6.7

$$A = (1+jx)/(1+jy) = (1+jx)(1-jy)/(1+y^2)$$

$$= [1 + xy + j(x-y)]/(1+y^2) \Rightarrow \angle A = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$A = [(1+x^2)^{1/2} \angle \tan^{-1}x]/[(1+y^2)^{1/2} \angle \tan^{-1}y] \Rightarrow \angle A = \tan^{-1}x - \tan^{-1}y$$

$$\therefore \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y$$

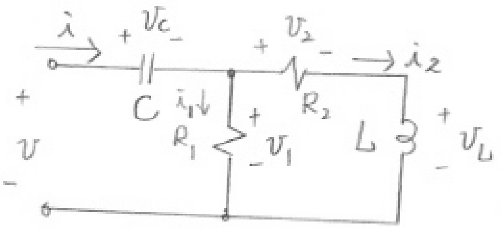
6.17

$$\underline{I} = 2.6\angle -22.6^\circ - 9\angle -90^\circ - 12\angle -66.4^\circ$$

$$= (2.4 - j1.0) - (0 - j9) - (4.8 - j11) = -2.4 + j19$$

$$= 19.2\angle 97.2^\circ \Rightarrow i = 19.2\cos(\omega t + 97.2^\circ) \text{ A}$$

6.24



$$Z_1 = 20 \parallel (4+j8) = 5+j5 = 7.07\angle 45^\circ$$

$$Z = -10j + Z_1 = 7.07\angle -45^\circ$$

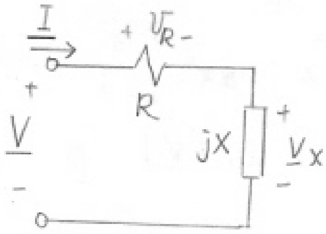
$$\underline{I} = 10/Z = 1.41\angle 45^\circ \Rightarrow i = 1.41\cos(2000t + 45$$

$$\underline{V_C} = -j10\,\underline{I} = 14.1\angle -45^\circ \Rightarrow v_C = 14.1\cos(2000t - 45^\circ) \text{ V}$$

$$\underline{V_1} = Z_1\underline{I} = 10\angle 90^\circ \Rightarrow v_1 = 10\cos(2000t + 90^\circ) \text{ V}$$

$$\underline{V_C} + \underline{V_1} = 10 - j10 + j10 = 10\angle 0^\circ = \underline{V}$$

6.32

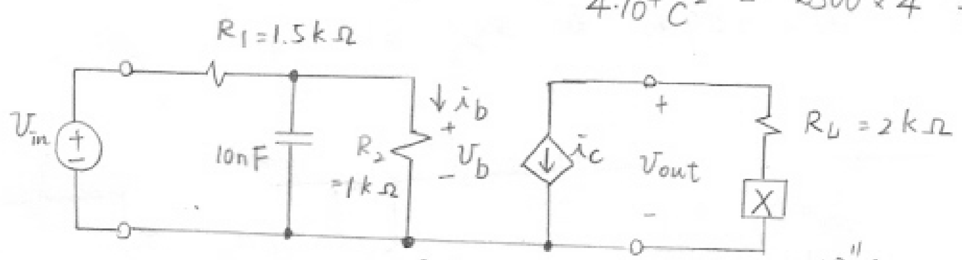


$X = -\frac{1}{\omega C}$ since $|\underline{V}_x|$ decreases as ω increases

$$\underline{V_R}/\underline{V} = 50/\left(50 - \frac{j}{200C}\right), \quad \left|\frac{\underline{V_R}}{\underline{V}}\right|^2 = \frac{50^2}{50^2 + \left(\frac{1}{200C}\right)^2} = \frac{1}{4}$$

$$\Rightarrow 2500 + \frac{1}{4\cdot 10^4 C^2} = 2500 \times 4 \Rightarrow C = 57.7\,\mu F$$

6.36



(a) Let $Z_{in} = 1500 + \frac{10^8}{j\omega} \parallel 1000 = 1500 + \frac{10^{11}/j\omega}{10^8/j\omega + 10^3} = \frac{2.5\times 10^8 + j\omega\cdot 1500}{10^5 + j\omega}$

$$\underline{I}_{in} = \underline{V_{in}}/Z_{in}, \quad \underline{I_b}/\underline{I}_{in} = \frac{10^8/j\omega}{10^3 + 10^8/j\omega} = \frac{10^5}{j\omega + 10^5}$$

$$\underline{I_c}/\underline{V_{in}} = 100\,\underline{I_b}/\underline{V_{in}} = 100\,\underline{I_b}/Z_{in}\underline{I}_{in} = 100\,(\underline{I_b}/\underline{I_{in}})/Z_{in} = \frac{0.04}{1 + j\cdot 6\times 10^{-6}\omega}$$

$$\underline{V_{out}}/\underline{V_{in}} = [2000 + jX(\omega)](-\underline{I}_c)/\underline{V_{in}}$$

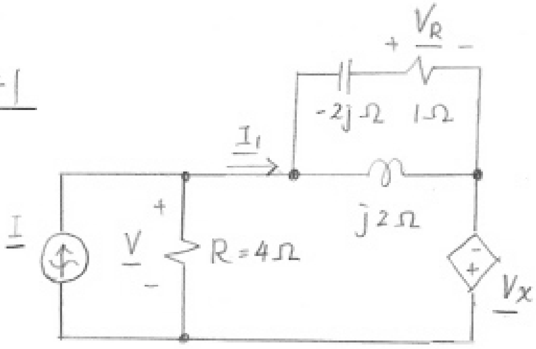
$$= \frac{-0.04\cdot 2000[1 + jX(\omega)/2000]}{1 + j\cdot 6\cdot 10^{-6}\omega} = \frac{-80[1 + j\cdot 5\cdot 10^{-4}X(\omega)]}{1 + j6\cdot 10^{-6}\omega}$$

(b) want $\frac{\underline{V}_{out}}{\underline{V}_{in}} = -K \Rightarrow 1 + j5\cdot10^{-4}X(\omega) = \quad 1 + j\,6\cdot10^{-6}\omega$

$X(\omega) = 0.012\,\omega$ so $X(\omega) = \omega L$, $L = 12$ mH

(p.s., $\frac{\underline{V}_{out}}{\underline{V}_{in}}$ should be $-K$)

6.41



take $\underline{V}_R = 1 \Rightarrow \underline{I}_R = 1$, $\underline{V}_x = 1$

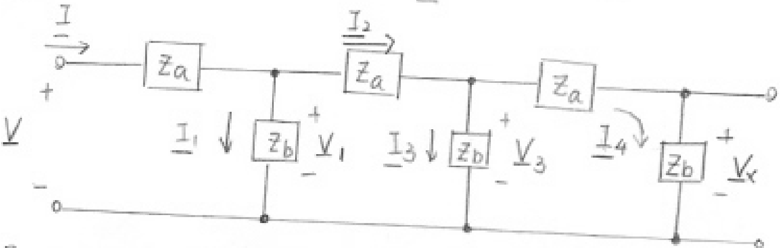
$\underline{I}_1 = \underline{I}_R + \frac{(-j2+1)}{j2}\cdot\underline{I}_R = -j0.5$

$\underline{V} = (-j2+1)\underline{I}_R - \underline{V}_x = -j2$

$\underline{I} = \underline{I}_1 + \underline{V}/R = -j$

$\frac{\underline{V}_R}{\underline{V}} = 0.5\angle 90^\circ$, $\quad \underline{V}/\underline{I} = 2\angle 0^\circ$

6.44



$\underline{V}_x = R\cdot 1 = R$, $\underline{V}_3 = R + jX$, $\underline{I}_3 = \underline{V}_3/R = 1 + jX/R$, $\underline{I}_2 = \underline{I}_3 + \underline{I}_4 = 2 + jX/R$

$\underline{V}_1 = jX\underline{I}_2 + \underline{V}_3 = R - \frac{X^2}{R} + j3X$, $\underline{I}_1 = \underline{V}_1/R = 1 - X^2/R^2 + j\,3X/R$

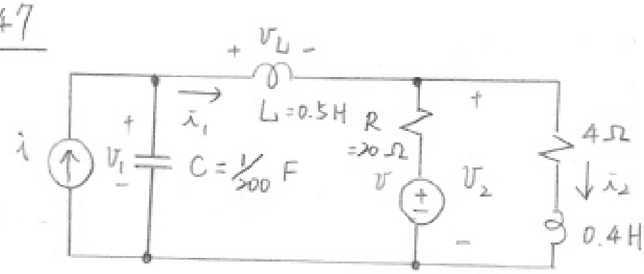
$\underline{I} = \underline{I}_1 + \underline{I}_2 = 3 - \frac{X^2}{R^2} + j\frac{4X}{R}$, $\underline{V} = jX\underline{I} + \underline{V}_1 = R - \frac{5X^2}{R} + j(6X - \frac{X^3}{R^2})$

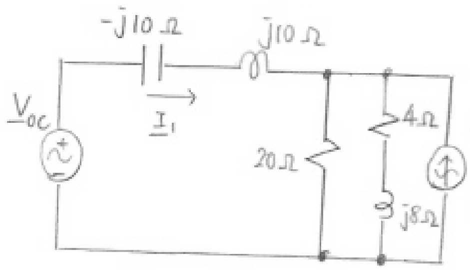
$\angle\underline{V}_x = 0$, $\angle(\underline{V}_x/\underline{V}) = -\angle\underline{V} = -180^\circ \Rightarrow \angle\underline{V} = 180^\circ \therefore Im[\underline{V}] = 0$

$\Rightarrow 6X - \frac{X^3}{R^2} = 0 \Rightarrow 1/(\omega CR)^2 = 6 \Rightarrow \omega_{osc} = 1/(\sqrt{6}RC)$

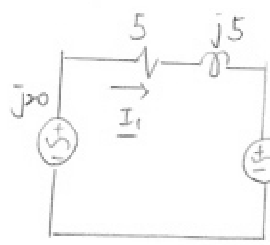
$\underline{V} = R - \frac{5}{(\omega C)^2 R} = R - 30R = -29R \Rightarrow \underline{V}_x/\underline{V} = -1/29$

6.47





80/20 = 4



4(5+j5)

= 20+j20

$\underline{V}_{oc} = (-j10)\cdot 2\angle 180^\circ = j20$, $20\,||\,(4+j8) = 5 + j5$

$\underline{I}_1 = [\,j20 - (20 + j20)\,]/(5+j5) = 2.83\angle 135^\circ$

$i_1 = 2.83\cos(20t + 135^\circ)$ A

6.51 $Y\underline{V} = \underline{I}_S$, $Y = 1/4 - j/2 + 1/(1-j2) = (9-j2)/20$

$\underline{I}_S = 1 - V_x/j2 - V_x/(1-j2) = \dfrac{20+8\underline{V}-j4\underline{V}}{20} = \frac{1}{20}(20) + \frac{1}{20}(8-j4)\underline{V}$

$\Rightarrow \frac{1}{20}[9-j2-(8-j4)]\underline{V} = \frac{1}{20}\cdot 20 \Rightarrow \underline{V} = \dfrac{20}{1-j2} = 4-j8 = 8.94\angle -63.4^\circ$

$\underline{I}_1 = 1 - V/4 = j2 = 2\angle 90^\circ$

6.53 $[Y] = \begin{bmatrix} \frac{j}{10} - \frac{j}{5} & \frac{j}{5} \\ \frac{j}{5} & -\frac{j}{5} + \frac{1}{20} + \frac{1}{4+j8} \end{bmatrix} = \frac{1}{20}\begin{bmatrix} -j2 & j4 \\ j4 & 2-j6 \end{bmatrix}$

$[\underline{I}_S] = \begin{bmatrix} 4 \\ 40/20 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 80 \\ 40 \end{bmatrix}$

$\begin{bmatrix} -j2 & j4 \\ j4 & 2-j6 \end{bmatrix}\begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 40 \end{bmatrix} \Rightarrow V_1 = 116.6\angle -30.9^\circ , V_2 = 70.7\angle -45^\circ$

6.57 $Z\underline{I} = \underline{V}_S$, $Z = 2 + j2 \parallel (-j2+1) = 6+j2$

$\underline{V_S} = 2\times 6 + \underline{V}_X = 12 + 4\underline{I}_1$

$(6+j2-4)\underline{I}_1 = 12 \Rightarrow \underline{I}_1 = 3-j3 = 4.24\angle -45^\circ$

$\underline{V} = 2(6-\underline{I}_1) = 6+j6 = 8.49\angle 45^\circ$

6.59

$[Z] = \begin{bmatrix} -8j+8j+2 & -2 \\ -2 & 2+4+j4 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 6+j4 \end{bmatrix}$

$[\underline{V}_S] = \begin{bmatrix} -8j\cdot 2 - (-10) \\ -10 \end{bmatrix} = \begin{bmatrix} 10-j16 \\ -10 \end{bmatrix}$

$\begin{bmatrix} 2 & -2 \\ -2 & 6+j4 \end{bmatrix}\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} 10-j16 \\ -10 \end{bmatrix}$

$\underline{I}_1 = 10.44\angle -73.3^\circ$, $\underline{I}_2 = 2.83\angle -135^\circ$