Chapter 2: Properties of Resistive Circuits

Chapter 2: Outline

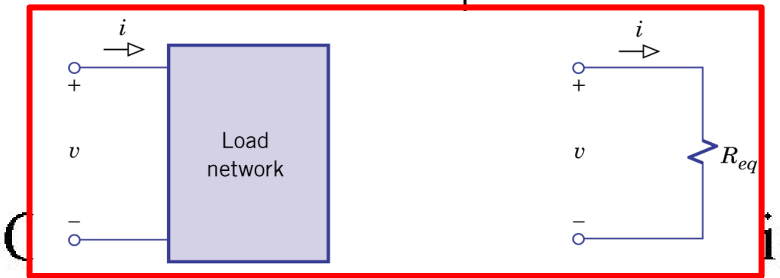
*Self-reading: 2.1 and 2.2.

Linear Resistor

Series and Parallel, Duality

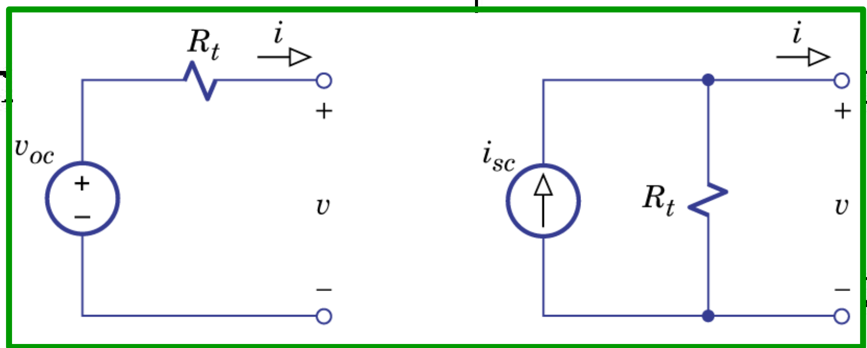
Real Dual

Structural Dual



C istive)

Load Network



Lin osition

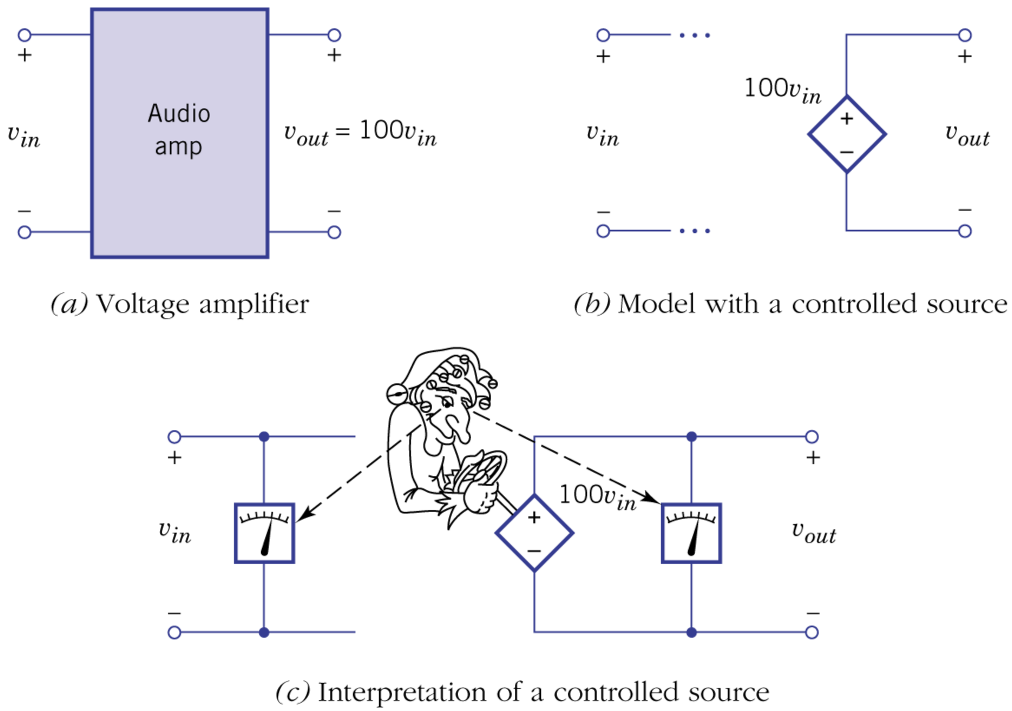
t

Source Network

Circuits with Controlled Sources

Independent vs. Dependent Sources

- <u>Independent sources:</u> the source voltage or current does not depend on any other voltage or current.
- <u>Dependent (or controlled) sources:</u> the source voltage or current depends on other voltage or current.

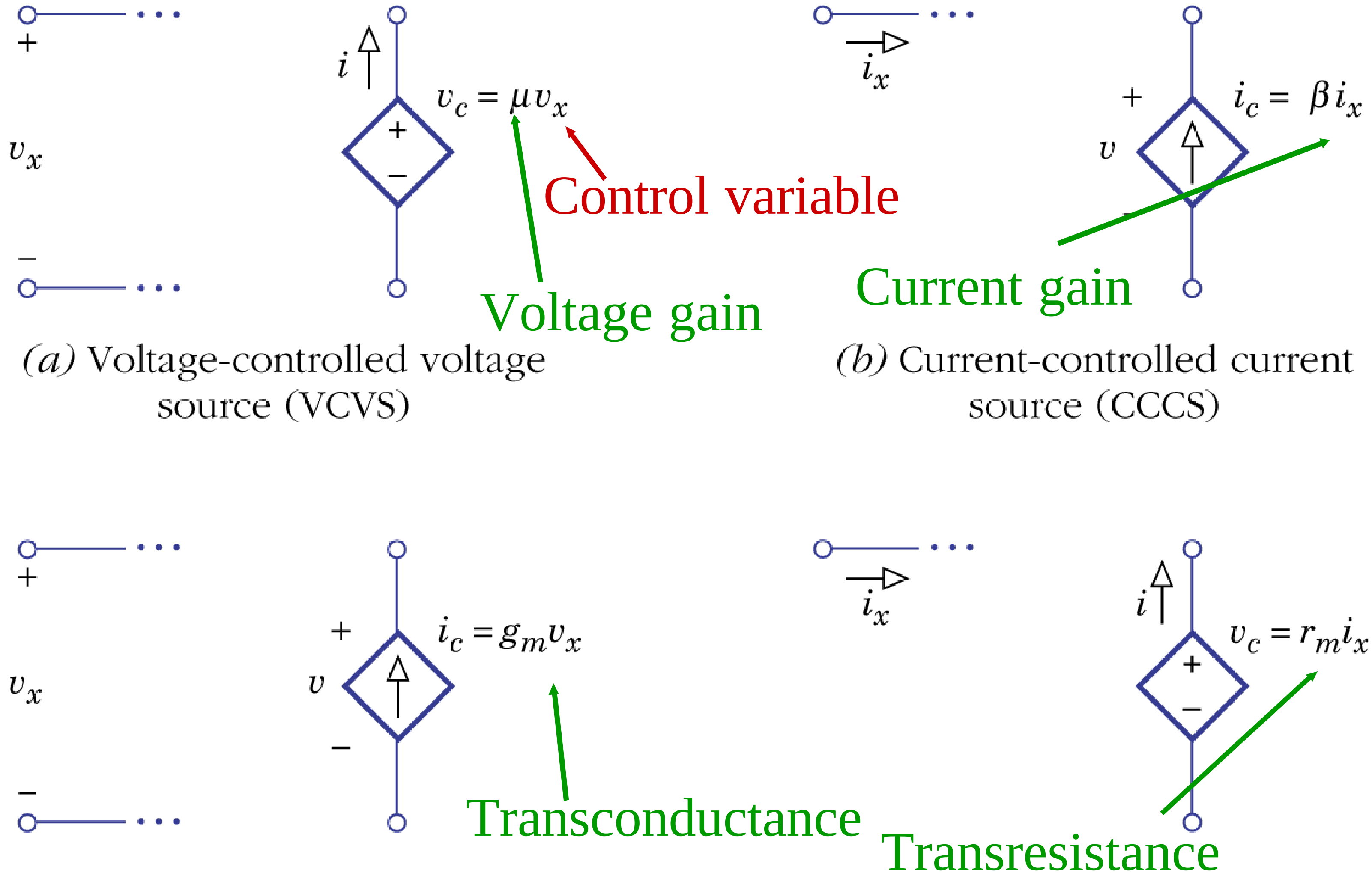


(a) Voltage amplifier

(b) Model with a controlled source

(c) Interpretation of a controlled source

Types of Controlled Sources



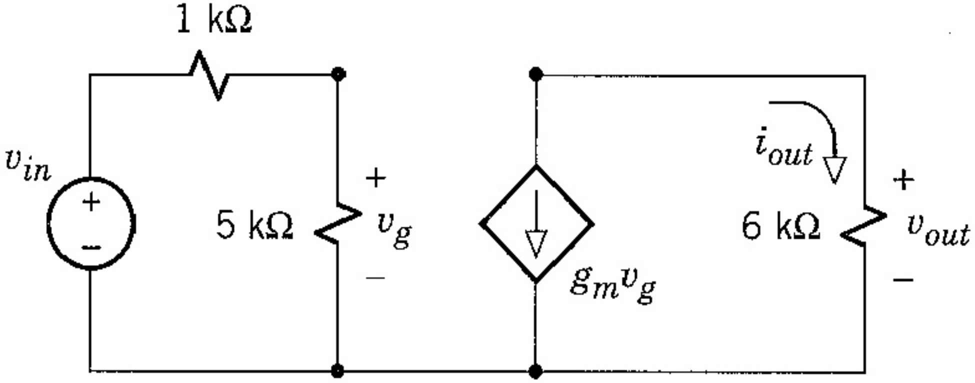
(a) Voltage-controlled voltage source (VCVS)

(b) Current-controlled current source (CCCS)

(c) Voltage-controlled current source (VCCS)

(d) Current-controlled voltage source (CCVS)

Example 2.6: Amplifier with a Field-Effect Transistor



g_m=5 mS
Find v_{out} in terms of v_{in}.
Note: Consistency of units.
(kΩ, V, mA, mS)

$$v_{out} = 6 \cdot i_{out}$$

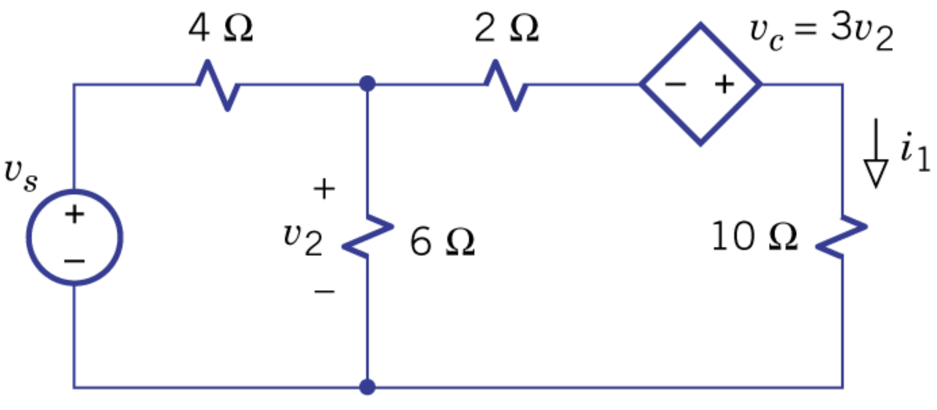
$$i_{out} = -g_m \cdot v_g$$

$$v_g = \frac{5}{6} v_{in}$$

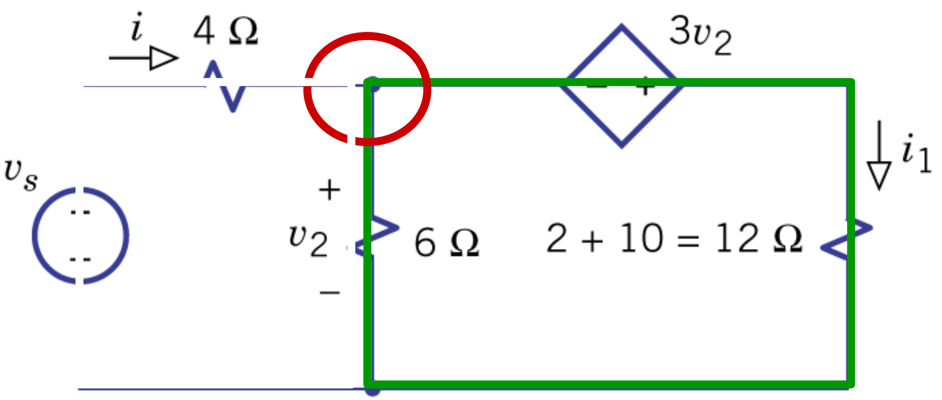
$$v_{out} = -25 \cdot v_{in}$$

Inverting amplifier

Example 2.7



(a) Circuit with a VCVS



(b) Simplified diagram

(1). KVL: $v_2=3i_1$

(2). KCL: $i=1.5i_1$

(3). KVL: $i_1=v_s/9$

<u>*All variables are zero if $v_s=0$.*</u>

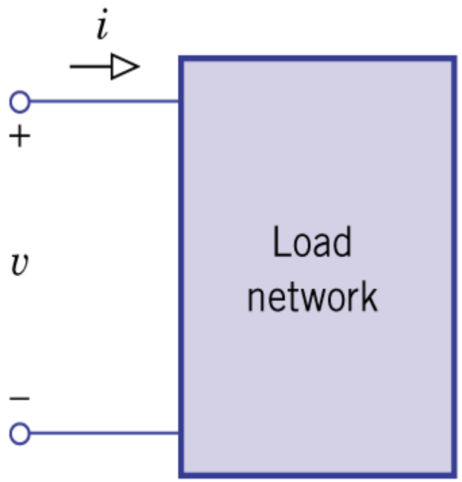
Load Network

- <u>Load network</u>: any two-terminal network that contains no independent sources. If controlled sources are included, the control variables must be within the same network.
- <u>Equivalent resistance theorem</u>: when a load network consists entirely of resistances or resistances and controlled sources, the terminal voltage and current are related by:

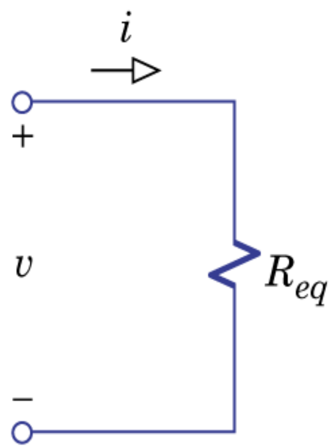
$$v = R_{eq} i, \text{ where } R_{eq} \text{ is a constant}$$

Load Network

$$v = R_{eq} i, \text{ where } R_{eq} \text{ is a constant}$$



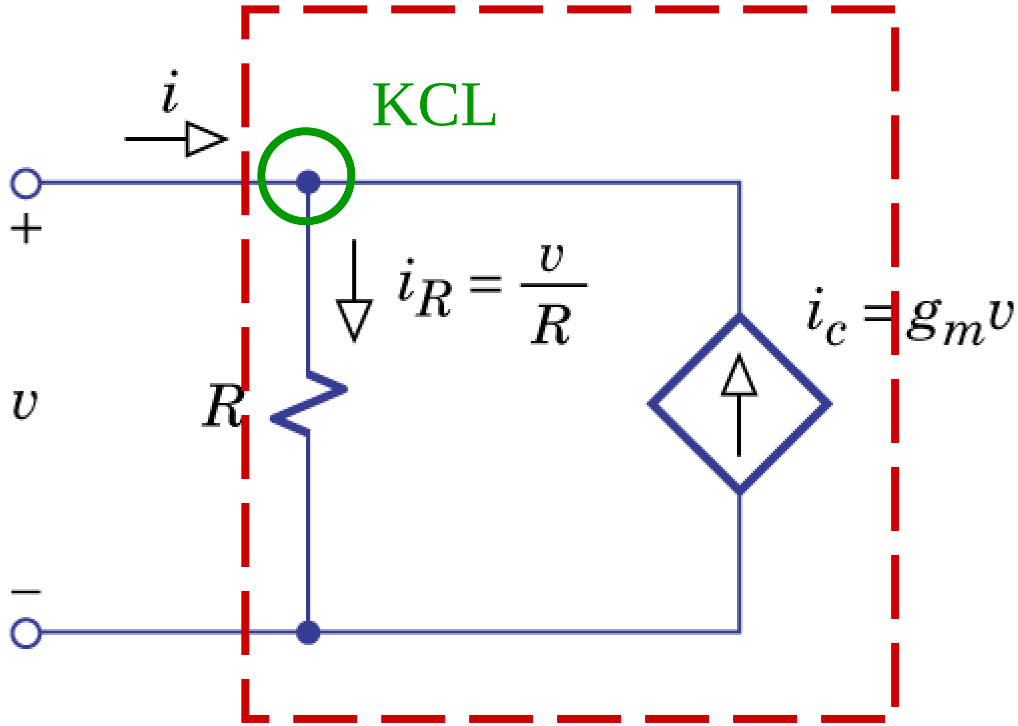
(a) Load network containing resistors and controlled sources



(b) Equivalent resistance $R_{eq} = v/i$

A combination of *i*-*v* curves.

Example 2.8: Equivalent Resistance



$$i = \frac{v}{R} - g_m v = (\frac{1}{R} - g_m)v$$

$$R_{eq} = \frac{v}{i} = \frac{R}{1 - g_m R}$$

Depending on g_m
R_{eq} can be positive,
infinite (open circuit),
negative (provide energy)

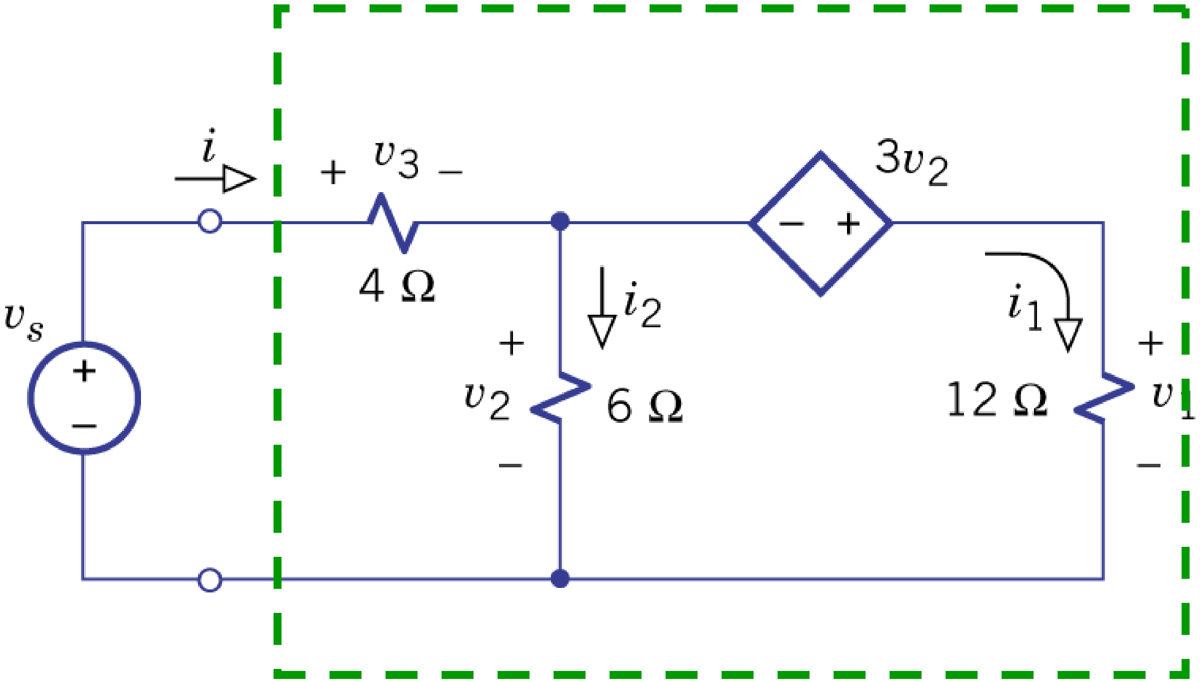
Linearity and Superposition

Linearity and Proportionality

- A circuit is linear if it consists entirely of linear elements (e.g. , controlled sources, linear resistors) and independent sources.
- For a linear function , where *x* is the input and *y* is the response, both the proportionality and the superposition properties need to be satisfied.
- Proportionality property:

$$f(Kx) = Kf(x) = Ky$$

Example 2.9: Circuit Analysis Using Proportionality



Set i_1=1

v_2=3, i_2=1/2

i=3/2, v_s=9

$$\frac{\hat{v}_s}{v_s} = 8 \Rightarrow \hat{i}_1 = 8i_1 = 8$$

In example 2.7, we found: $i_1 = v_s/9$

$\hat{v}_s = 72V$, find the other variables.

$$R_{eq} = \frac{v_s}{i} = \frac{\hat{v}_s}{\hat{i}} = 6\Omega$$

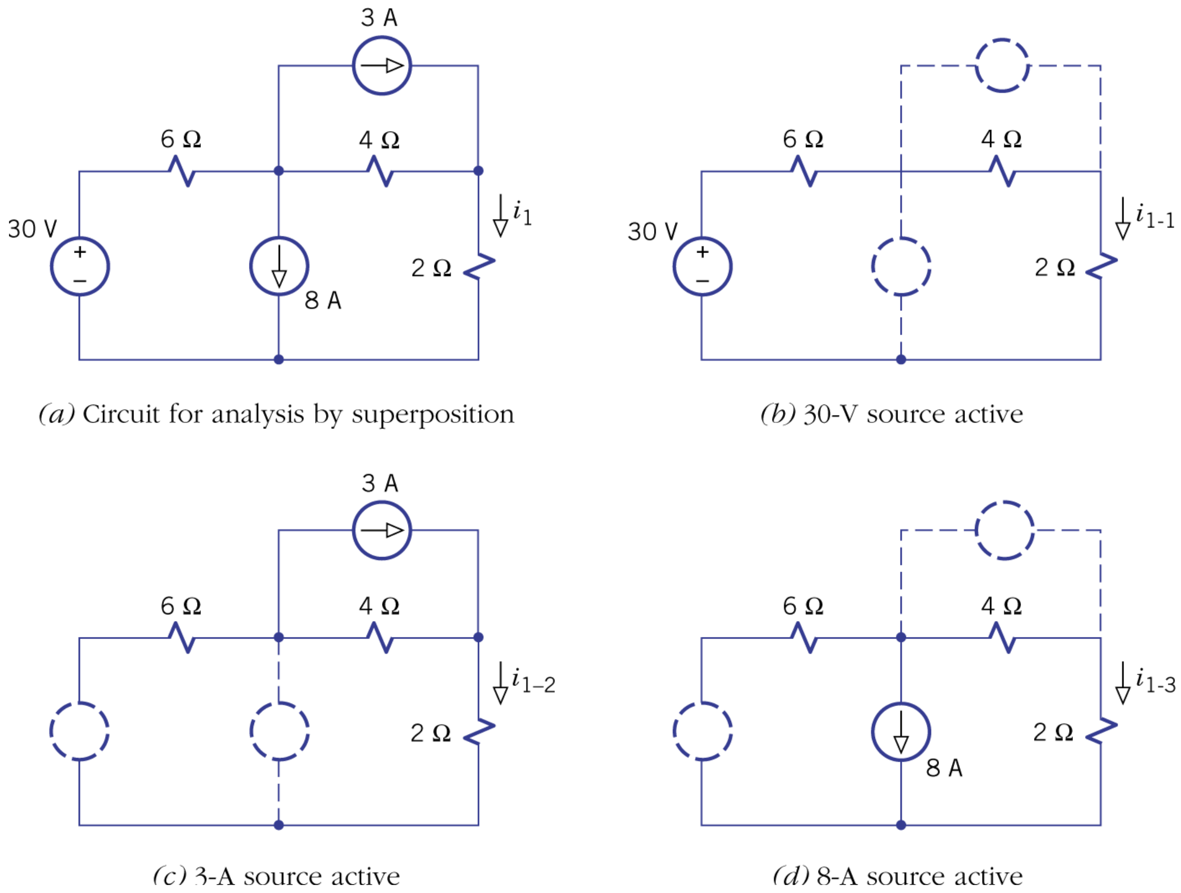
Equivalent resistance

Superposition

$$f(x_a + x_b) = f(x_a) + f(x_b) = y_a + y_b$$

- For a linear circuit containing two or more independent sources, the value of any branch variable is the algebraic sum of the individual contributions from each source with all other independent sources set to zero.
- Suppressed sources: Zero independent voltage source is short circuit, zero independent current source is open circuit. Controlled sources are not suppressed.

Example 2.10: Superposition



$$i_{1-1} = \frac{30}{12} = 2.5$$

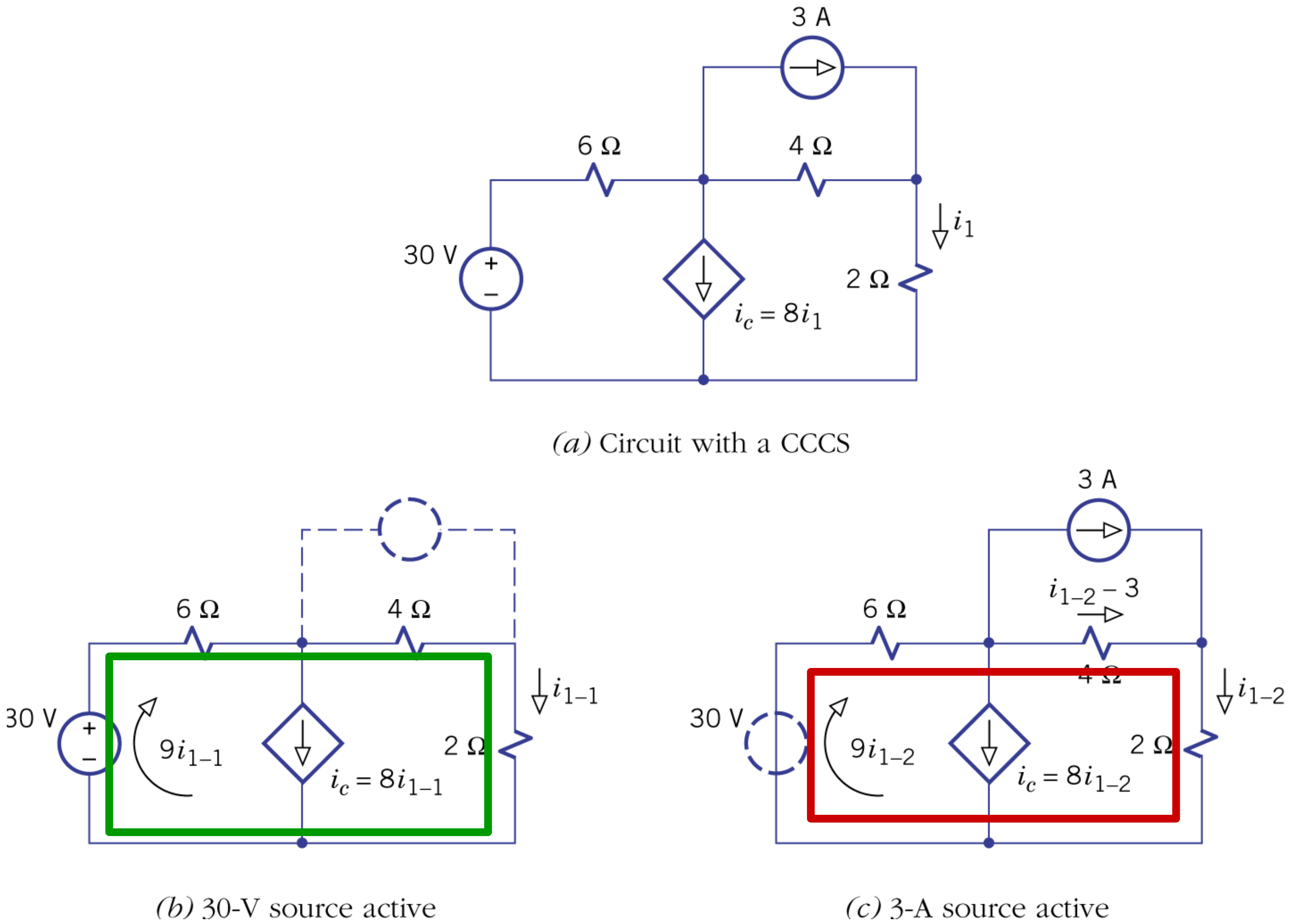
$$i_{1-2} = 3 \times \frac{4}{12} = 1$$

$$i_{1-3} = -\frac{8}{2} = -4$$

$$i_1 = i_{1-1} + i_{1-2} + i_{1-3} = -0.5A$$

Find i_1.

Example 2.11: Superposition with a Controlled Source



$i_{1\text{-}1}=0.5$

$i_{1\text{-}2}=0.2$

$i_1=0.5+0.2=0.7\text{A}$

Linear Circuits:
Proportionality and Superposition

$$f(K_a x_a + K_b x_b) = K_a f(x_a) + K_b f(x_b) = K_a y_a + K_b y_b$$

$$\Downarrow$$

$$v\,(\text{or } i) = \sum_{i=1}^{a} H_i \cdot v_{si} + \sum_{j=1}^{b} K_j \cdot i_{sj}$$

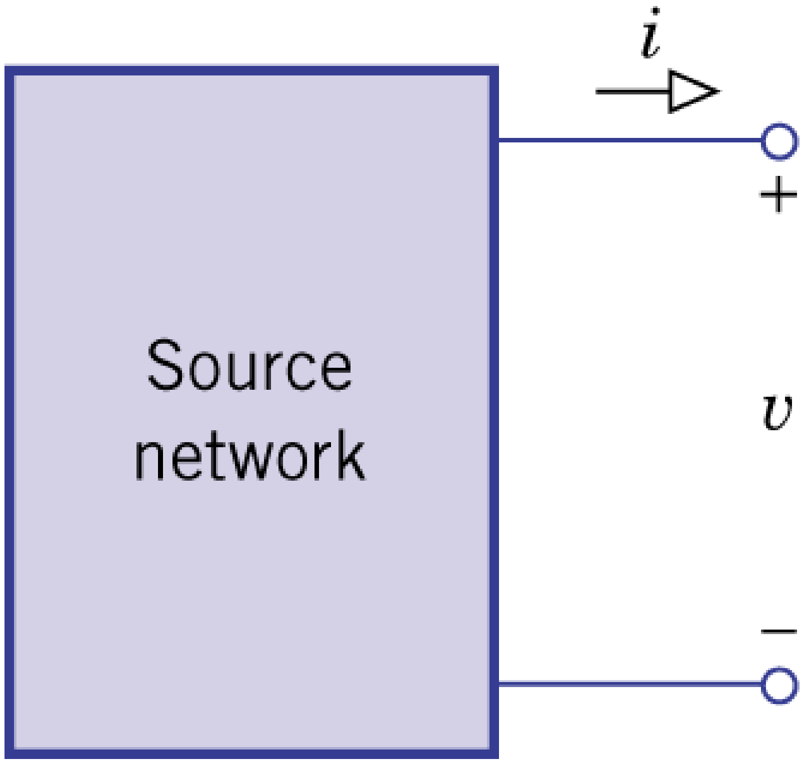
Power dissipation ($p=vi=Ri^2$) by a linear resistor is not linear.

$$R(i_1 + i_2)^2 \neq Ri_1^2 + Ri_2^2$$

Thévenin and Norton Networks

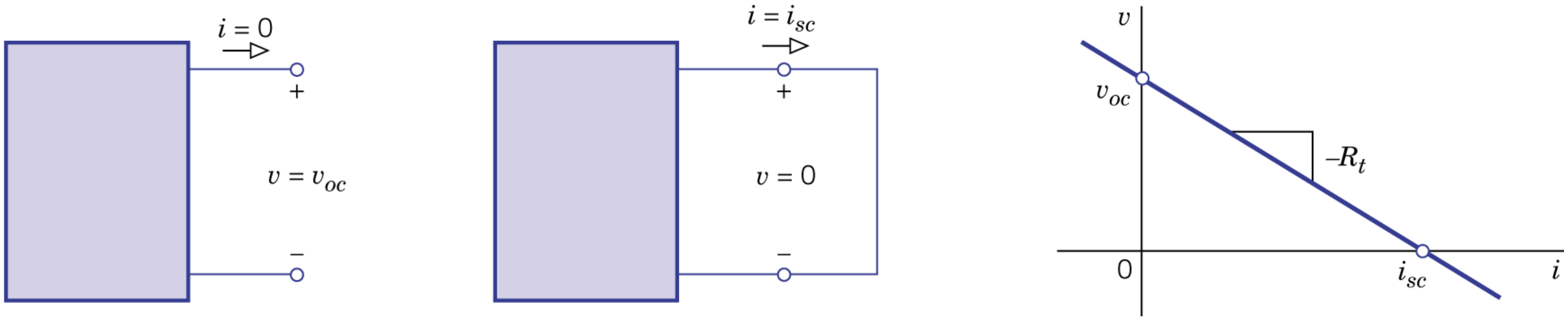
Source Network

- A source network is any two-terminal network that consists entirely of linear elements and <u>at least one independent source</u>. The control variables of all controlled sources (if any) must be within the same network.



Thévenin Parameters

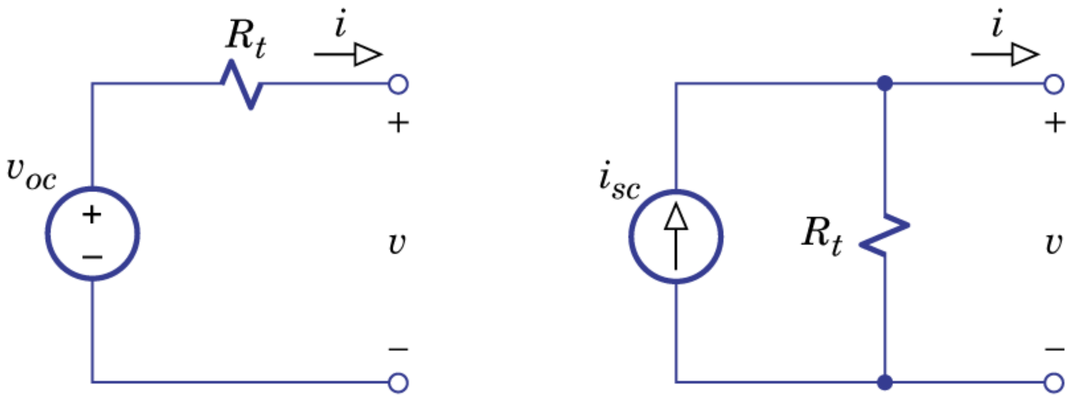
- Open-circuit voltage: $v_{oc} \equiv v\big|_{i=0}$
- Short-circuit current: $i_{sc} \equiv i\big|_{v=0}$
- Thévenin resistance: $R_t \equiv v_{oc} / i_{sc}$



(a) Open-circuit voltage *(b)* Short-circuit current *(c)* v–i curve

Thévenin’s and Norton’s Theorem

- *Thévenin's theorem*: Any linear resistive source network acts at its terminals like an ideal voltage source of value v_{oc} in series with a resistor of R_t, i.e., $v = v_{oc} - R_t i$.
- *Norton's theorem*: Any linear resistive source network acts at its terminals like an ideal current source of value i_{sc} in parallel with a resistor of R_t, i.e., $i = i_{sc} - v/R_t$.



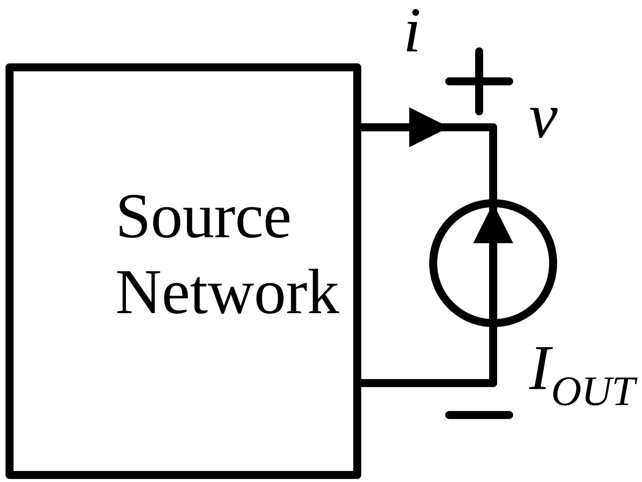
(a) Thévenin equivalent network

(b) Norton equivalent network

v_{oc}, i_{sc} and R_t are Thévenin parameters.

Thévenin's and Norton's Theorem

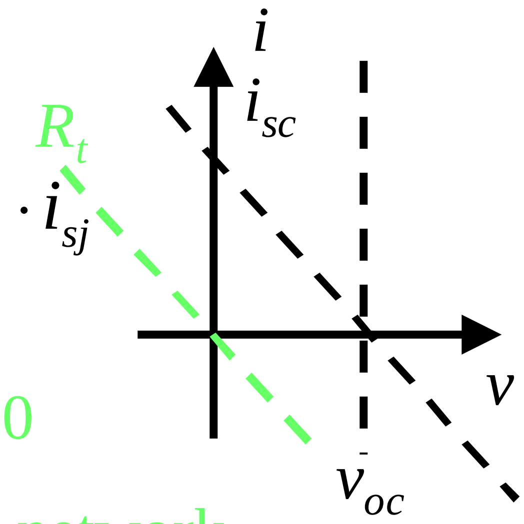
- Duality between Thévenin and Norton.
- Proof of Thévenin's theorem by superposition:



a voltage sources
b current sources

(1). $I_{OUT} = 0$

$$v_{\mathrm{oc}} = \sum_{i=1}^{a} H_i \cdot v_{si} + \sum_{j=1}^{b} K_j \cdot i_{sj}$$



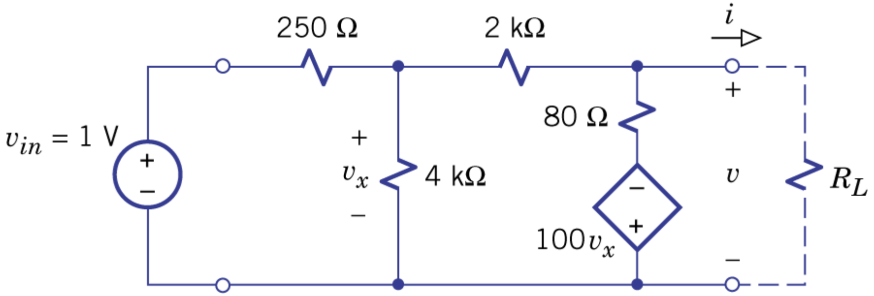
(2). All inside sources = 0

Source network → Load network

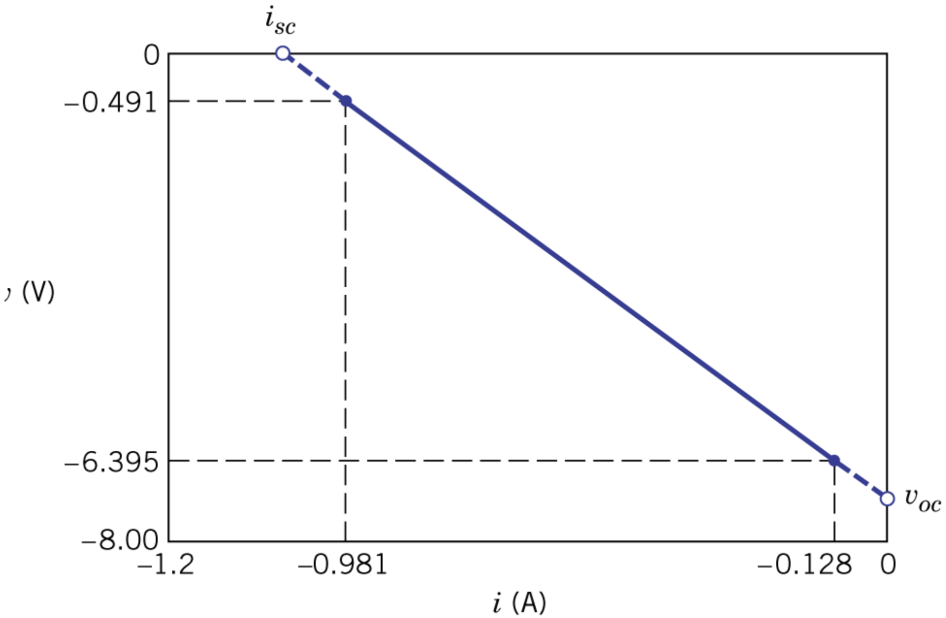
Reduces to a single resistor

(3). Superposition

Example 2.12: Thévenin Parameters from a *v*-*i* Curve



(a) Circuit diagram of an inverting voltage amplifier

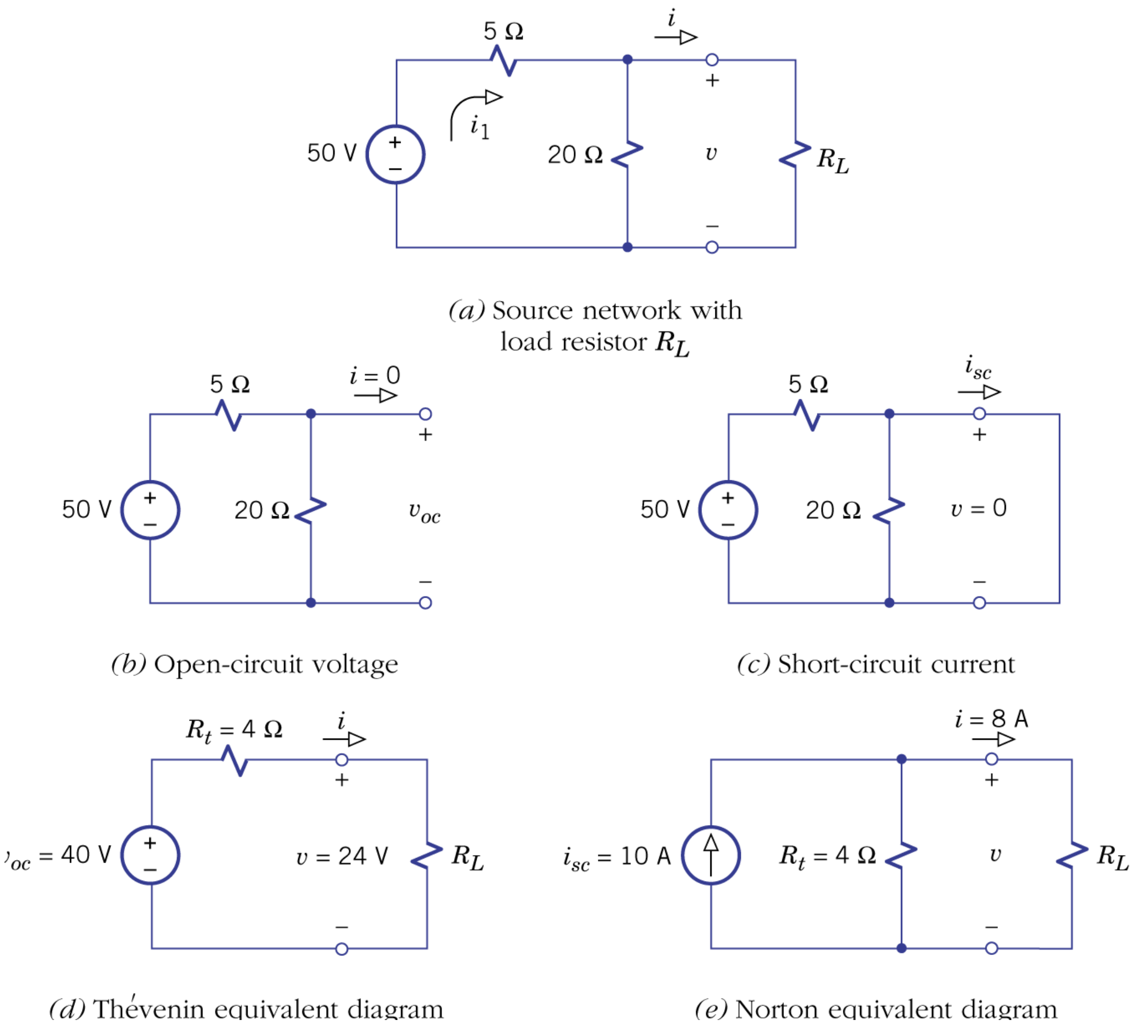


(b) *v*-*i* curve obtained by PSpice simulation

Use PSpice simulations

Find $v = v_{oc} - R_t i$ at two different R_L's.

Example 2.13: Equivalent Source Network



(a) Source network with load resistor R_L

(b) Open-circuit voltage

(c) Short-circuit current

(d) Thévenin equivalent diagram

(e) Norton equivalent diagram

$$R_t = 5 \| 20 = 4\Omega$$

$$v_{OC} = 40V$$

$$i_{SC} = 10A$$

$R_L = 6\ \Omega$

or

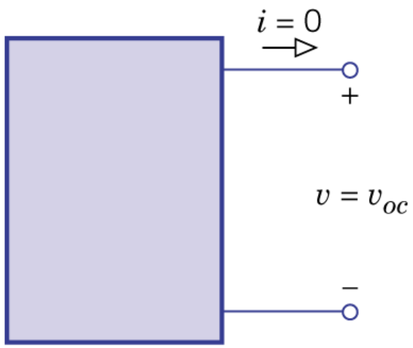
$R_L = 1\ \Omega$

Find R_L such that v=24V or i=8A.

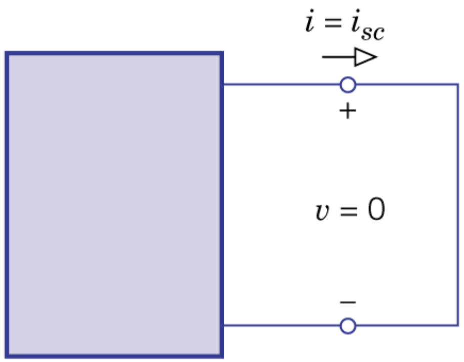
Thévenin Parameters

- Thévenin resistance: the equivalent resistance of a source network when all independent sources have been suppressed (i.e., turned off).

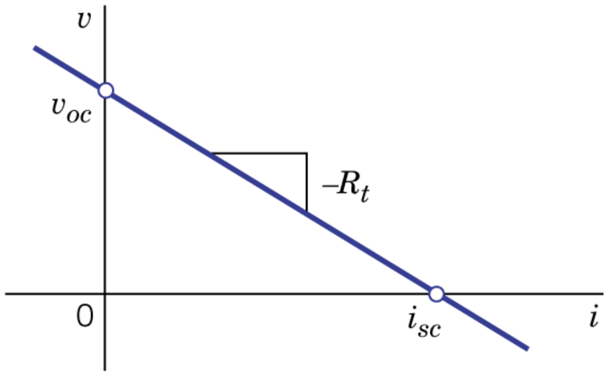
$$v_{oc} = R_t i_{sc} \qquad i_{sc} = v_{oc} / R_t$$



(a) Open-circuit voltage

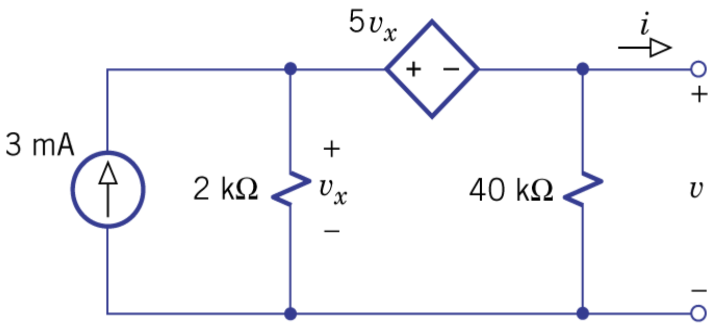


(b) Short-circuit current

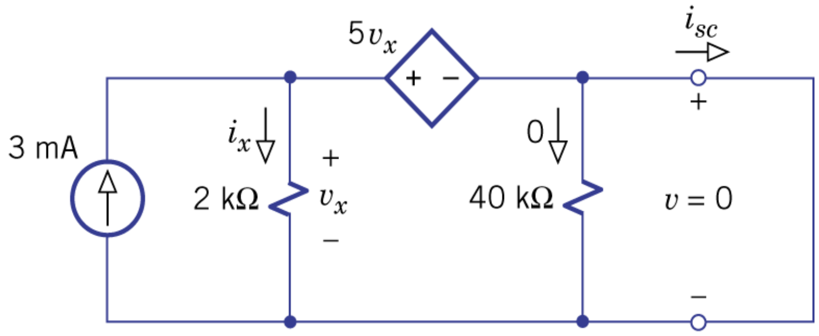


(c) v–i curve

Example 2.14: Calculating Thévenin Parameters

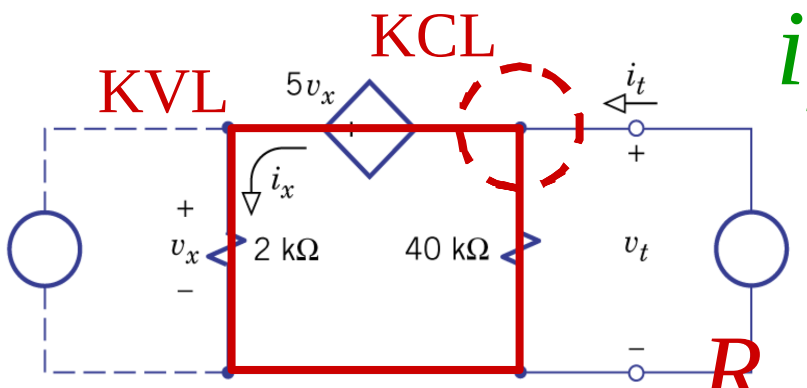


(a) Source network with a VCVS



(b) Short circuit at the output

$$i_{sc} = 3mA$$

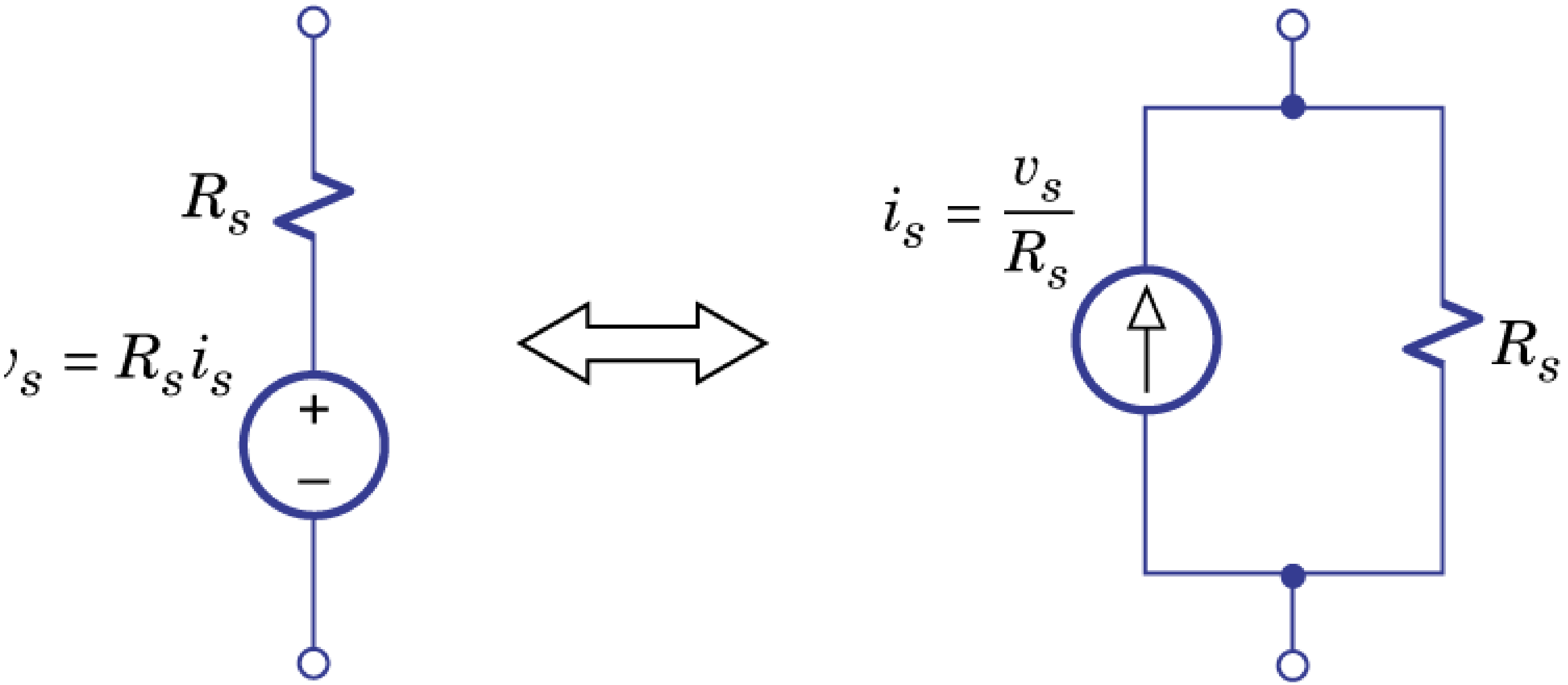


(c) Dead network with test source

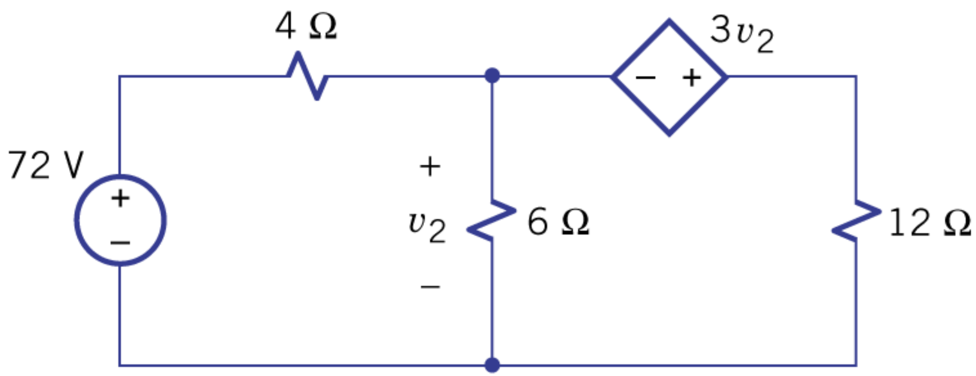
$$R_t = -10k\Omega$$

$$v_{oc} = R_t \cdot i_{sc} = -30V$$

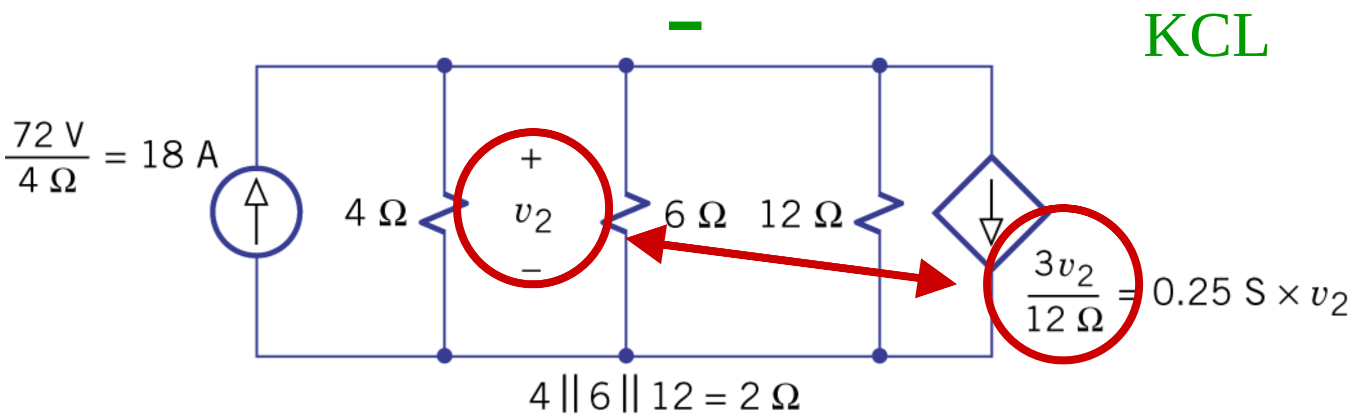
Source Conversions



Example 2.15:
Circuit Reduction by Source Conversion



(a) Circuit for analysis by source convertion



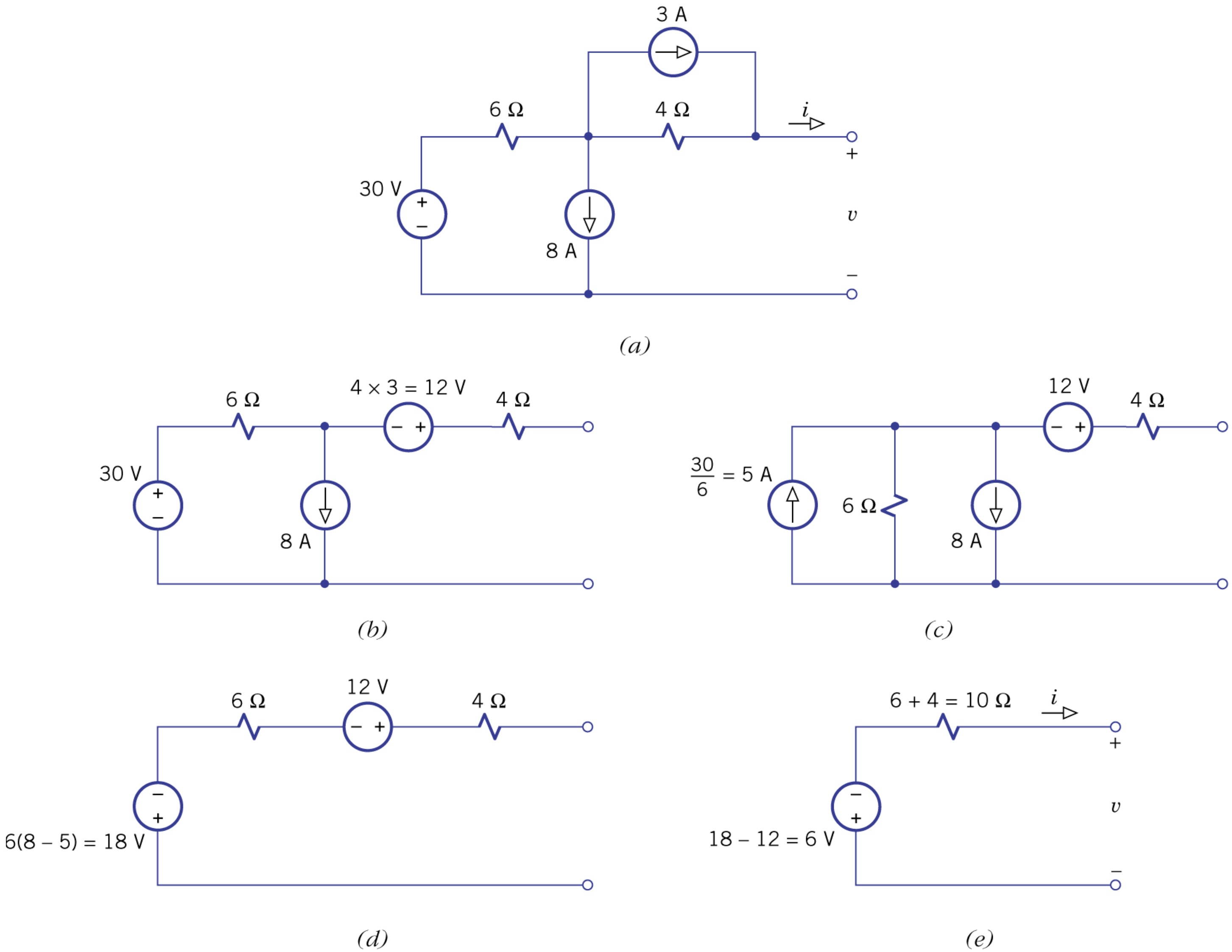
(b) Equivalent parallel circuit

KCL

$18 = v_2/2 + v_2/4$

$v_2 = 24V$

Example 2.16:
Thévenin Network via Source Conversions



Chapter 2: Problem Set

- 23, 29, 35, 36, 40, 46, 55, 57, 61, 67, 71, 77