left(\frac{1}{R_a}+\frac{1}{R_b}+\frac{1}{R_c}\right)v_1-\frac{1}{R_c}v_2=i_s+\frac{v_s}{R_a}$$

$$-\frac{1}{R_c}v_1+\left(\frac{1}{R_c}+\frac{1}{R_d}+\frac{1}{R_e}\right)v_2-R_ev_3=0$$

$$-\frac{1}{R_e}v_2+\left(\frac{1}{R_e}+\frac{1}{R_f}\right)v_3=-i_s$$

$$\Rightarrow\begin{bmatrix}\frac{1}{R_a}+\frac{1}{R_b}+\frac{1}{R_c} & -\frac{1}{R_c} & 0\\ -\frac{1}{R_c} & \frac{1}{R_c}+\frac{1}{R_d}+\frac{1}{R_e} & -\frac{1}{R_e}\\ 0 & -\frac{1}{R_e} & \frac{1}{R_e}+\frac{1}{R_f}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}i_s+\frac{v_s}{R_a}\\ 0\\ -i_s\end{bmatrix}$$

Matrix Node Equations

sum -(in between) net into

$$\begin{bmatrix} G_a + G_b + G_c & -G_c & 0 \\ -G_c & G_c + G_d + G_e & -G_e \\ 0 & -G_e & G_e + G_f \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_s + G_a v_s \\ 0 \\ -i_s \end{bmatrix}$$

$$\Rightarrow \quad [G]\cdot[v] = [i_s] \quad \Rightarrow \quad [v] = [G]^{-1}\cdot[i_s]$$

Conductance Matrix

$$[G] = \begin{bmatrix} G_{11} & -G_{12} & \cdots & -G_{1N} \\ -G_{21} & G_{22} & \cdots & -G_{2N} \\ \vdots & \vdots & & \vdots \\ -G_{N1} & -G_{N2} & \cdots & G_{NN} \end{bmatrix}$$

where G_{nn}= sum of conductance connected to node *n*,
$G_{nm} = G_{mn}$ =equivalent conductance directly connected nodes *n* and *m*.

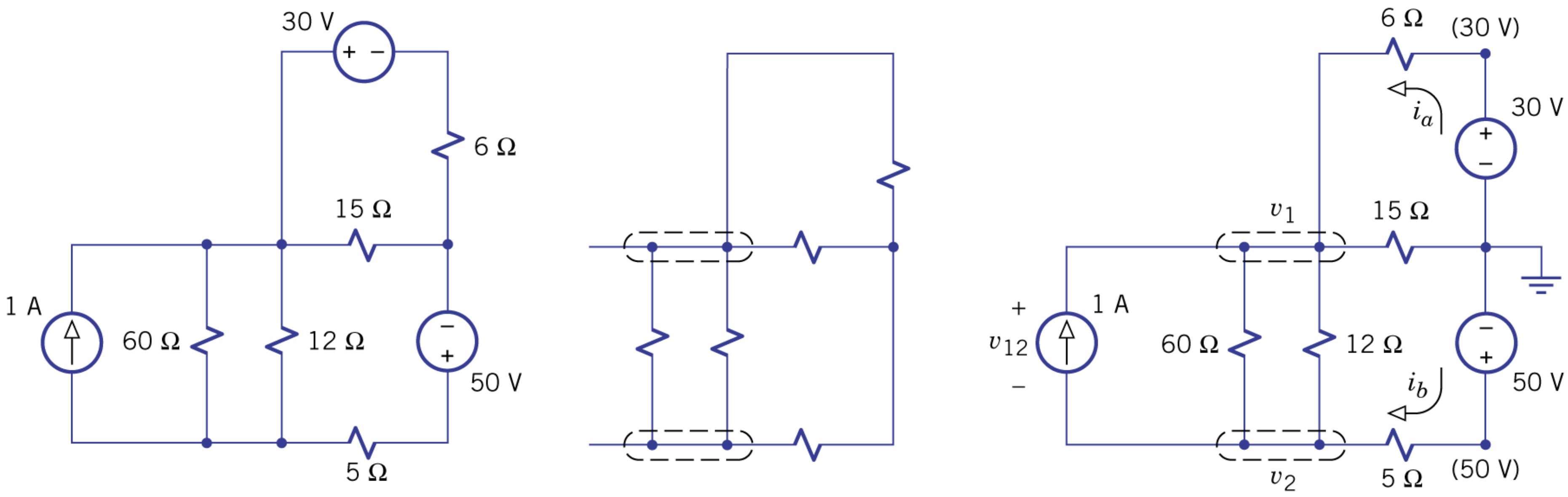
How Many Node Equations?

- The minimum number of unknown node voltages is determined by suppressing all sources, counting the remaining sources and subtracting one for the reference node.

Node Analysis: General Approach

- Determine the number of node KCL equations.
- Write down G and i_{s}:
  - By inspection.
  - *Treatment for non-voltage controlled devices.*
  - *Controlled sources.*
- Solve node voltages.
- Solve branch voltages and branch currents.

Example 4.2: Two Unknowns



(a) Circuit for node analysis

(b) Source suppression leaving three nodes

(c) Diagram with two unknown node voltages

<u>By inspection:</u>

$$G_{11} = \frac{1}{60} + \frac{1}{12} + \frac{1}{15} + \frac{1}{6} = \frac{1}{3}$$

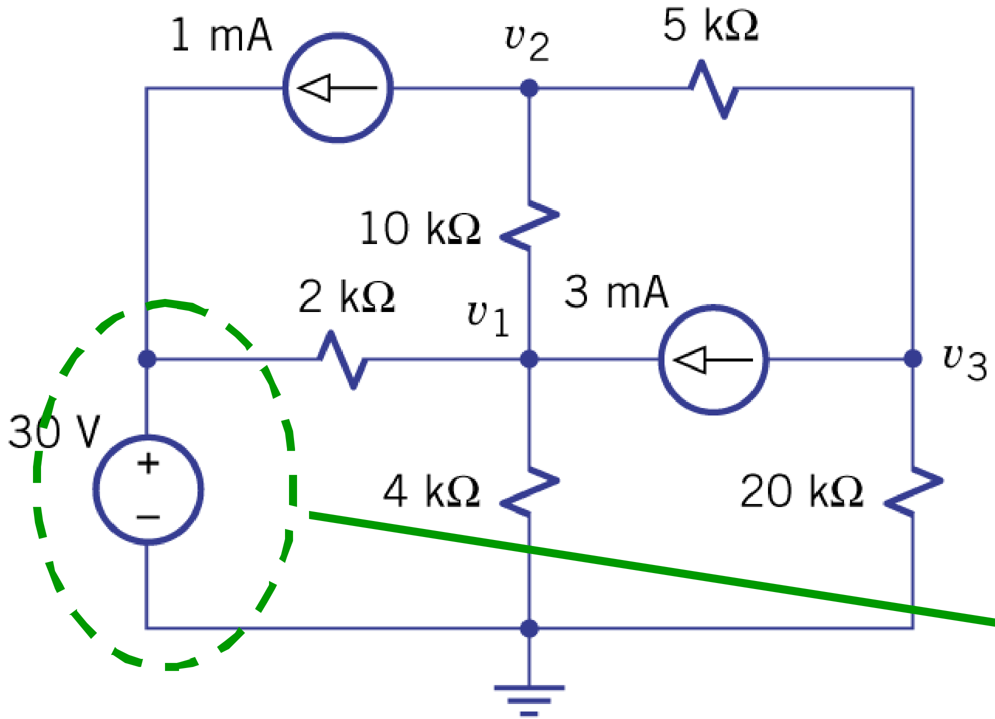
$$G_{12} = \frac{1}{60} + \frac{1}{12} = \frac{1}{10}$$

$$G_{22} = \frac{1}{60} + \frac{1}{12} + \frac{1}{5} = \frac{3}{10}$$

$$i_{s1} = 6$$

$$i_{s2} = 9$$

Example 4.3: Three Unknowns



$$[G]=\begin{bmatrix} \frac{1}{4}+\frac{1}{2}+\frac{1}{10} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5}+\frac{1}{20} \end{bmatrix}$$

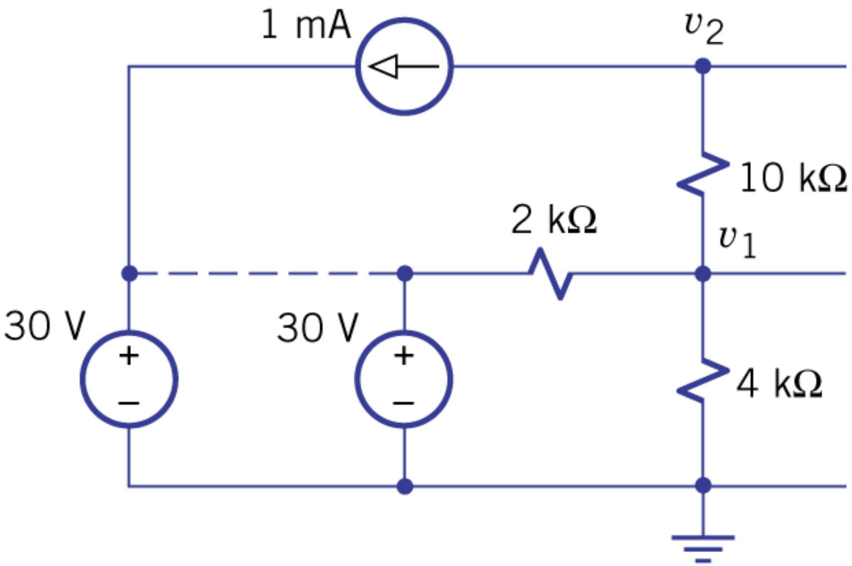
$$[i_s]=\begin{bmatrix} \frac{30}{2}+3 \\ -1 \\ -3 \end{bmatrix}$$

$$\text{node } v_1: \left(\frac{v_1-30}{2}\right)+\frac{v_1}{4}+\frac{v_1-v_2}{10}-3=0$$

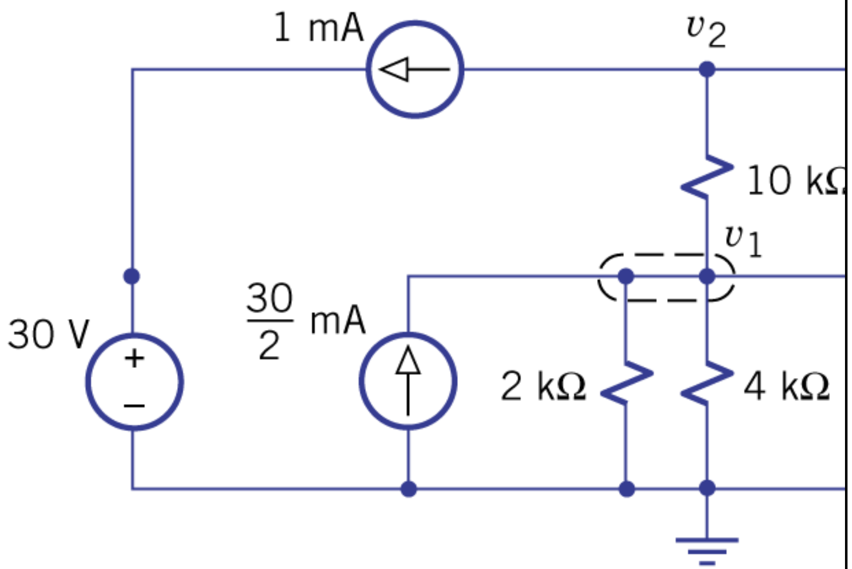
How to Handle Voltage Sources?

- A voltage source is non-voltage controlled, i.e., one cannot specify its current given a voltage.
- Source node splitting.
- Floating voltage sources:
  - With series resistance→ Source conversion.
  - Floating ideal sources→ Introduce a new variable (or use supernode KCL).

Source Node Splitting



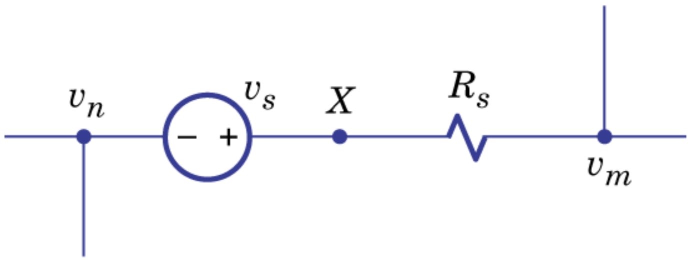
(a) Adding parallel voltage source



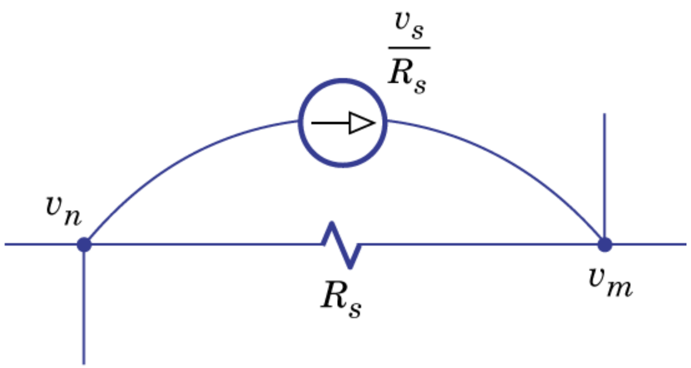
(b) After source conversion

Floating Voltage Sources

- Floating Voltage Sources: a voltage source is floating if it lacks direct connections to the reference node.
- The reference node should be chosen such that it ties to one terminal of as many voltage sources as possible.
- For floating sources with series resistance, perform source conversion.

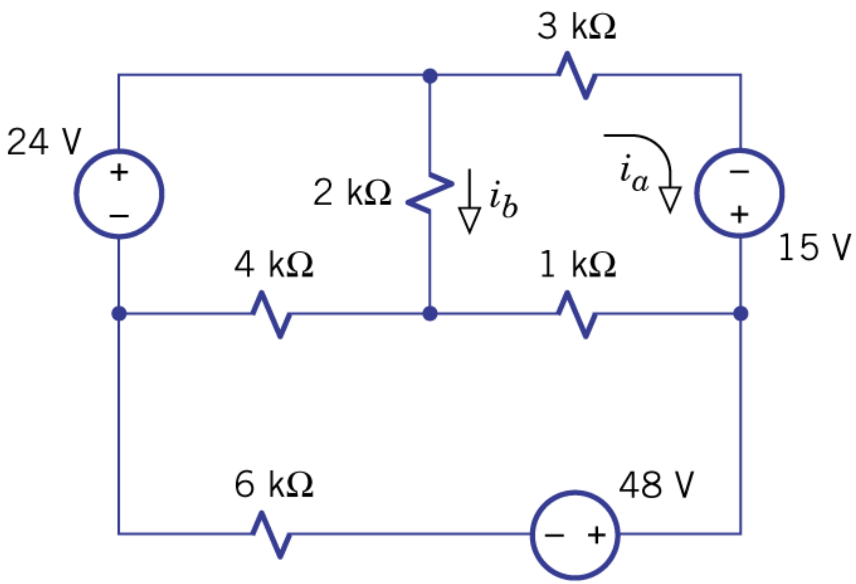


(a) Floating voltage source with series resistance

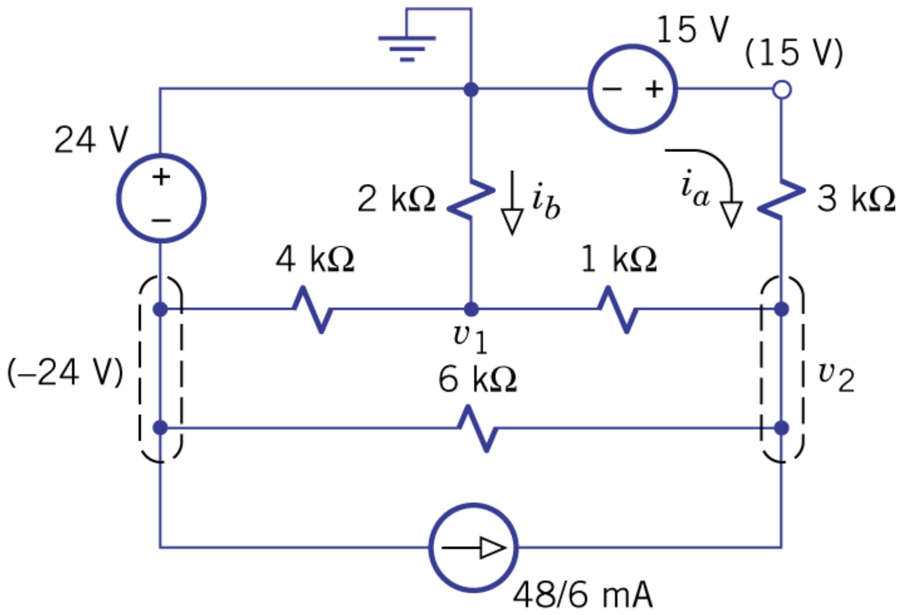


(b) Source conversion eliminating node X

Example 4.4



(a) Circuit with four nodes

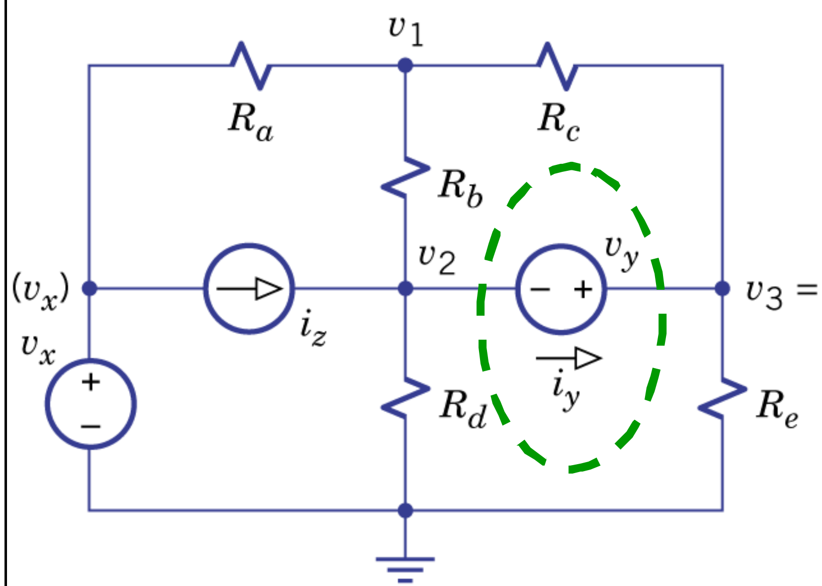


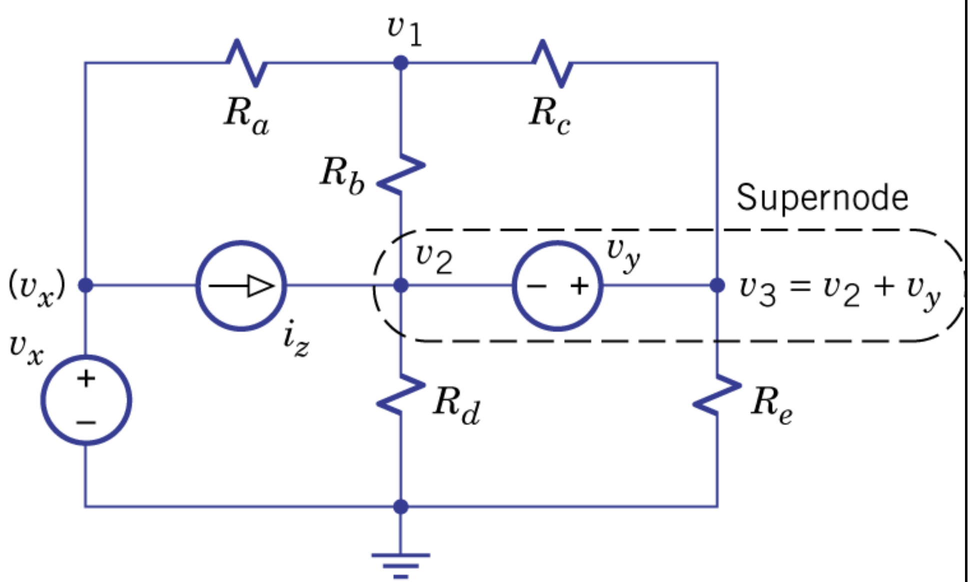
(b) Diagram for node analysis

$$[G]=\begin{bmatrix} 1+\frac{1}{2}+\frac{1}{4} & -1 \\ -1 & 1+\frac{1}{3}+\frac{1}{6} \end{bmatrix} \qquad [i_s]=\begin{bmatrix} -6 \\ 8+5-4 \end{bmatrix}$$

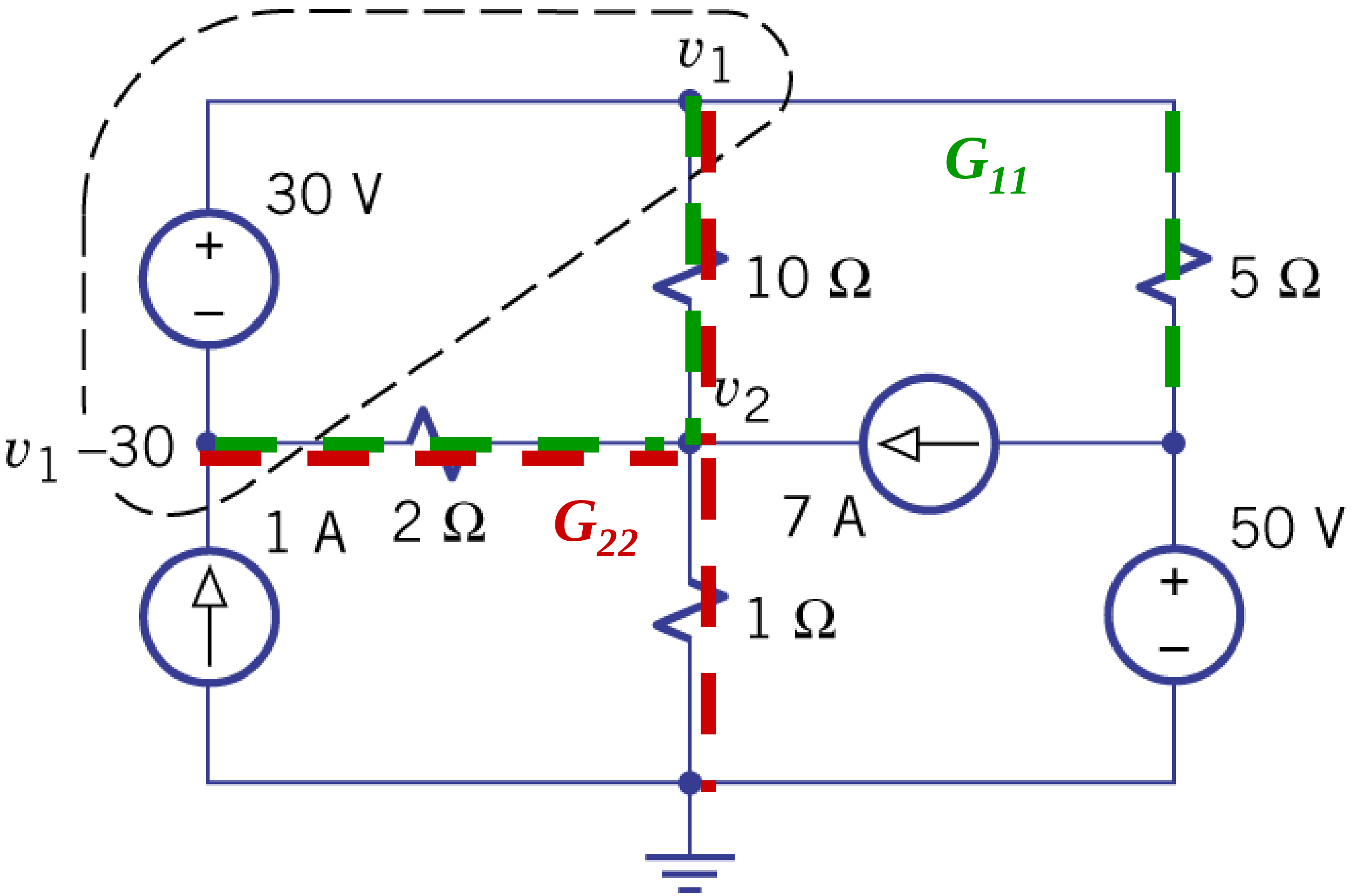
Floating Ideal Sources

- For floating ideal sources, introduce a new circuit variable (a fictitious source current) and an extra node equation. Alternatively, use a supernode to encircle the floating ideal voltage source.





Example 4.5



$$i_{s1} = \frac{30}{2} + 1 + \frac{50}{5}$$

$$i_{s2} = -\frac{30}{2} + 7$$

If any doubt, you can always go back to the original KCL equations,…

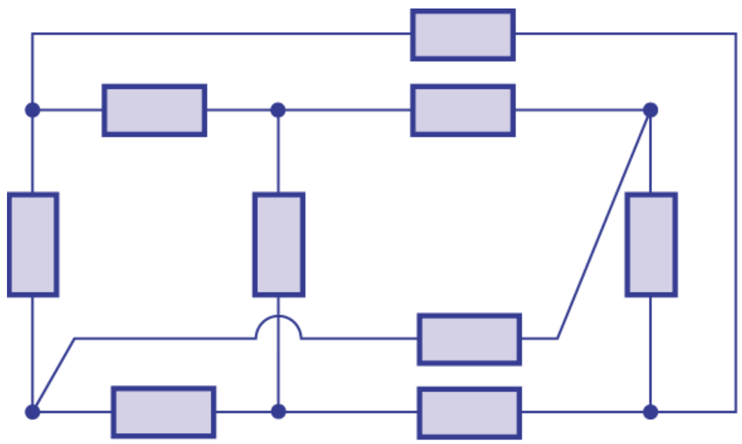
Mesh Analysis

Mesh Analysis

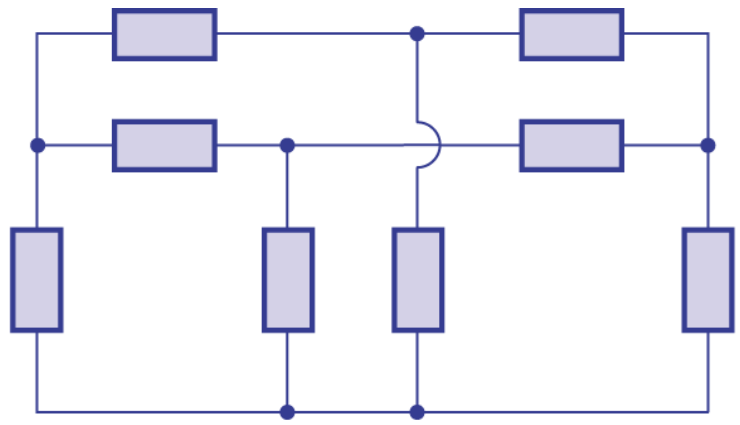
- Node analysis and mesh analysis:
  - Structural dual
  - Node vs. Mesh
  - KCL vs. KVL
  - Non-voltage controlled vs. Non-current controlled
  - Voltage source vs. Current source
  - Conductance matrix ***G*** vs. Resistance matrix ***R***

Mesh Analysis

- Structural dual of node analysis. Node voltage vs. Mesh current.
- Planar circuit: the diagram of the circuit can be drawn without hop-overs for crossing branches.



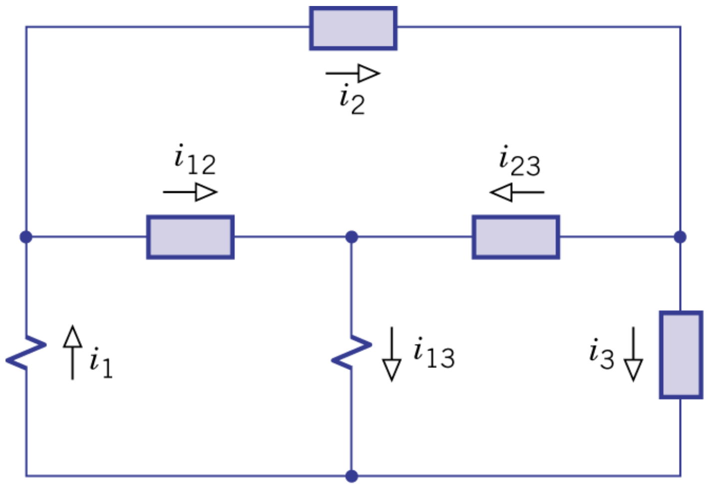
(a) Diagram of a nonplanar circuit



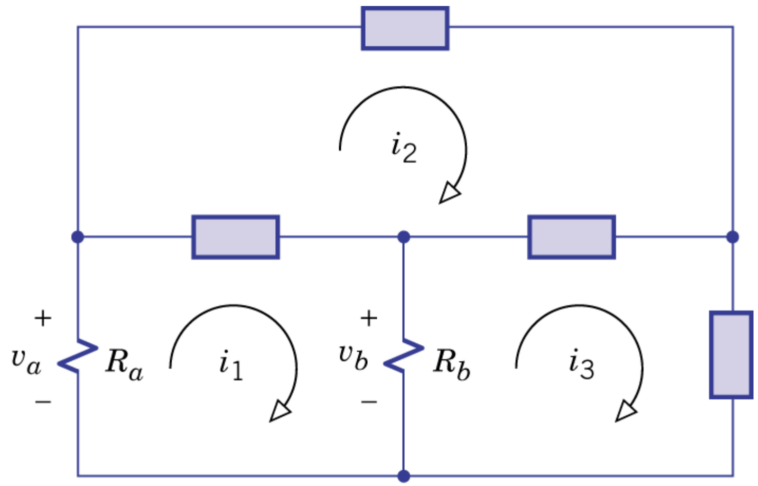
(b) Diagram of a planar circuit that could be redrawn without a hop-over

Mesh Analysis

- Mesh: a closed current path that contains no closed paths within it. Note the difference between a mesh and a loop.
- Mesh current: the current that circulates completely around a mesh.
- Reference convention: all mesh currents circulate in the same direction, either clockwise or counterclockwise.

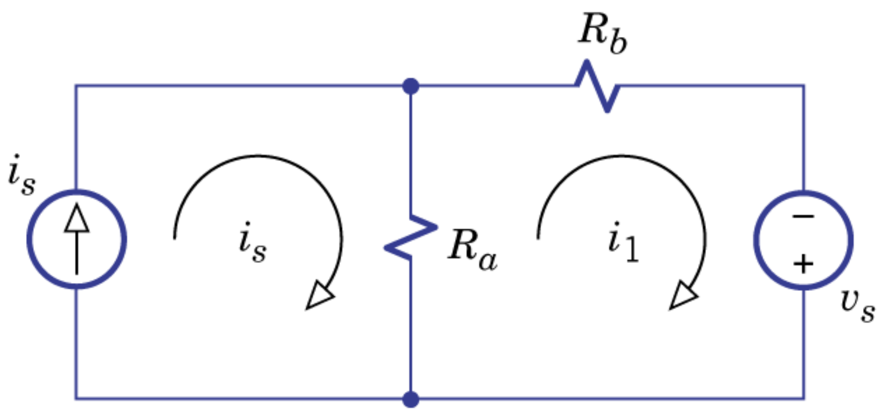


(a) Circuit with three meshes

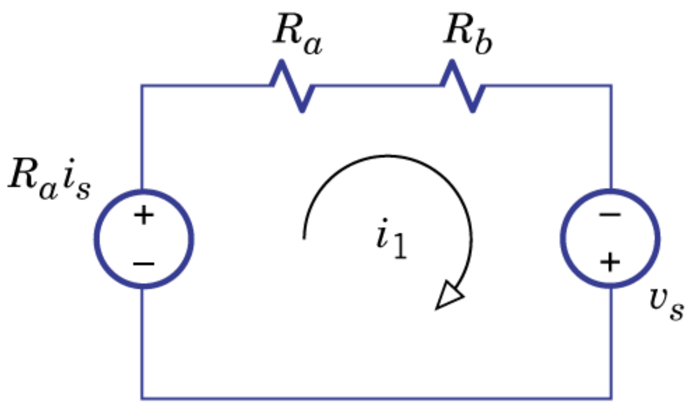


(b) Mesh currents

An Example with Source Conversion



(a) Circuit with one unknown mesh current



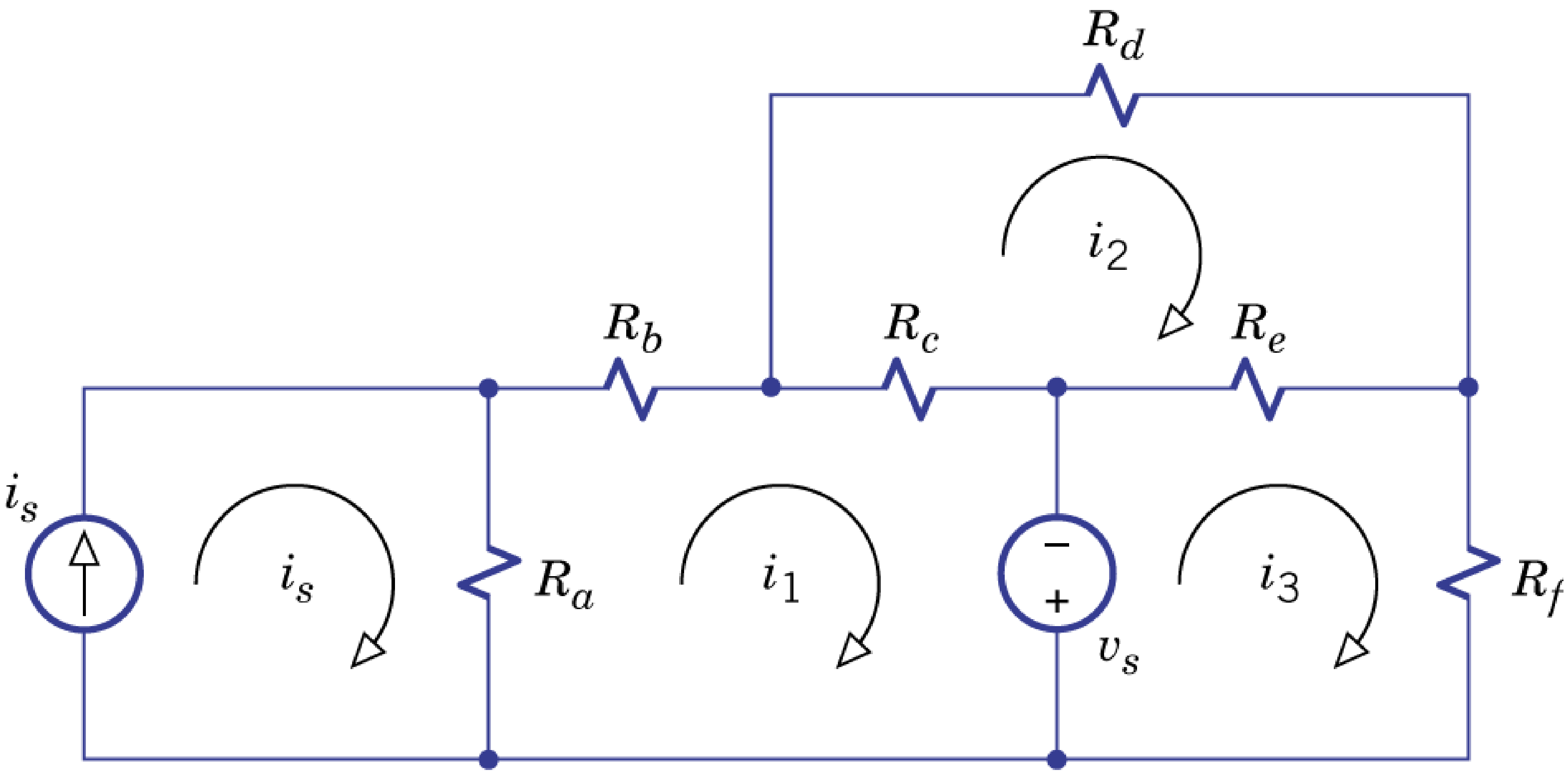
(b) Equivalent series circuit

Mesh equation: $(R_a + R_b) i_1 = v_s + R_a i_s$

Mesh Analysis: General Approach

- Determine the number of mesh KVL equations.
- Write down R and $v_{s:}$
  - By inspection.
  - *Treatment for non-current controlled devices.*
  - *Controlled sources.*
- Solve mesh currents.
- Solve branch voltages and branch currents.

Matrix Mesh Equations



$$\begin{bmatrix} R_a + R_b + R_c & -R_c & 0 \\ -R_c & R_c + R_d + R_e & -R_e \\ 0 & -R_e & R_e + R_f \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} R_a i_s + v_s \\ 0 \\ -v_s \end{bmatrix}$$

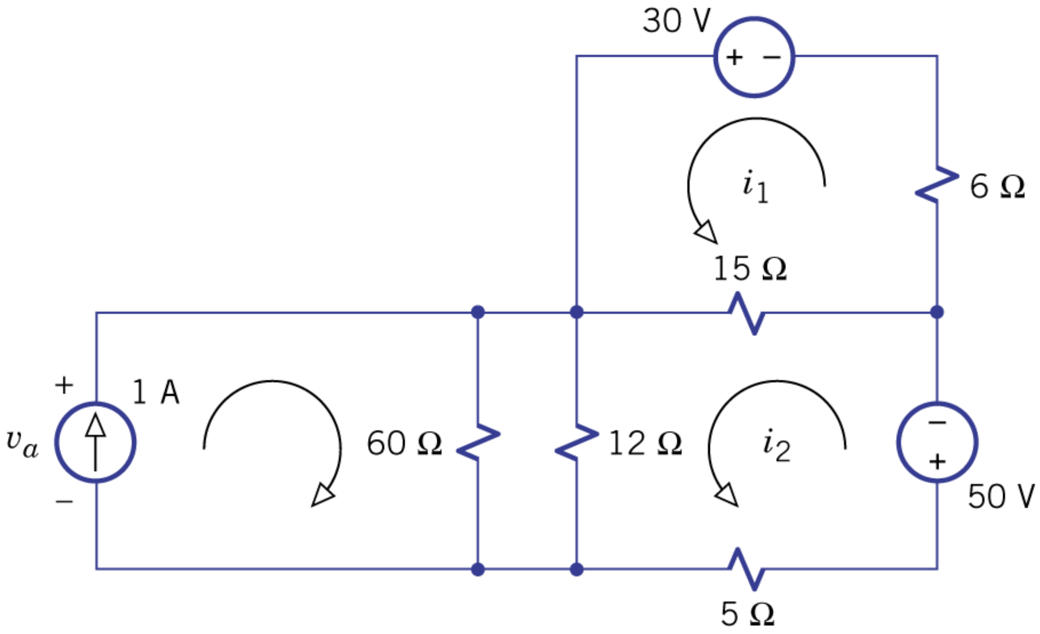
Mesh Analysis: General Form

$$[R][i]=[v_s]$$

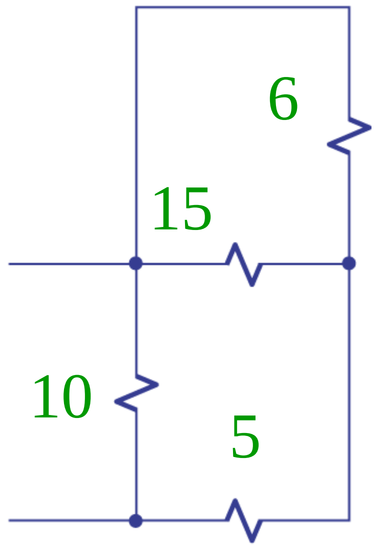
$$[R]=\begin{bmatrix} R_{11} & -R_{12} & \cdots & -R_{1N} \\ -R_{12} & R_{22} & \cdots & -R_{2N} \\ \vdots & \vdots & & \vdots \\ -R_{N1} & -R_{N2} & \cdots & R_{NN} \end{bmatrix} \qquad [i]=\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \qquad [v_s]=\begin{bmatrix} v_{s1} \\ v_{s2} \\ \vdots \\ v_{sN} \end{bmatrix}$$

- $[R]$ is the resistance matrix, R_{nn}= sum of resistances around mesh n, $R_{nm}=R_{mn}$= equivalent resistance shared by meshes n and m.
- Similar to node analysis, the minimum number of unknown mesh currents is determined by suppressing all sources and counting the remaining meshes.

Example 4.6: Two unknowns



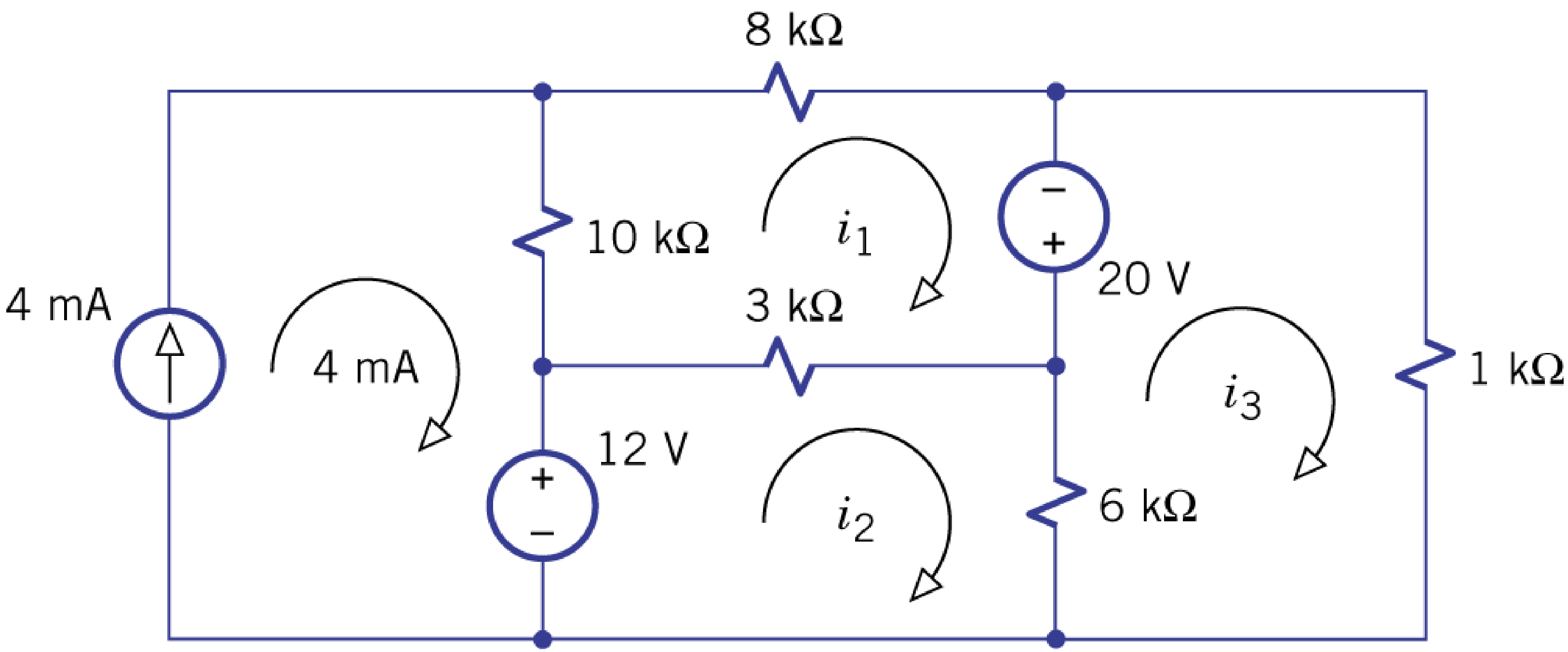
(a) Circuit for mesh analysis



(b) Source suppression leaving two meshes

$$\begin{bmatrix} 6+15 & -15 \\ -15 & 15+10+5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 30 \\ -60 \end{bmatrix}$$

Example 4.7: Three Unknowns

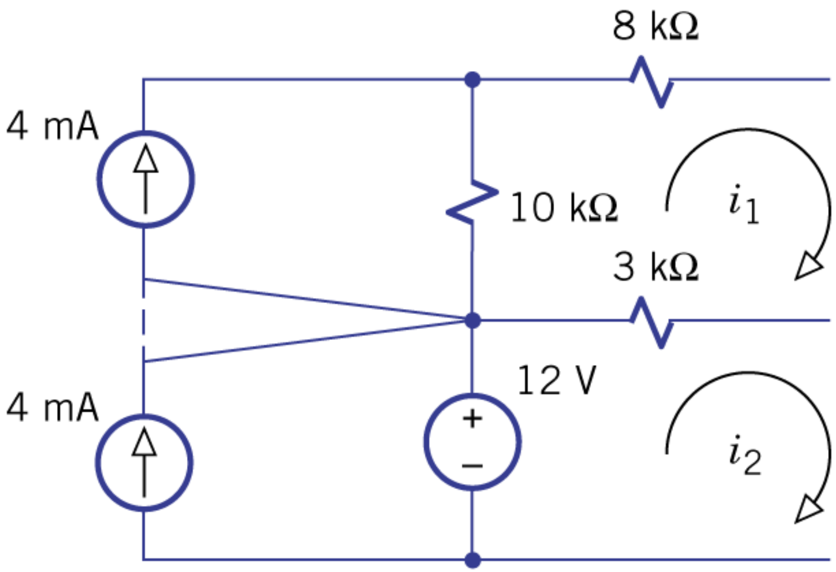


$$\begin{bmatrix} 10+8+3 & -3 & 0 \\ -3 & 3+6 & -6 \\ 0 & -6 & 6+1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 20+40 \\ 12 \\ -20 \end{bmatrix}$$

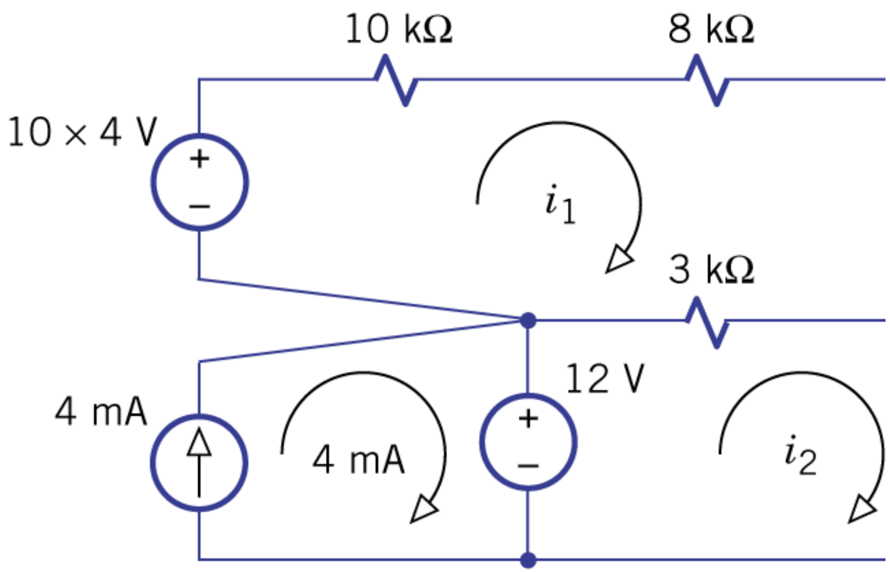
How to Handle Current Sources?

- A current source is non-current controlled, i.e., one cannot specify its voltage given a current.
- Source mesh splitting.
- Interior current sources:
  - With parallel resistance → Source conversion.
  - Interior ideal sources → Introduce a new variable (or use supermesh KVL).

Source Mesh Splitting

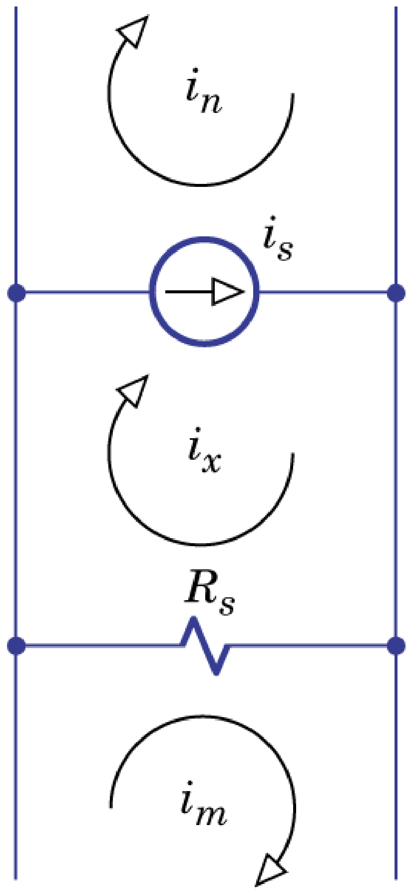


(a) Adding series current source

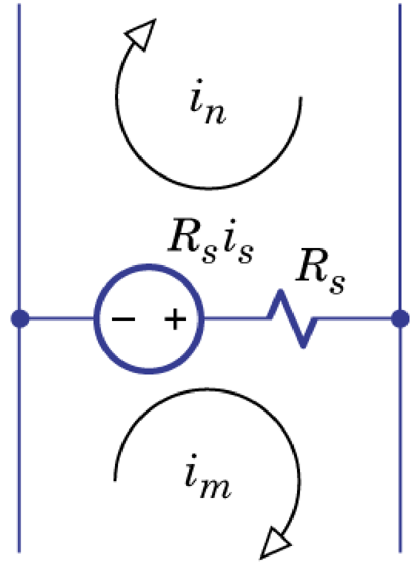


(b) After source conversion

Interior Current Source with Parallel Resistance → Source Conversion

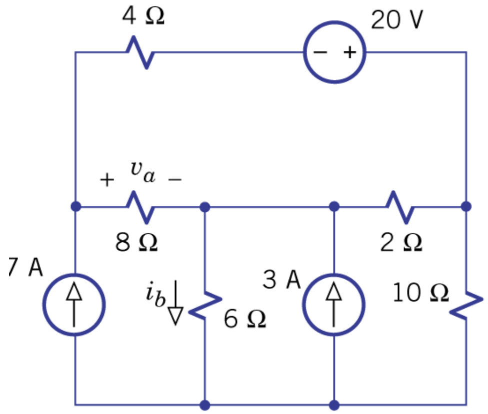


(a) Interior current source with parallel resistance

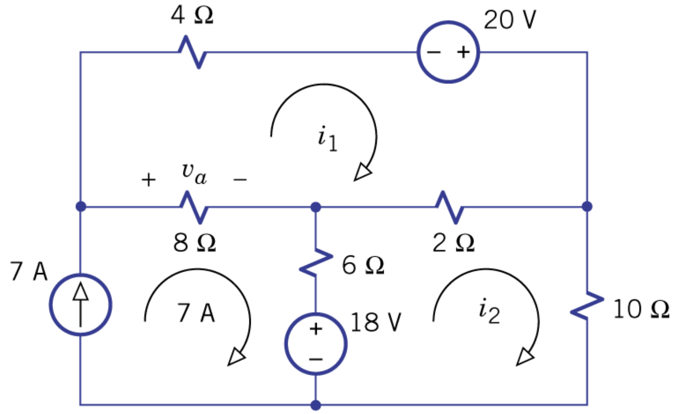


(b) Source conversion eliminating i_x

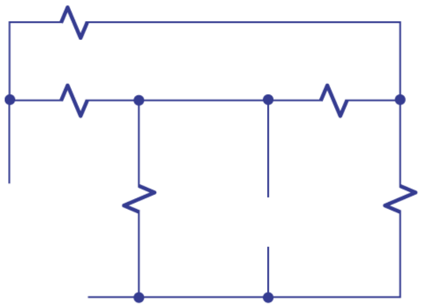
Example 4.8: Source Conversion



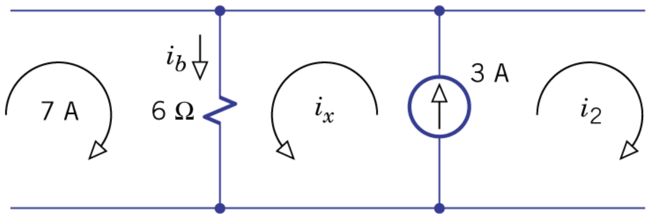
(a) Circuit with four meshes



(c) Diagram for mesh analysis



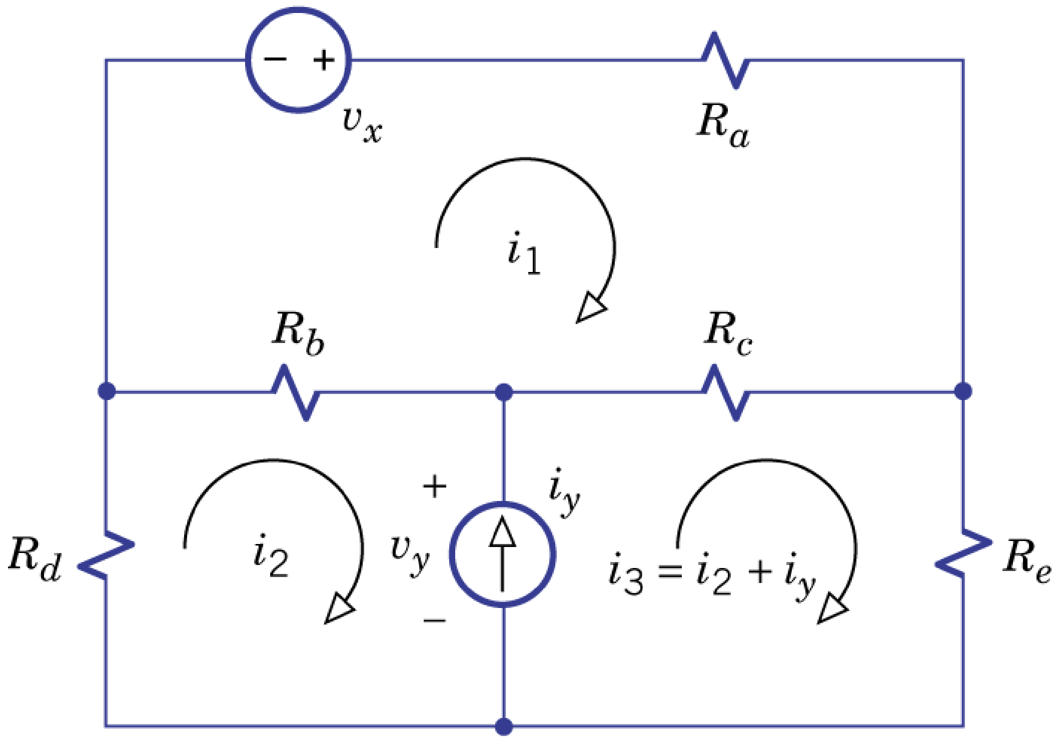
(b) Source suppression leaving two meshes



(d) Partial diagram for i_x

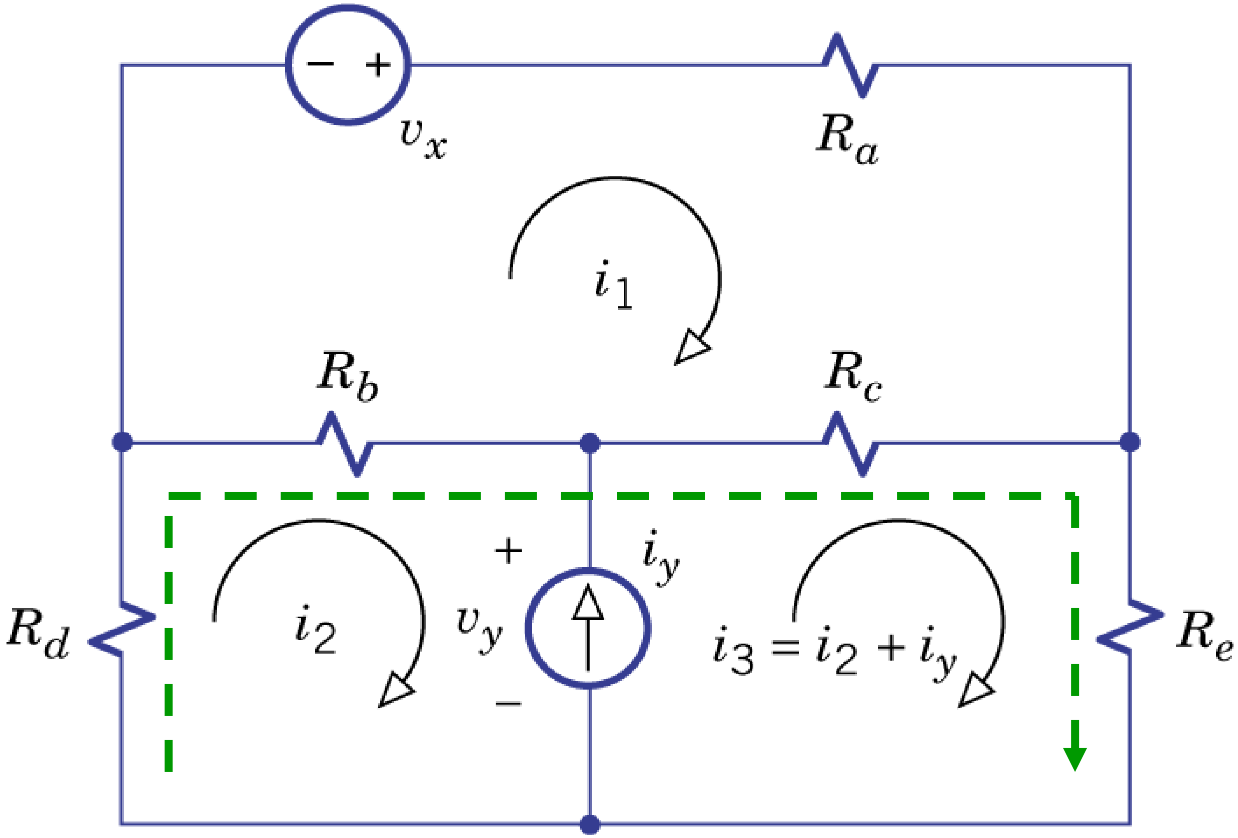
$$\begin{bmatrix} 4+2+8 & -2 \\ -2 & 6+2+10 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 20+56 \\ 18+42 \end{bmatrix}$$

Interior Ideal Sources: A New Variable



- v_y as a fictitious source voltage.
- Three mesh equations with three unknowns: i_1, i_2 and v_y.
- v_y can be eliminated.

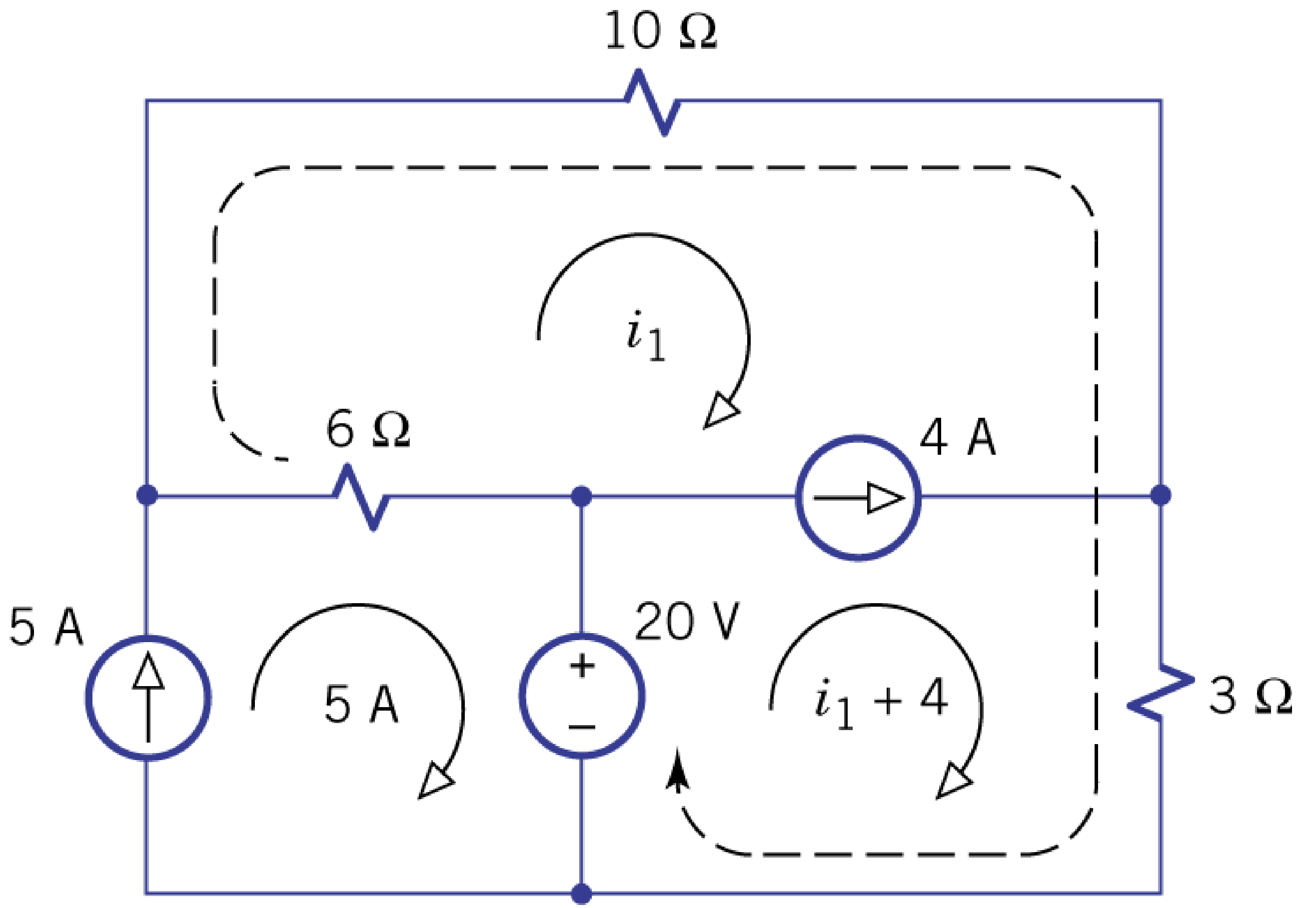
Interior Ideal Sources: Supermesh



$$R_d i_2 + R_b(i_2 - i_1) + R_c(i_2 + i_y - i_1) + R_e(i_2 + i_y) = 0$$

Same as the sum of the previous two mesh equations (2 an 3).

Example 4.9: Supermesh



$$19i_1 = 20 + 5\times 6 - 3\times 4 = 38$$

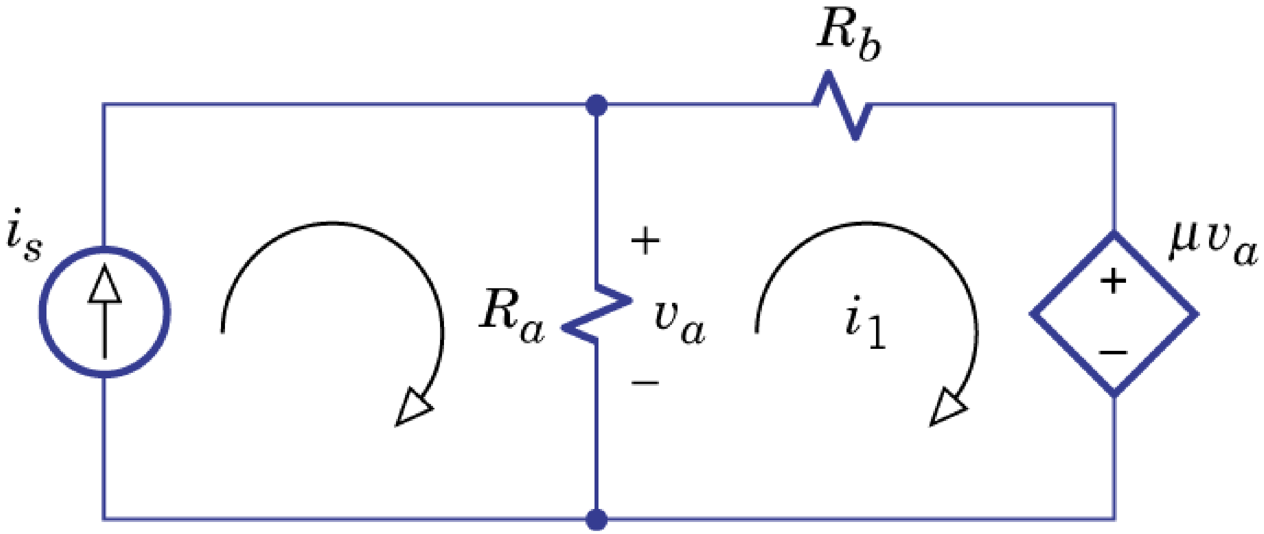
$$i_1 = 2$$

Systematic Analysis with Controlled Sources

Circuits with Controlled Sources

- In the presence of controlled sources, symmetry of the resistance/conductance matrices does not exist.
- Since controlled sources introduce new variables, new equations (constraint equations) are needed. The constraint equations are written in terms of known constants and/or unknown mesh currents/node voltages.

Mesh Analysis with a Controlled Source



$$(R_a + R_b)i_1 = R_a i_s - m v_a$$ unknown

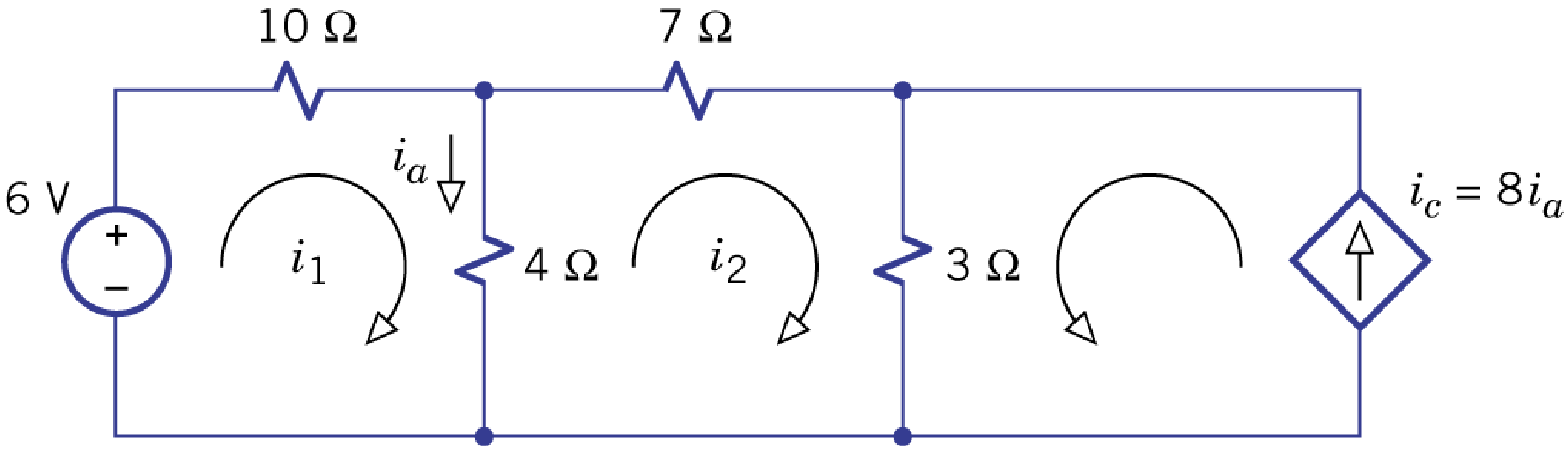
$$v_a = R_a(i_s - i_1)$$ constraint equation

$$(R_a + R_b)i_1 = R_a i_s - mR_a(i_s - i_1) = (1 - m)R_a i_s + mR_a i_1$$

$$(R_{11} - \tilde{R})i_1 = \tilde{v}_s$$

$$\text{General form}: \left[R - \tilde{R}\right][i] = [\tilde{v}_s]$$

Example 4.10: Mesh Analysis with CCCS



$$[R]=\begin{bmatrix}14 & -4\\ -4 & 14\end{bmatrix}$$

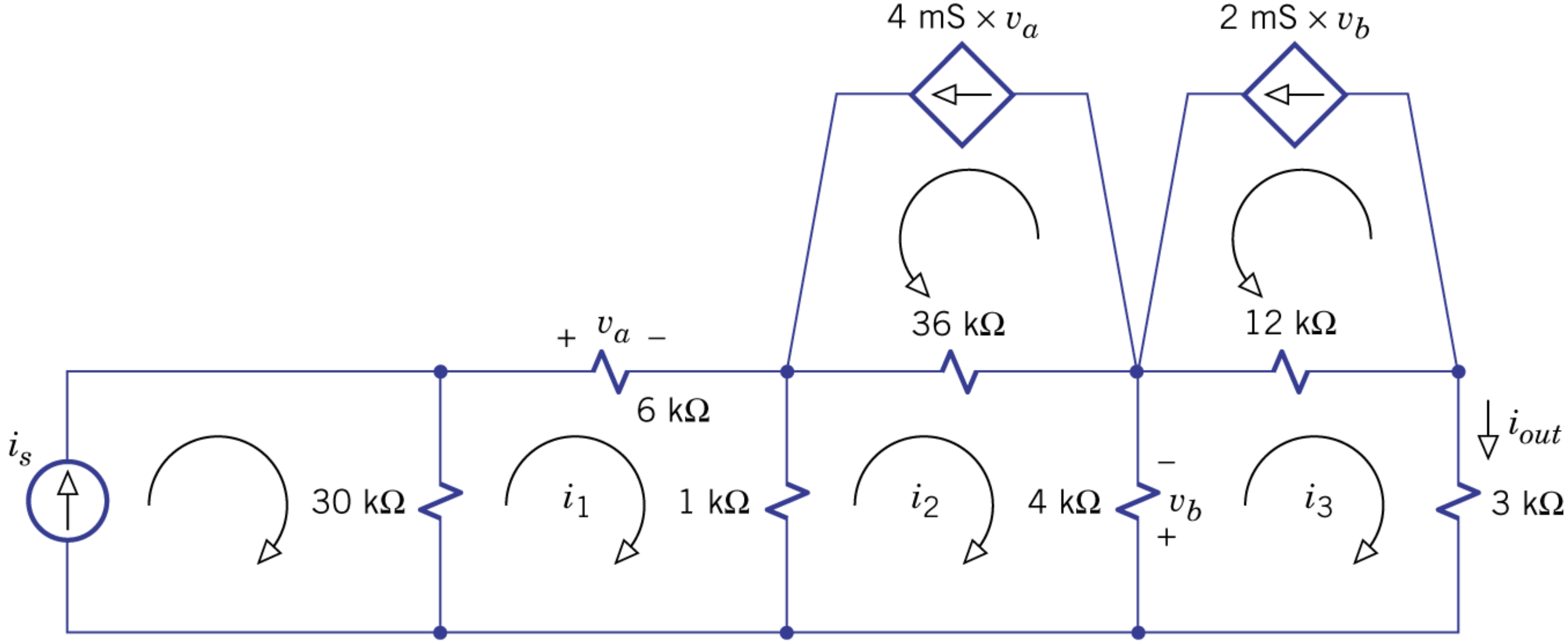
constraint equation: $i_a = i_1 - i_2$

$$[v_s]=\begin{bmatrix}6\\ -3\times 8i_a\end{bmatrix}=\begin{bmatrix}6\\ -24i_1+24i_2\end{bmatrix}=\begin{bmatrix}6\\ 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ -24 & 24\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}=[\tilde{v}_s]+[\tilde{R}][i]$$

$$[R-\tilde{R}]=\begin{bmatrix}14 & -4\\ 20 & -10\end{bmatrix}, \qquad \begin{bmatrix}14 & -4\\ 20 & -10\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}=\begin{bmatrix}6\\ 0\end{bmatrix}$$

$$i_1 = 1A,\ i_2 = 2A$$

Example 4.11: Current Amplifier



$$[R]=\begin{bmatrix}37 & -1 & 0\\ -1 & 41 & -4\\ 0 & -4 & 19\end{bmatrix}$$

$$[v_s]=\begin{bmatrix}30i_s\\ -144v_a\\ -24v_b\end{bmatrix}=\begin{bmatrix}30i_s\\ -864i_1\\ 96i_2-96i_3\end{bmatrix}=\begin{bmatrix}30i_s\\ 0\\ 0\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ -864 & 0 & 0\\ 0 & 96 & -96\end{bmatrix}\begin{bmatrix}i_1\\ i_2\\ i_3\end{bmatrix}$$

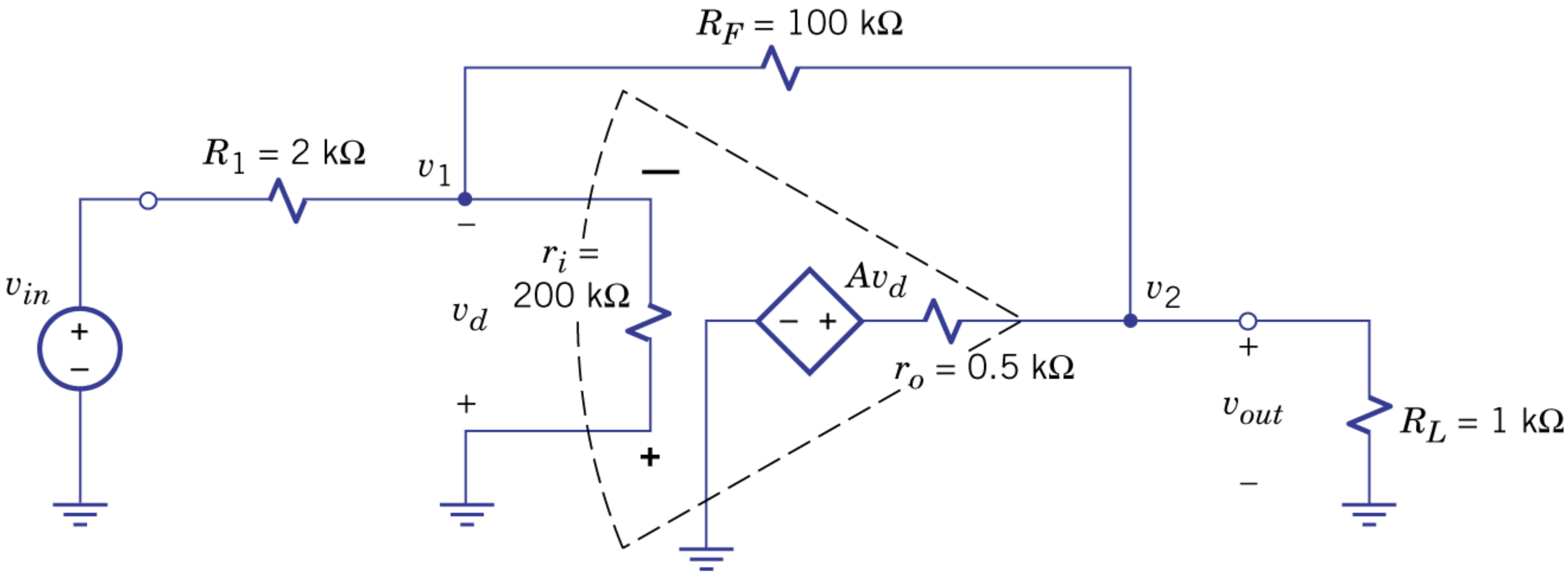
constraint equations: $v_a=6i_1$, $v_b=4(i_3-i_2)$

$$\left[R-\tilde{R}\right][i]=\left[\tilde{v}_s\right]$$

Node Analysis with Controlled Sources

$$\left[G - \tilde{G}\right]\left[v\right] = \left[\tilde{i}_s\right]$$

Example 4.12: Inverting Amplifier



$$[G]=\begin{bmatrix} \frac{1}{2}+\frac{1}{100}+\frac{1}{200} & -\frac{1}{100} \\ -\frac{1}{100} & 1+\frac{1}{100}+\frac{1}{0.5} \end{bmatrix} \qquad v_d=-v_1$$

$$[i_s]=\begin{bmatrix} \frac{v_{in}}{2} \\ \frac{A(-v_1)}{0.5} \end{bmatrix}=\begin{bmatrix} 0.5v_{in} \\ 0 \end{bmatrix}+\begin{bmatrix} 0 & 0 \\ -2A & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Chapter 4: Problem Set

- 1, 5, 8, 11, 17, 23, 32, 34, 38, 44, 49, 51, 60