Introduction to Biomedical Engineering

Device/Instrumentation III

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Outline

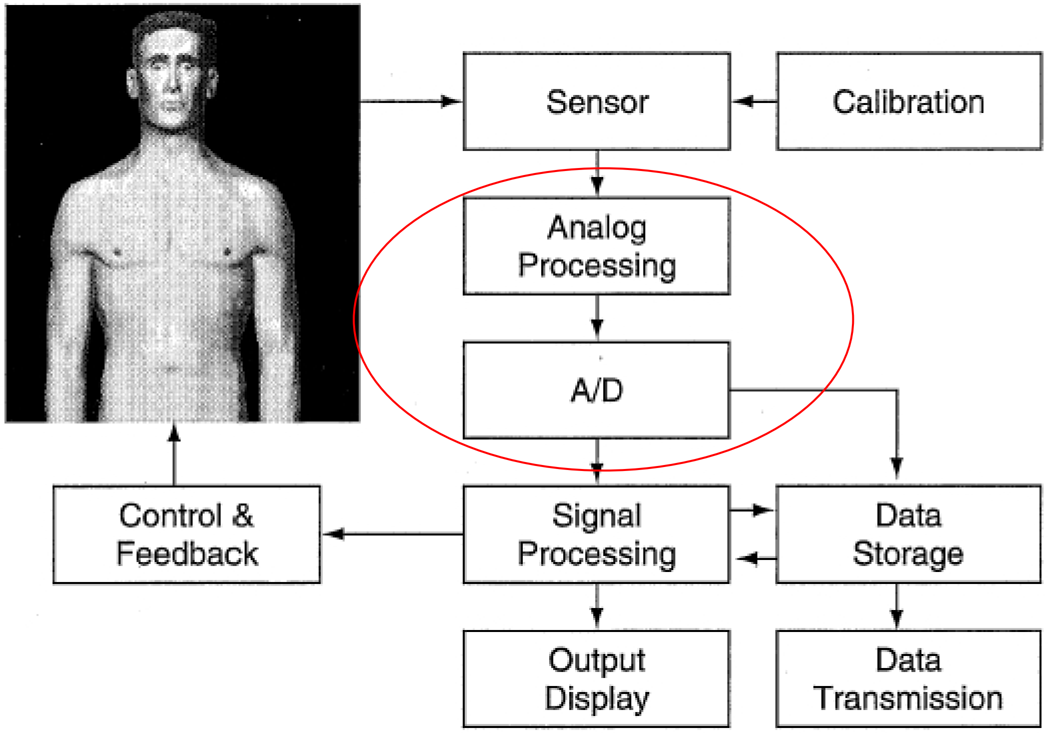
- Chapter 8 and chapter 5 of 1st edition: Bioinstrumentation
  - Review of signals and system, analog circuits
  - Operational amplifiers, instrumentation amplifiers
  - Transfer function, frequency response
  - Filters
  - Non-ideal characteristics of op-amps
  - Noise and interference
  - Data acquisition (sampling, digitization)

Types of medical instrumentation

- Biopotential
- Blood (pressure, flow, volume, etc)
- Respiratory (pressure, flow rate, lung volume, gas concentration)
- Chemical (gas, electrolytes, metabolites)
- Therapeutic and prosthetic devices
- Imaging (X-ray, CT, ultrasound, MRI, PET, etc.)
- Others

Overview of bioinstrumentation

Basic instrumentation system



Emphasis of this module will be on instruments that measure or monitor physiological activities/functions

Characteristics of bio-signals

Parameter	Voltage	Frequency (Hz)
ECG (skin)	0.5-4 mV	0.01-250
EEG (scalp)	5-200 μV	DC-150
EGG (skin)	10-1000 μV	DC-1
EGG (stomach)	0.5-80 mV	DC-1
EMG (needle)	0.1-5 mV	DC-10,000
EOG (contact)	50-3500 μV	DC-50
ERG (contact)	0-900 μV	DC-50
Nerve	0.01-3 mV	DC-10,000

EGG (electrogastrogram): measures muscular activity of the stomach

EOG (**E**lectro**O**culo**G**ram): measures the resting potential of retina

ERG (ElectroRetinoGram): measures the electrical response of retina to light stimuli

Signal amplification

- Gain up to 10^7
- Cascade (series) of amplifiers, with gain of 10-10000 each
- DC offset must be removed (ex. by HPF with a cutoff frequency of 1Hz)
- Further reduction of the common-mode signal

Time-varying signals

Sinusoidal signals have amplitude, frequency and phase

$$v_1(t) = V_1 \cos(\omega t + \theta) = V_1 \cos(2\pi f t + \theta)$$

Phasors: complex numbers (magnitude and phase angle) representing the sinusoidal signal (without the frequency)

$$\hat{V}_1 = V_1 e^{j\theta} = V_1 \angle \theta \qquad e^{j\theta} = \cos\theta + j\sin\theta$$

Since capacitors and inductors introduce phase shift to the signal, their impedances Z can be expressed in phasors as following

$$\hat{V}_1 = Z\hat{I} \qquad Z_R = R$$

$$Z_L = j\omega L = \omega L e^{j\pi/2}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C} e^{-j\pi/2}$$

Laplace transform

Definition

$$L\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

Some properties of Laplace transform

Operation	Time Function	Laplace Transform
Linear combination	$Af(t) + Bg(t)$	$AF(s) + BG(s)$
Multiplication by e^{-at}	$e^{-at}f(t)$	$F(s+a)$
Multiplication by t	$tf(t)$	$-dF(s)/ds$
Time delay	$f(t-t_0)u(t-t_0)$	$e^{-st_0}F(s)$
Differentiation	$f'(t)$	$sF(s) - f(0^-)$
	$f''(t)$	$s^2F(s) - sf(0^-) - f'(0^-)$
Integration	$\int_{0^-}^{t} f(\lambda)\, d\lambda$	$\frac{1}{s}F(s)$

Laplace domain analysis

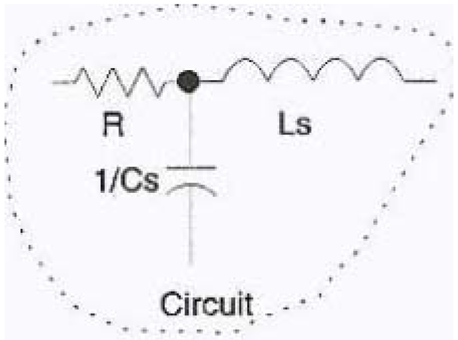
Use Laplace transform to describe time-varying signals $\Rightarrow$
Differential equations become algebraic equations

Let $s = j\omega$

$$Z_R = R$$

$$Z_L = sL$$

$$Z_C = \frac{1}{sC}$$



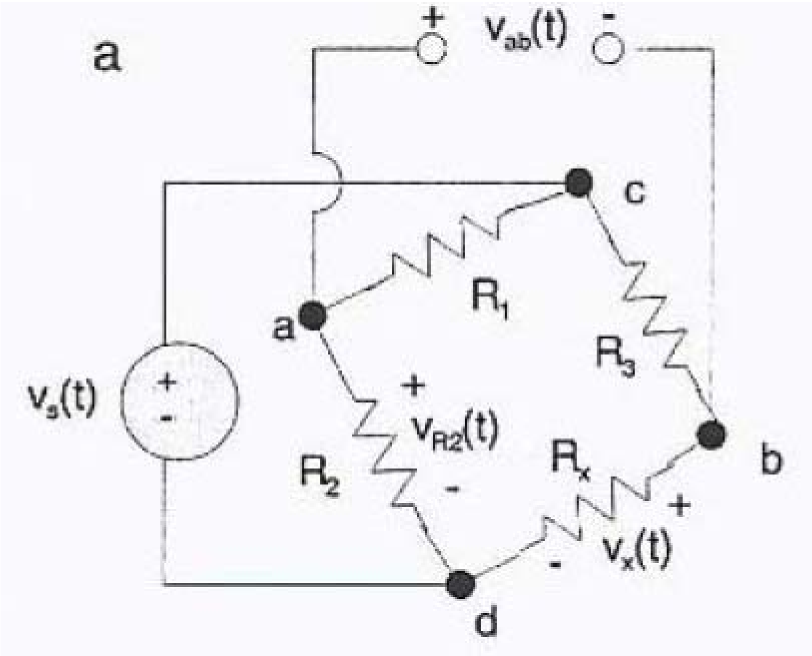
Inverse Laplace transform is used when we want to obtain the time-domain signals (ex. transient response of circuits)

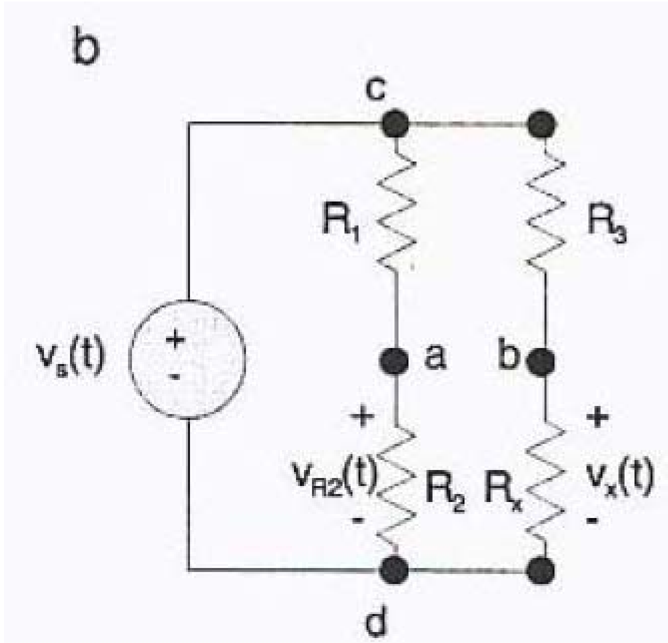
Laplace transform pairs

$f(t)$	$F(s)$
A	$\frac{A}{s}$
$u(t) - u(t - D)$	$\frac{1 - e^{-sD}}{s}$
t	$\frac{1}{s^2}$
t^r	$\frac{r!}{s^{r+1}}$
e^{-at}	$\frac{1}{s + a}$
te^{-at}	$\frac{1}{(s + a)^2}$
$t^r e^{-at}$	$\frac{r!}{(s + a)^{r+1}}$
$\sin \beta t$	$\frac{\beta}{s^2 + \beta^2}$
$\cos (\beta t + \phi)$	$\frac{s \cos \phi - \beta \sin \phi}{s^2 + \beta^2}$
$e^{-at} \cos (\beta t + \phi)$	$\frac{(s + a) \cos \phi - \beta \sin \phi}{(s + a)^2 + \beta^2}$

Analog circuits

Wheatstone bridge circuit

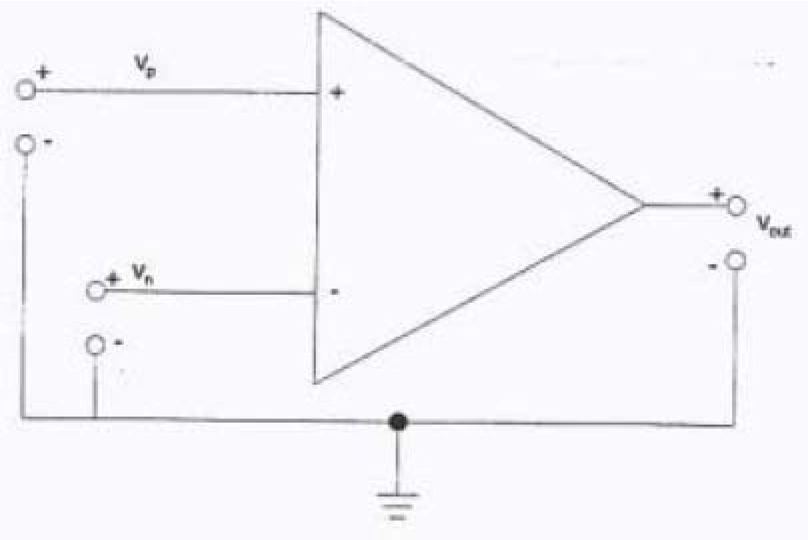




$$V_{ab} = V_{R2} - V_{Rx} = V_s\left(\frac{R_1}{R_1 + R_2}\right) - V_s\left(\frac{R_x}{R_3 + R_x}\right)$$

The measured V_{ab} can be used to get R, which represents unknown resistance of devices such a strain gauge and a thermistor

Operational amplifier (op-amp)



$$V_{out} = A(v_p - v_n)$$

Open-loop voltage gain

$A \sim 10^6$

For ideal op-amps:

- No current flows into or out of input terminals (input impedance = infinity)
- $v_p = v_n$ since $A \sim 10^6$
- Output impedance = 0

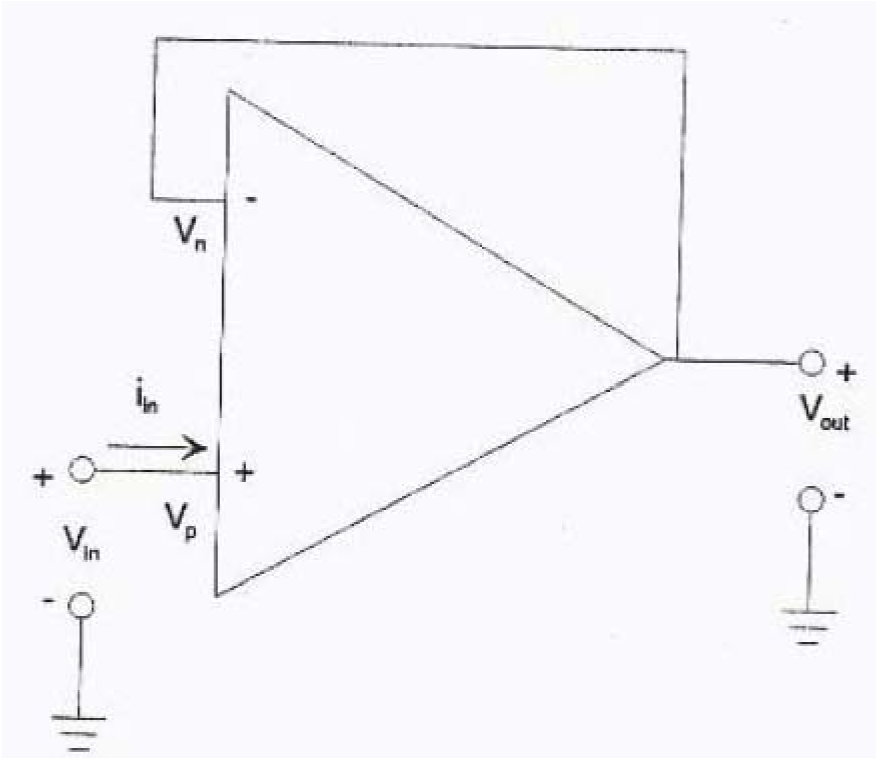
Cautions for op-amp circuits

Op-amps are used with (negative) feedback loops for stability

Must be in the active region (input and output not saturated)

Op-amp circuits

Voltage follower or unity buffer



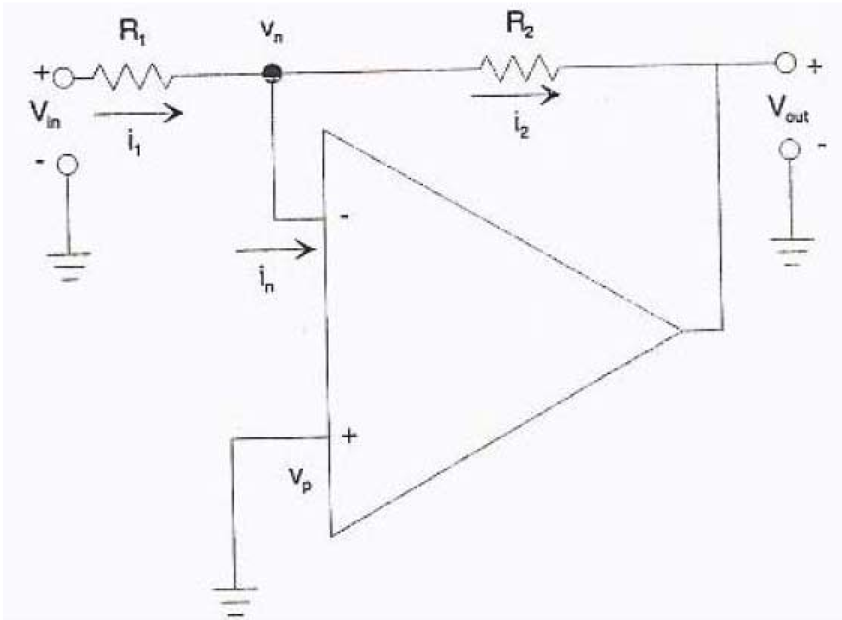
$V_{out} = V_{in}$

G=1

Advantage: input current is ~0, high input impedance. Output current drawn from the op-amp can drive a load (Z_L) or next stage of circuit; particularly suitable as the first stage for physiological measurements

Op-amp circuits

Inverting amplifier

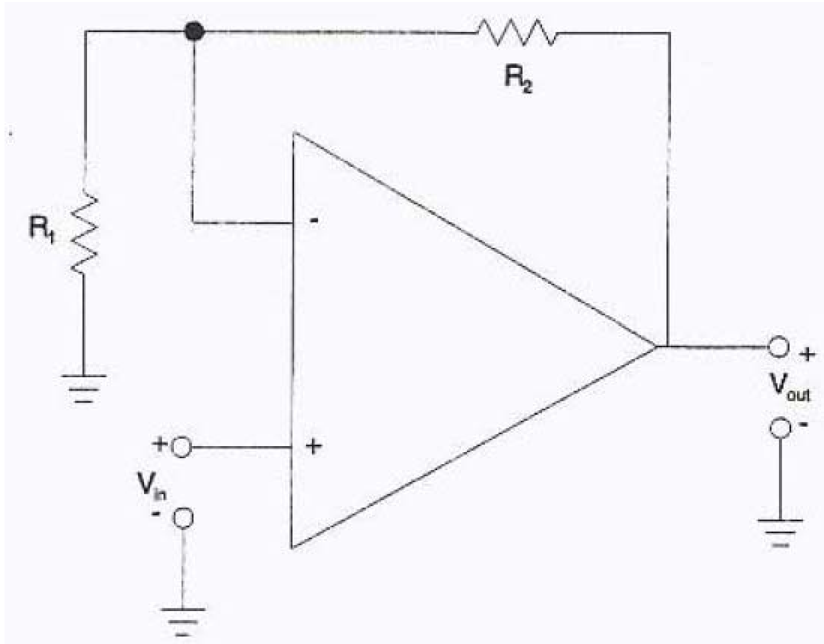


$$v_{out} = -i_2 R_2 = -i_1 R_2 = -\frac{v_{in}}{R_1} R_2$$

$$G = -\frac{R_2}{R_1}$$

Input impedance = R_1

Non-inverting amplifier



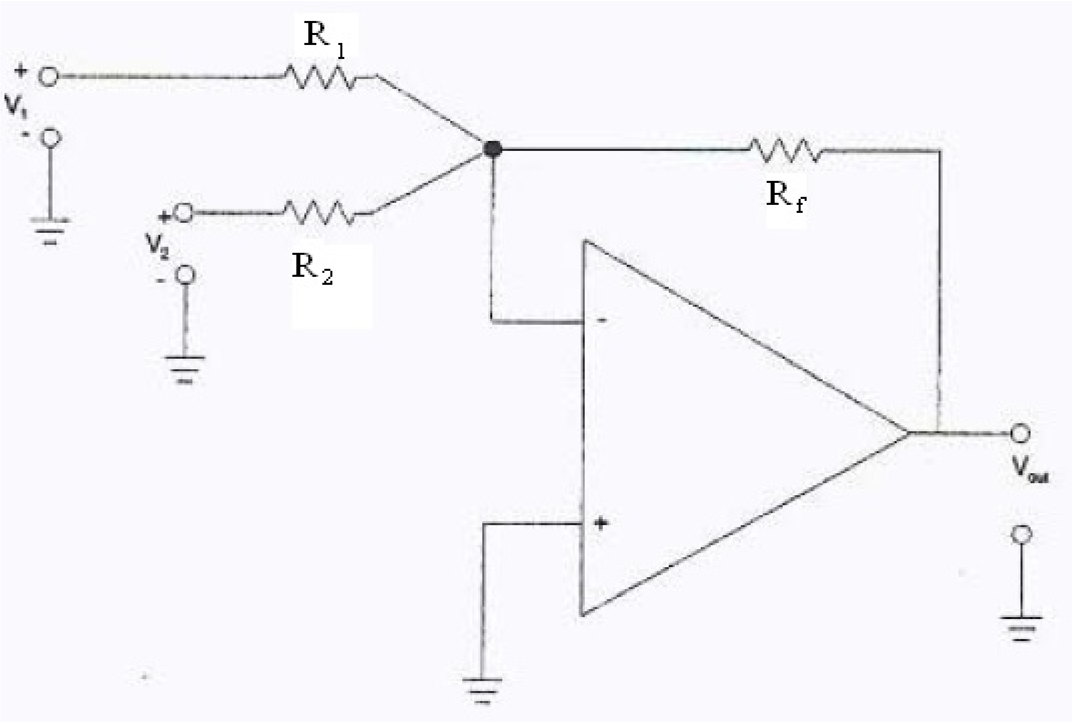
$$v_{out} = v_{in} + \frac{v_{in}}{R_1} R_2$$

$$G = 1 + \frac{R_2}{R_1}$$

Input impedance = Z_{in} of the op-amp

Op-amp circuits

Summing amplifier

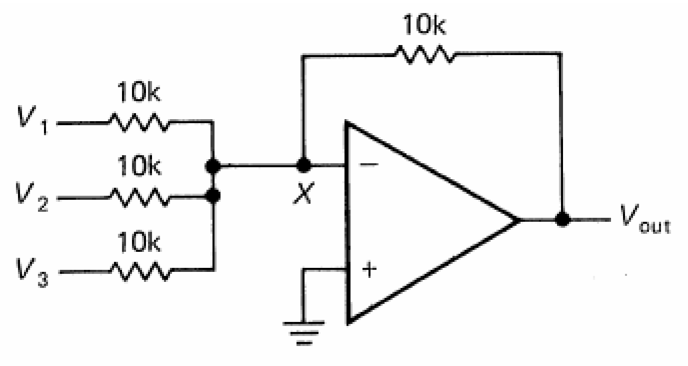


$$v_{out} = i_f R_f = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

$$G = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2\right)$$

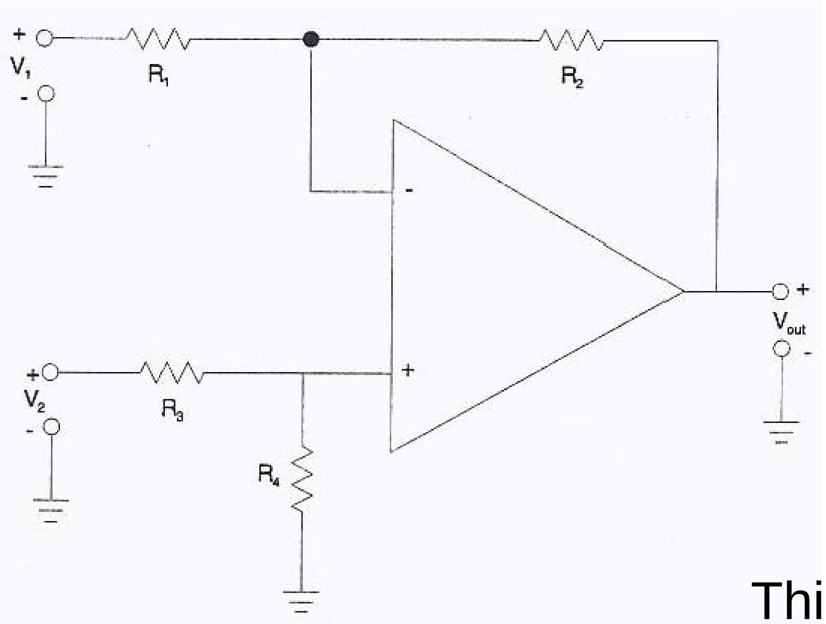
You can add more input voltages…

$V_{out} = -(V_1+V_2+V_3)$



Op-amp circuits

Subtractor



$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)V_2$$

If $R_1=R_3$, $R_2=R_4$

$$V_{out} = \frac{R_2}{R_1}\left(V_2 - V_1\right)$$

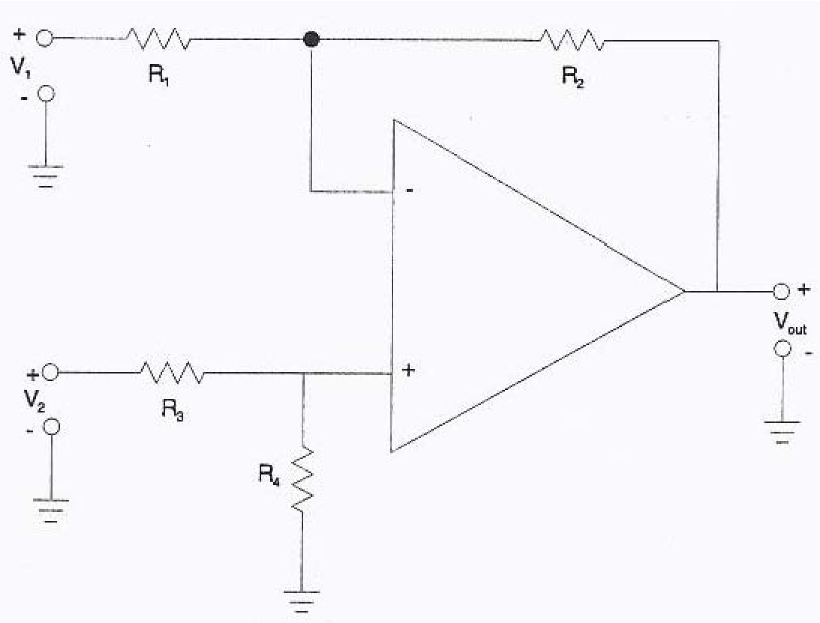
This is called a **differential amplifier**

If a differential signal (ex. biopotential) is measured across the input terminals

Differential gain $G_d = \frac{V_{out}}{V_2 - V_1} = \frac{R_2}{R_1}$

Op-amp circuits

Common-mode rejection ratio CMRR of the differential amplifier



If a common-mode voltage at both inputs is $V_{cm}=(V_1+V_2)/2$

Then the common-mode gain =

$$G_{cm} = \frac{V_{out}}{V_{cm}} = \frac{R_1 R_4 - R_2 R_3}{R_1(R_3 + R_4)}$$

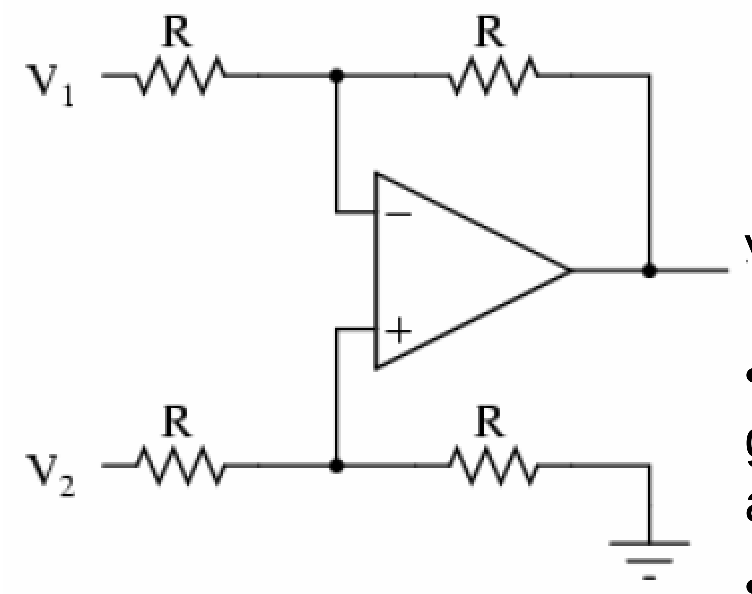
CMRR is defined as:

$$CMRR = 20\log\left|\frac{G_d}{G_{cm}}\right|$$

Homework:

1. Derive the expression for $G_d=V_{out}/V_d$ with a differential input $V_d=V_2-V_1$
2. Suppose you use 4 resistors 100K$\Omega\pm$0.01%, simulate the CMRR using random numbers for errors in resistance

More on differential amplifier



$V_{out} = V_2 - V_1$

• For measuring biopotentials, voltage gain can be obtained by subsequent amplifier stages

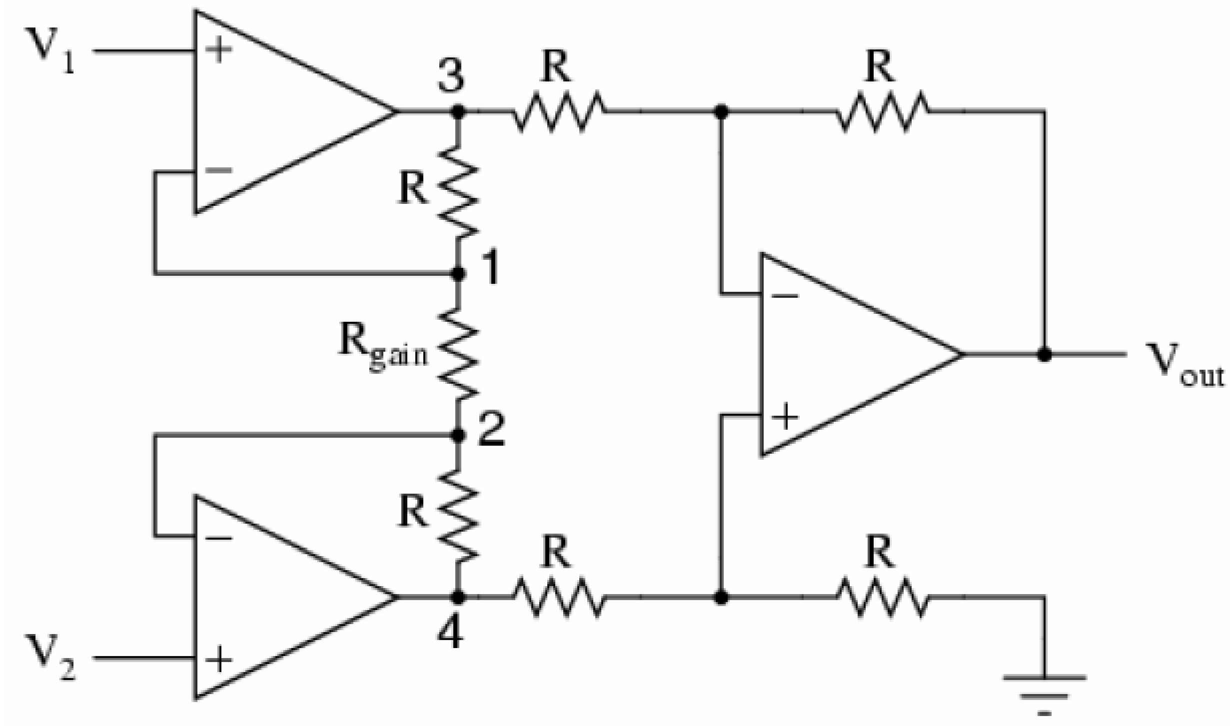
• Input impedance is small ~R

• In ECG, the impedance of skin is ~MΩ (can be lowered by applying electrolyte gel to 15-100KΩ)

• Mismatches in R reduce the CMRR

Add unity buffers in the inputs

Instrumentation amplifier

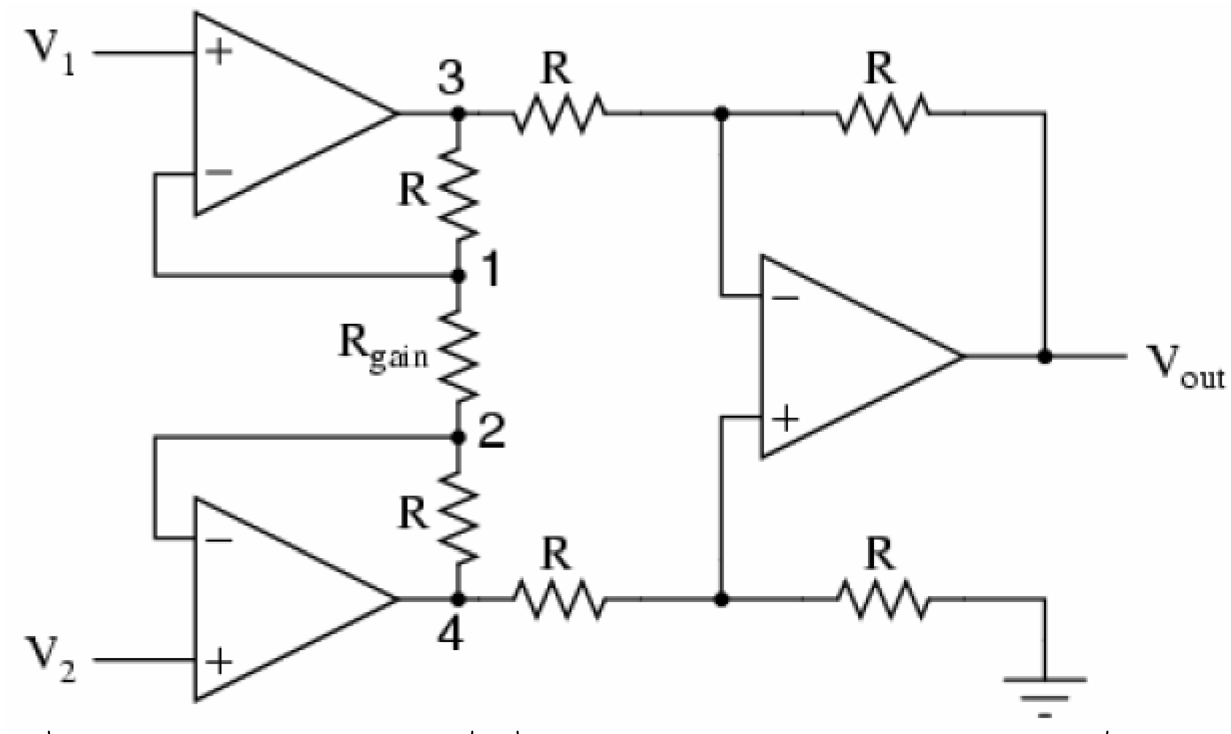


$$V_3 - V_4 = \frac{R_{gain} + 2R}{R_{gain}}\left(V_1 - V_2\right) \qquad V_{out} = V_3 - V_4$$

$$G_d = 1 + \frac{2R}{R_{gain}}$$

In practice, R_{gain} is external and used to select gain which is typically 1-1000

Instrumentation amplifier



$$G_d = 1 + \frac{2R}{R_{gain}}$$

$$G_{cm} = 1$$

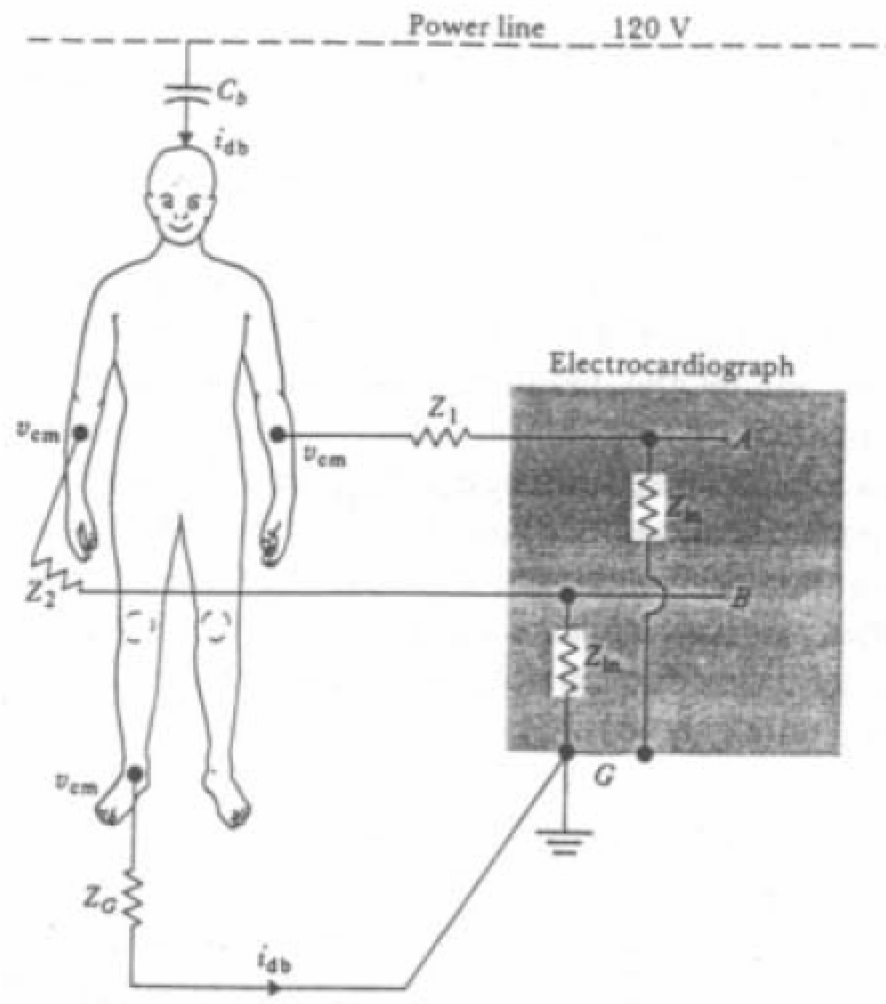
$$G_d = 1$$

$$G_{cm} \approx 0$$

Provides good CMRR without the need for precisely matching resistors

Example of common-mode voltage

Interference from power line (60Hz) can induce current i_{db}



$$v_{cm} = i_{db} \cdot Z_G$$

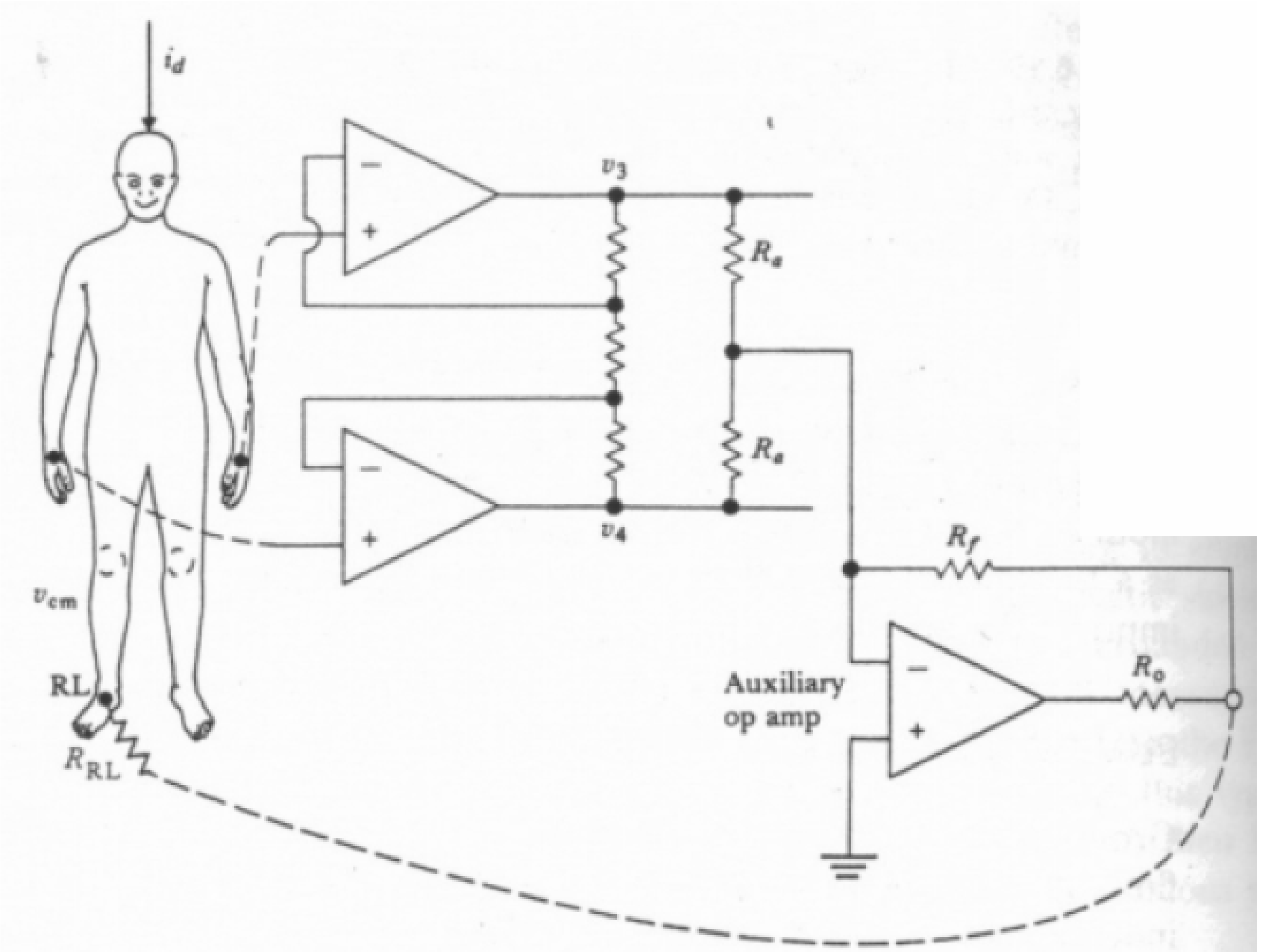
For i_{db} = 0.2 μA

Z_G = 50 kΩ

v_{cm} = 10 mV

Driven-right-leg circuit

Output is connected to the right leg through a surface electrode, which provides negative feedback



Driven-right-leg circuit

Current at inverting input: $$\frac{2v_{cm}}{R_a} + \frac{v_o}{R_f} = 0$$

$$v_{cm} = R_{RL} i_d + v_o$$

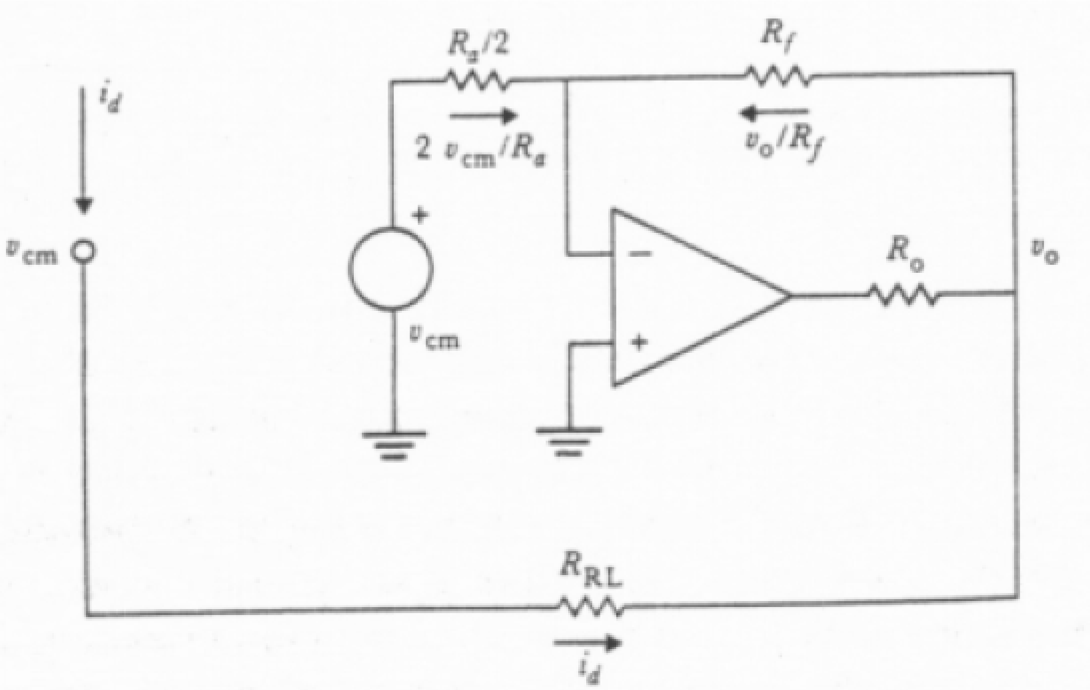
$$v_{cm} = \frac{R_{RL} i_d}{1 + 2R_f / R_a}$$

Typical values:

$$R_f = 5M\Omega, R_a = 25k\Omega$$

$$R_{RL} = 100k\Omega, i_d = 0.2\mu A$$

$$v_{cm} = 50\mu V$$



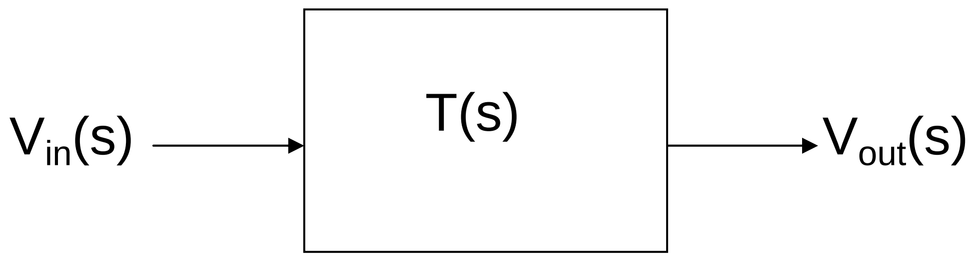
Transfer function

- Relationship between the input and output

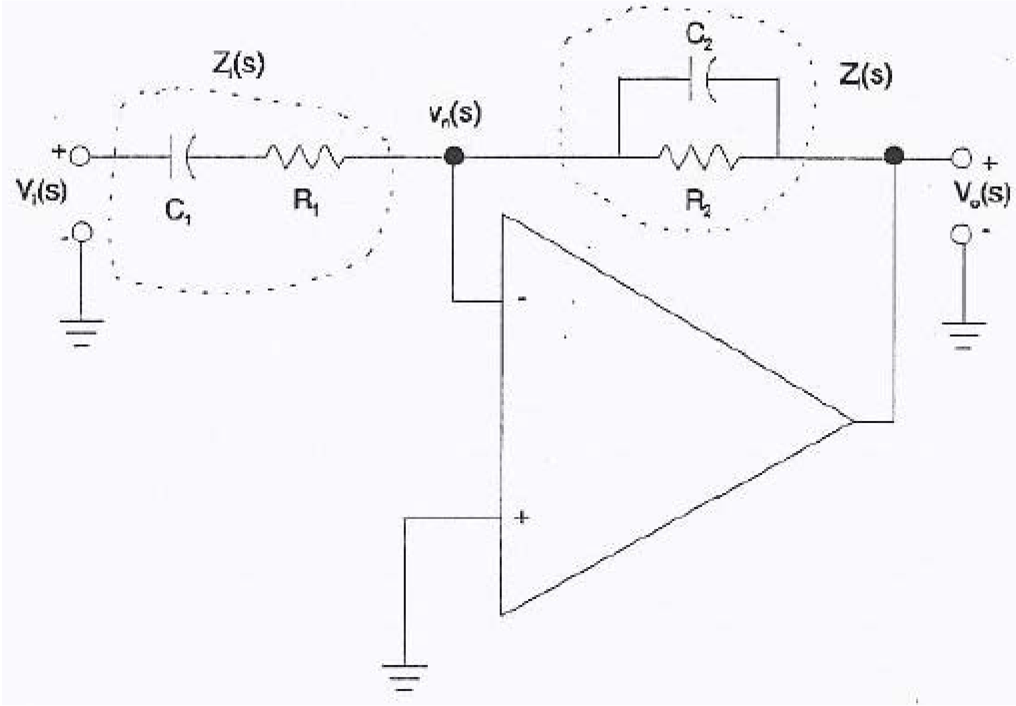
$$T(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

- Since $s = j\omega = j2\pi f$

  T(s) also provides information on the frequency and phase of the instrument – frequency response



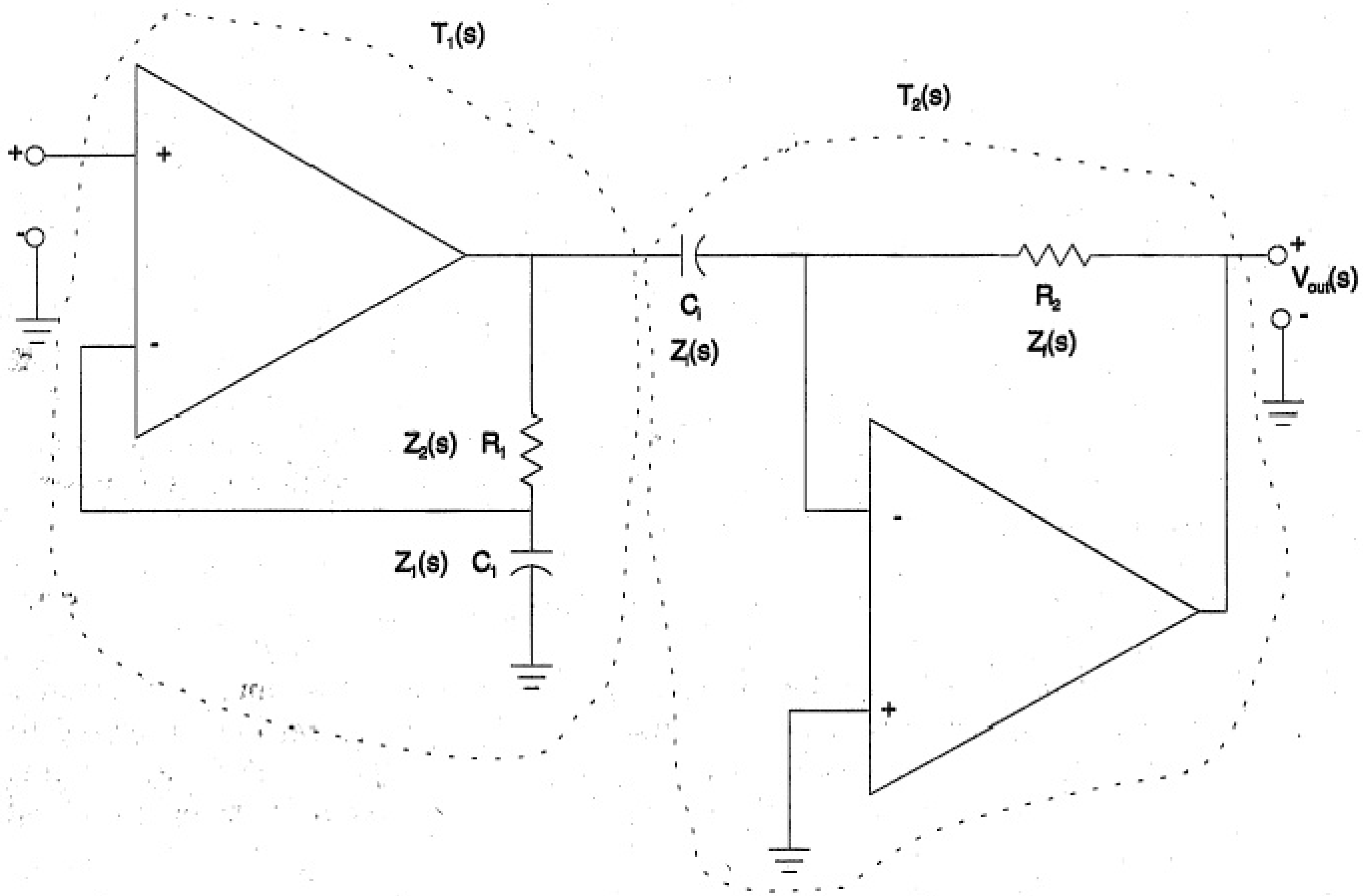
Transfer function – example



$$Z_i(s) = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1} \qquad Z_f(s) = \frac{1}{\frac{1}{R_2} + sC_2} = \frac{R_2}{1 + sR_2C_2}$$

$$T(s) = -\frac{Z_f(s)}{Z_i(s)} = \frac{sR_2C_1}{(1 + sR_1C_1)(1 + sR_2C_2)}$$

Transfer function – example2



$$T_1(s) = \frac{Z_1(s) + Z_2(s)}{Z_1(s)} = sR_1C_1 + 1 \qquad T_2(s) = -\frac{Z_f(s)}{Z_i(s)} = -sR_2C_2$$

$$T(s) = T_1(s)T_2(s) = -(1 + sR_1C_1)(sR_2C_2)$$

Frequency response

- The transfer function can be factored into poles and zeros

$$T(s) = K \frac{(s+z_1)(s+z_2)\cdots}{(s+p_1)(s+p_2)\cdots}$$

- Alternatively

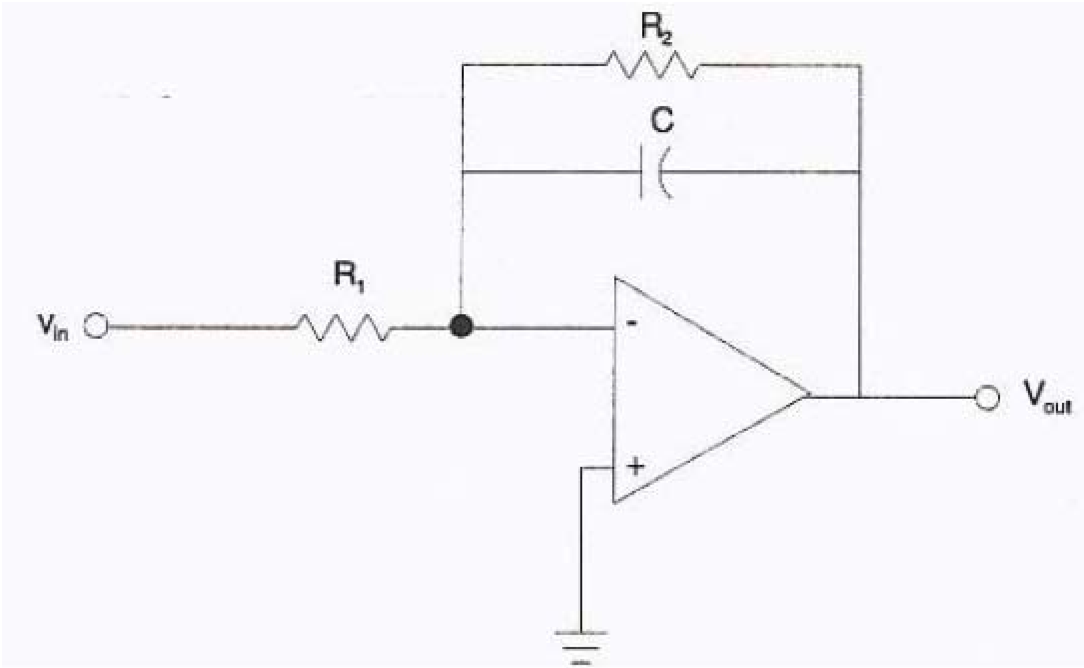
$$T(s) = K' \frac{(1+s/z_1)(1+s/z_2)\cdots}{(1+s/p_1)(1+s/p_2)\cdots} = K' \frac{(1+j\omega/z_1)(1+j\omega/z_2)\cdots}{(1+j\omega/p_1)(1+j\omega/p_2)\cdots}$$

$$T(j\omega) = \left|T(j\omega)\right| e^{j\theta(\omega)}$$

Magnitude response ($\left|T(j\omega)\right|$)

Phase response ($\theta(\omega)$)

Frequency response – LPF



$$T(s) = -\frac{R_2}{R_1(1 + sR_2C)}$$

$$K' = -\frac{R_2}{R_1}$$

$$p_1 = \frac{1}{R_2C}$$

$$T(j\omega) = K'\frac{1}{(1 + j\omega / p_1)}$$

Magnitude response

$$|T(j\omega)| = |K'|\frac{1}{\sqrt{1 + \frac{\omega^2}{p_1^{\,2}}}}$$

Gain:
- $|K'| = R_2/R_1$ — DC ($\omega=0$)
- $\frac{K'}{\sqrt{2}}$ — When $\omega = p_1$

Frequency response– LPF

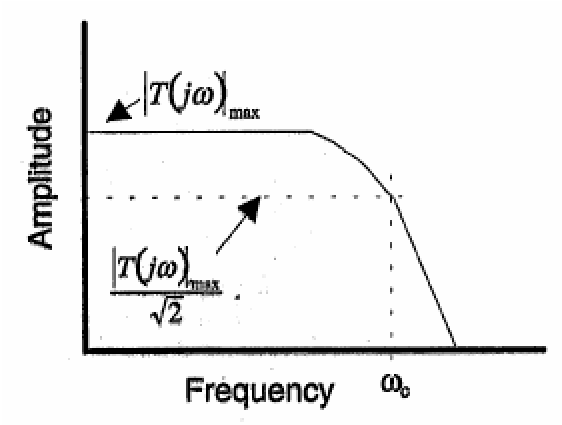
Cut-off frequency f_c: the magnitude response is

$$\left|T(j2\pi f_c)\right| = \frac{\left|T(j\omega)\right|_{\max}}{\sqrt{2}}$$ (-3dB power attenuation)

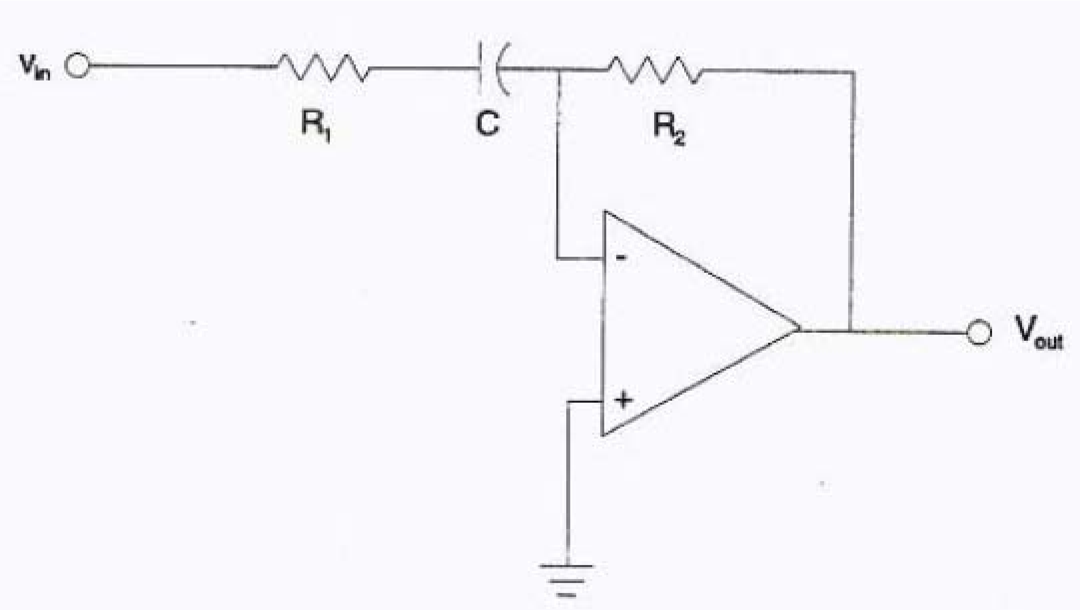
In this example

$$\omega_c = 2\pi f_c = p_1$$

$$\therefore f_c = \frac{1}{2\pi R_2 C}$$



Frequency response – HPF



$$T(s) = -\frac{sR_2C}{1+sR_1C}$$

$$T(j\omega) = -\frac{j\omega R_2C}{1+j\omega/p_1}$$

$$p_1 = \frac{1}{R_1C}$$

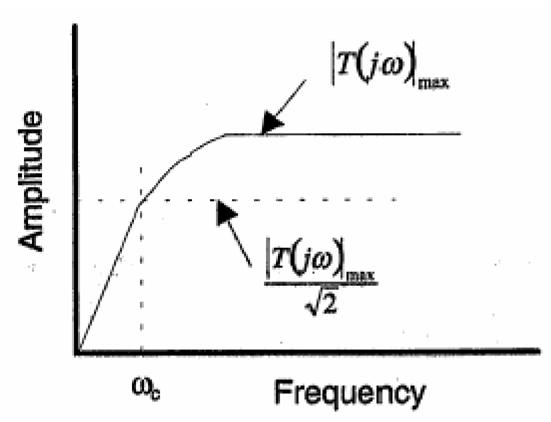
$$|T(j\omega)| = \frac{\omega R_2C}{\sqrt{1+\omega^2R_1{}^2C^2}}$$

Gain $\begin{cases} 0 & \text{DC } (\omega=0) \\ -\dfrac{R_2}{R_1} & \text{When } \omega \rightarrow \infty \end{cases}$

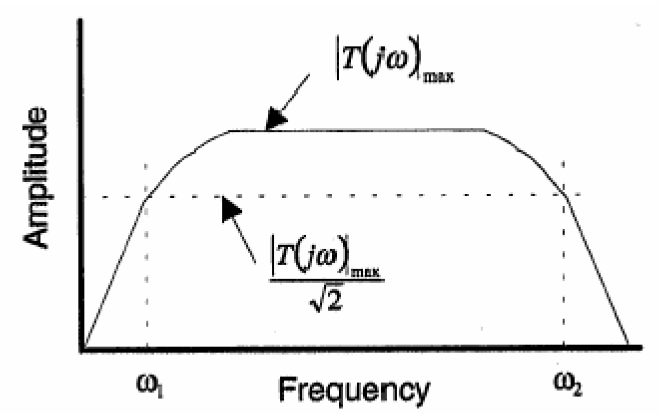
$$\omega_c = 2\pi f_c = p_1 \qquad \therefore f_c = \frac{1}{2\pi R_1C}$$

Frequency response – HPF, BPF

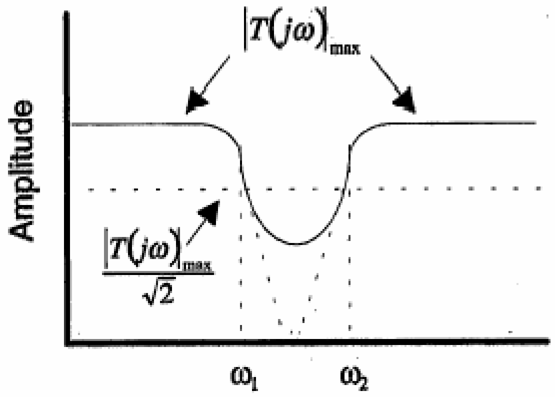
High pass filter



Band pass filter



Band stop filter



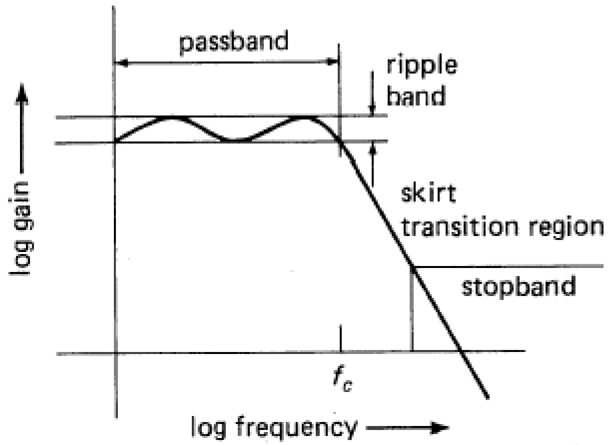
Homework:

- For the circuit on slide 25, find out the cut-off frequencies corresponding to ω_1 and ω_2, respectively
- What modifications can you do to make a <u>band-stop</u> filter?

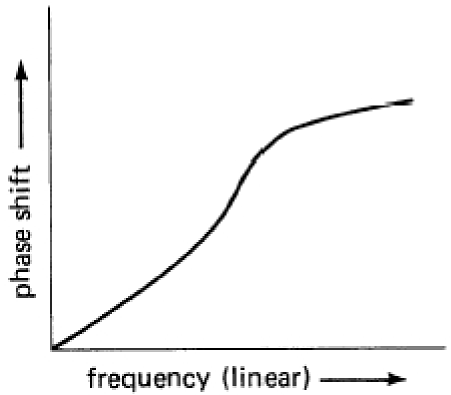
Active filters

Frequency characteristics of filters

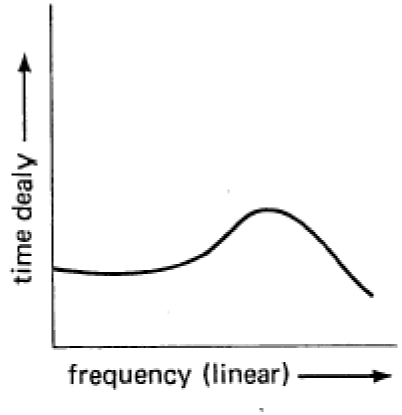
Amplitude



Phase shift



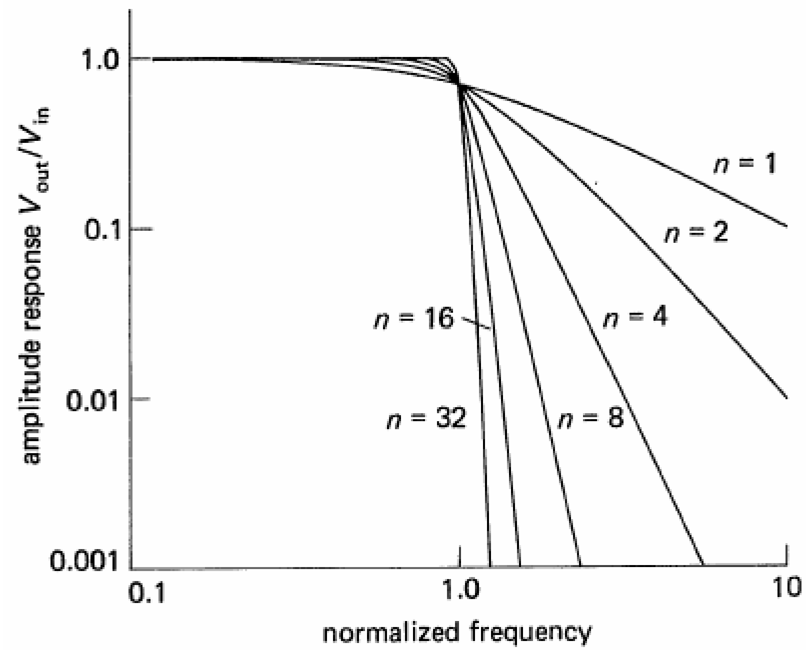
Time delay



The most important is the amplitude response which represents how the amplitudes of different frequency components will be modified by the filter

Active filters

Response of low-pass Butterworth filters with different orders (-3dB frequency is normalized at 1)



Sharper knee with higher orders

Butterworth filter of order n

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1+(f/f_c)^{2n}}}$$

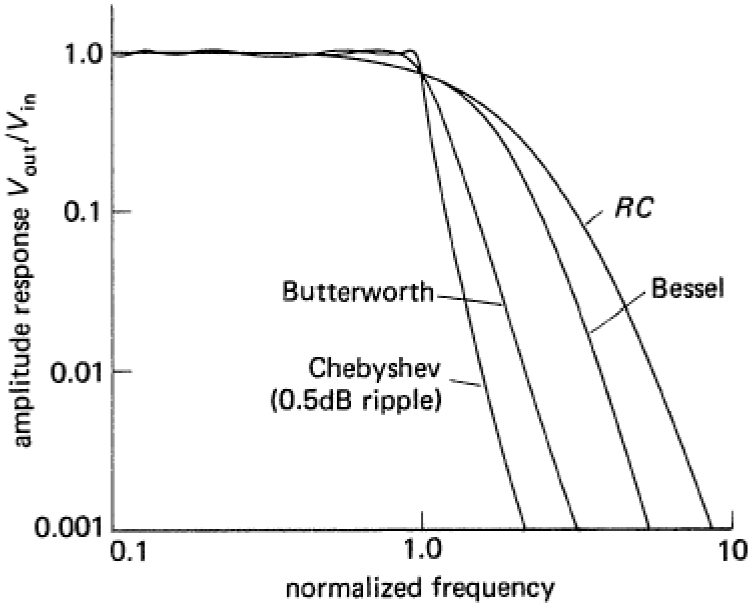
Chebyshev filter of order n

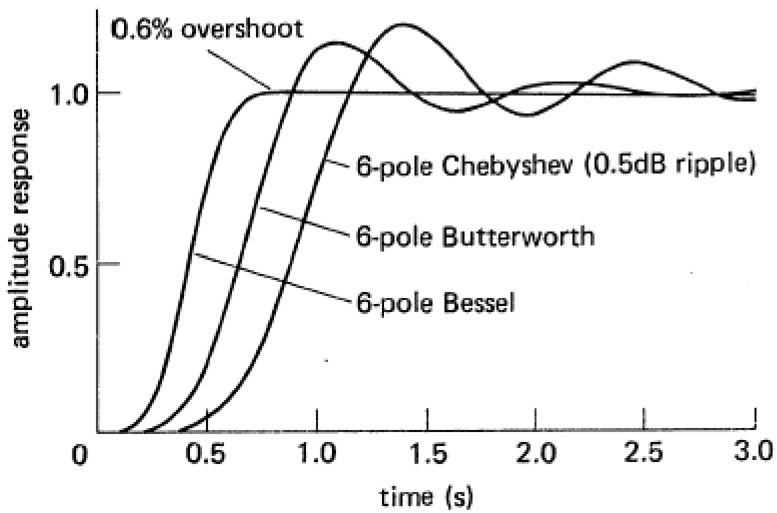
$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2(f/f_c)}}$$

C_n is the Chebyshev polynomial of the first kind of degree n, ε is a constant that sets the passband ripple

Active filters

Comparison of several 6-pole low-pass filters

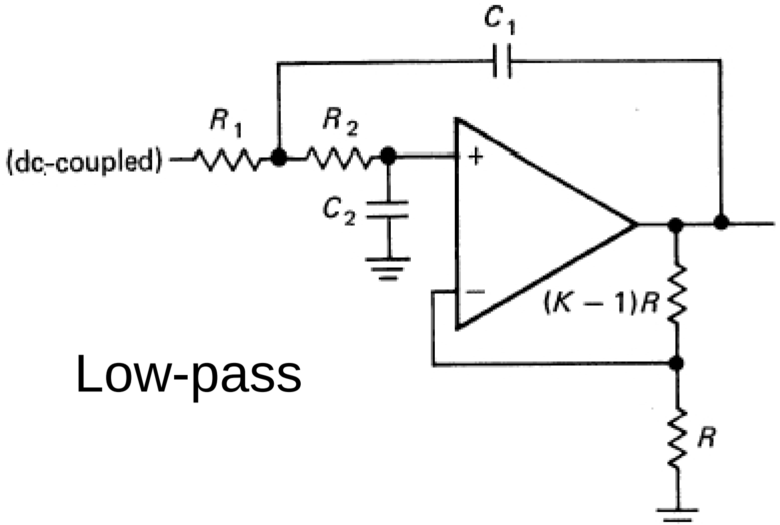




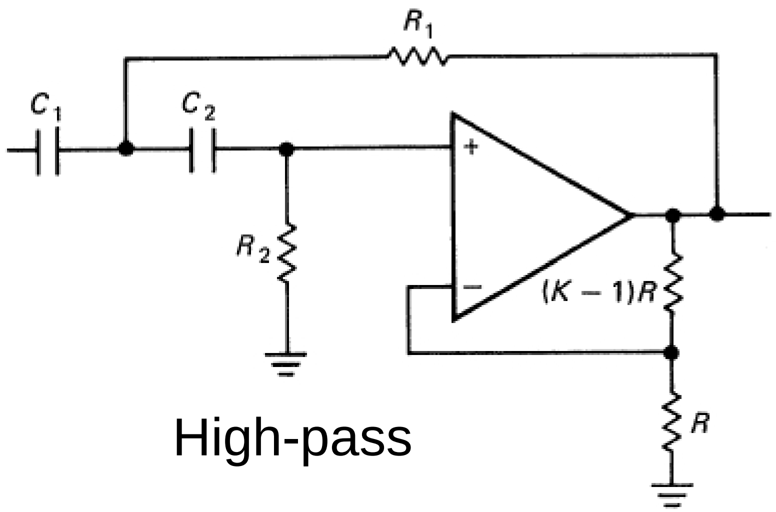
Step response (-3dB at 1Hz)

Transfer Function	Frequency-Domain Characteristics		Time-Domain Characteristics	
	Ripple	Stopband	Phase	Group Delay
Chebyshev	Equal ripple flat	Steep	Poor	Poor
Butterworth	Smooth	Moderate	Moderate	Moderate
Bessel	Maximum smoothness	Weak	Very flat	Very flat

Active filter circuits – VCVS



Low-pass



High-pass

Poles	Butter-worth K	Bessel f_n	Bessel K	Chebyshev (0.5dB) f_n	Chebyshev (0.5dB) K	Chebyshev (2.0dB) f_n	Chebyshev (2.0dB) K
2	1.586	1.272	1.268	1.231	1.842	0.907	2.114
4	1.152	1.432	1.084	0.597	1.582	0.471	1.924
	2.235	1.606	1.759	1.031	2.660	0.964	2.782
6	1.068	1.607	1.040	0.396	1.537	0.316	1.891
	1.586	1.692	1.364	0.768	2.448	0.730	2.648
	2.483	1.908	2.023	1.011	2.846	0.983	2.904
8	1.038	1.781	1.024	0.297	1.522	0.238	1.879
	1.337	1.835	1.213	0.599	2.379	0.572	2.605
	1.889	1.956	1.593	0.861	2.711	0.842	2.821
	2.610	2.192	2.184	1.006	2.913	0.990	2.946

VCVS filter design

- Each circuit is a 2-pole filter; i.e. for an n-pole filter, you need to cascade $n/2$ VCVS sections
- Within each section, set $R_1=R_2=R$ and $C_1=C_2=C$
- Set the gain K according to the table
- For Butterworth filters

$$RC = \frac{1}{2\pi f_c}$$

f_c is the -3dB frequency

- For Bessel and Chebyshew low-pass filters

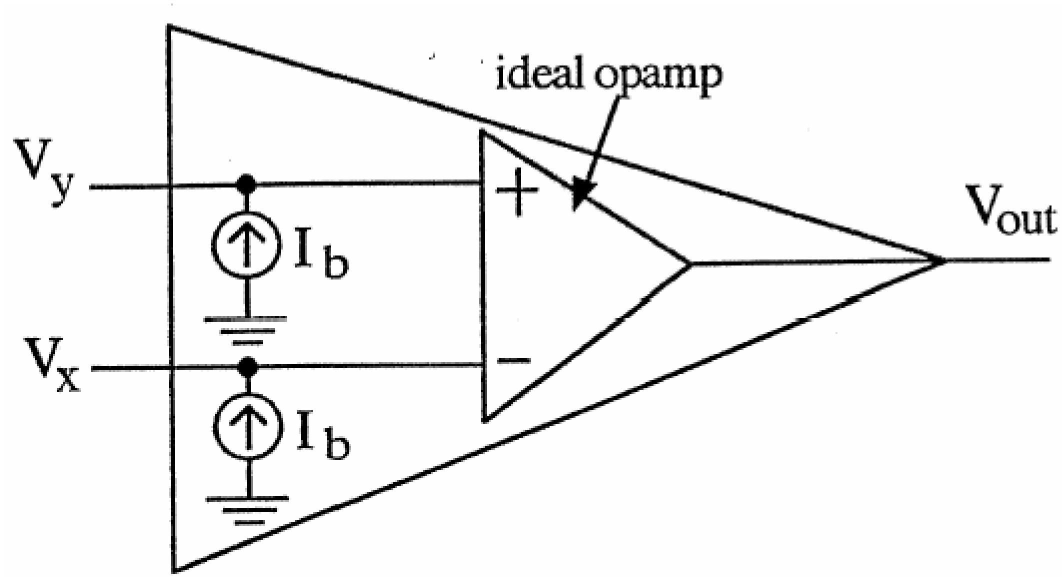
$$RC = \frac{1}{2\pi f_n f_c}$$

- For Bessel and Chebyshew high-pass filters

$$RC = \frac{1}{2\pi f_c / f_n}$$

Non-ideal op-amp

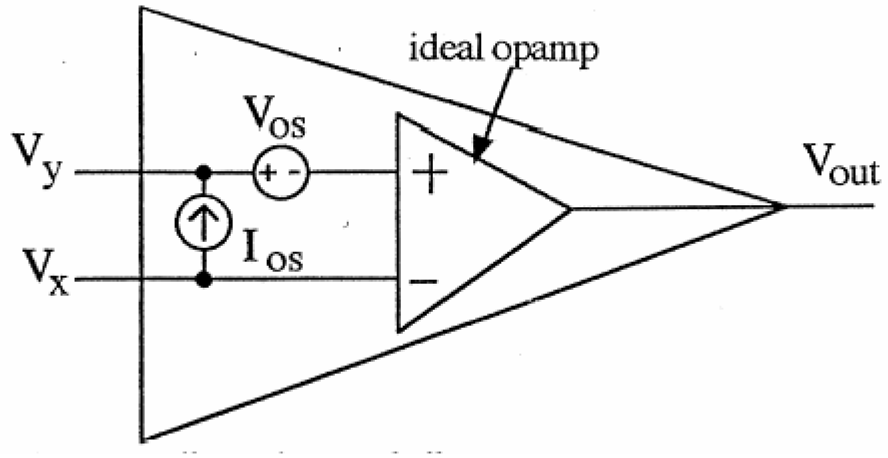
Input bias current I_B: simply the base or gate currents of the input transistors (could be current source or sink) – the effect of I_B can be reduced by selecting resistors to equalize the effective impedance to ground from the two inputs



Non-ideal op-amp

Input offset current I_{OS}: difference in input currents between two inputs; typically 0.1~0.5 I_B

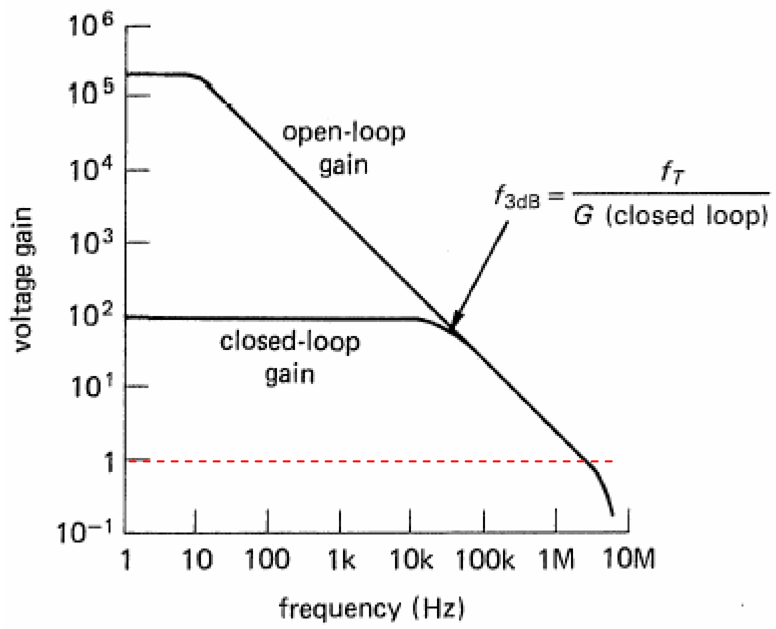
Input offset voltage: the difference in input voltages necessary to bring the output to zero (due to imperfectly balanced input stages)

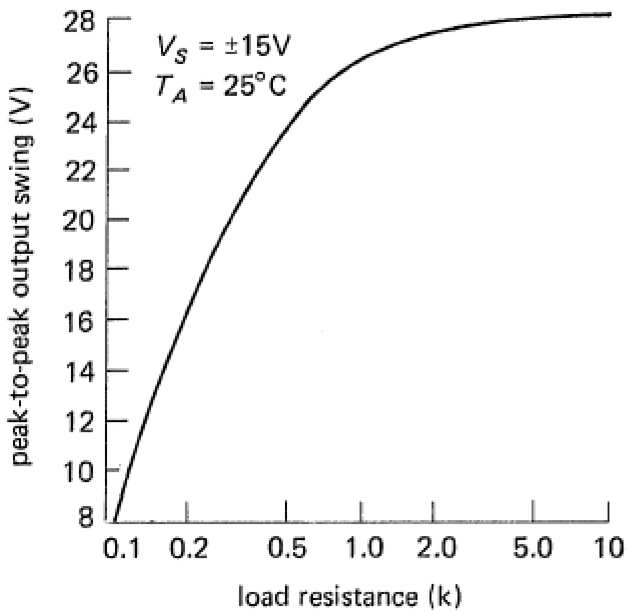


The offset voltage can be eliminated by adjusting null offset pots on some op-amps (with inputs connecting to ground through resistors)

Non-ideal op-amp cont.

Voltage gain: typically 10^5-10^6 at dc and drops to 1 at some f_T (~ 1-10 MHz); when used with feedback (closed-loop gain = G), the bandwidth of the circuit will be f_T/G





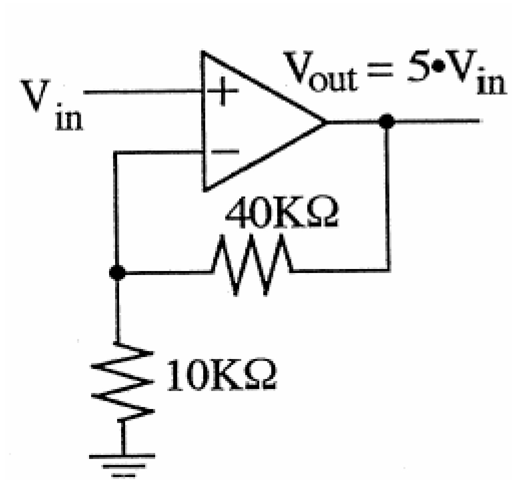
Output current: due to limited output current capability, the max. output voltage range (swing) of op-amp is reduced at small load resistances

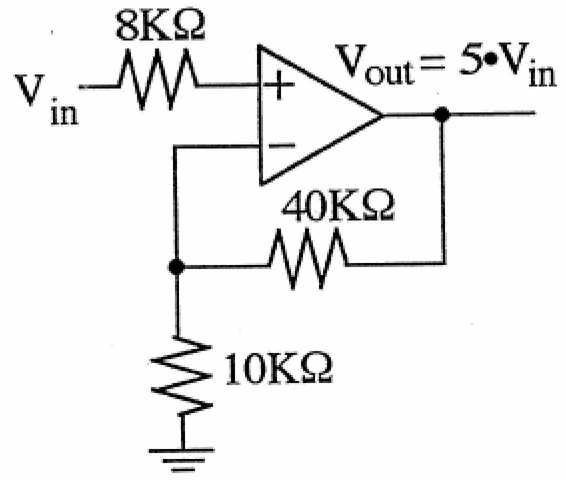
Practical considerations

- Negative feedback (resistor between the output and the inverted input terminal) provides a linear input/output response and in general stability of the circuit
- Choose resistor values 1kΩ-1MΩ (best 10kΩ–100kΩ)
- Match input impedances of the two inputs to improve CMRR
- Equalize the effective resistance to ground at the two input terminals to minimize the effects of I_B

Matching effective impedance to ground

The voltage gain is 5 for both circuits





$40K\Omega \,\|\, 10K\Omega = 8K\Omega$

So the effective impedance to ground from both input terminals is the same

Noise

- Interference from outside sources
  - Power lines, radio/TV/RF signals
  - Can be reduced by filtering, careful wiring and shielding
- Noise inherent to the circuit
  - Random processes
  - Can be reduced by good circuit design practice, but not completely eliminated

Signal-to-noise ratio

$$SNR = 20\log\left|\frac{V_{s(rms)}}{V_{n(rms)}}\right| \quad \text{dB} \qquad V_{rms} = \left[\frac{1}{T}\int_0^T v^2(t)dt\right]^{1/2}$$

Noise

- Types of fundamental (inherent) noise:
  - Thermal noise (Johnson noise or white noise)
  - Shot noise
  - Flicker (1/f) noise
  - Transducer limitations

Noise

Thermal noise: generated in a resistor due to thermal motion of atoms/molecules

$$V_{noise}(rms) = \sqrt{4kTRB}$$

k: Boltzmann's constant
T: absolute temperature (°K)
R: resistance (Ω)
B: bandwidth f_{max}-f_{min}

Thermal noise contains superposition of all frequencies ⇒ white noise

Shot noise: arises from the statistical uncertainty of counting discrete events

$$\text{Shot noise} = \sqrt{\frac{dn}{dt}\Delta t} \approx \sqrt{n}$$

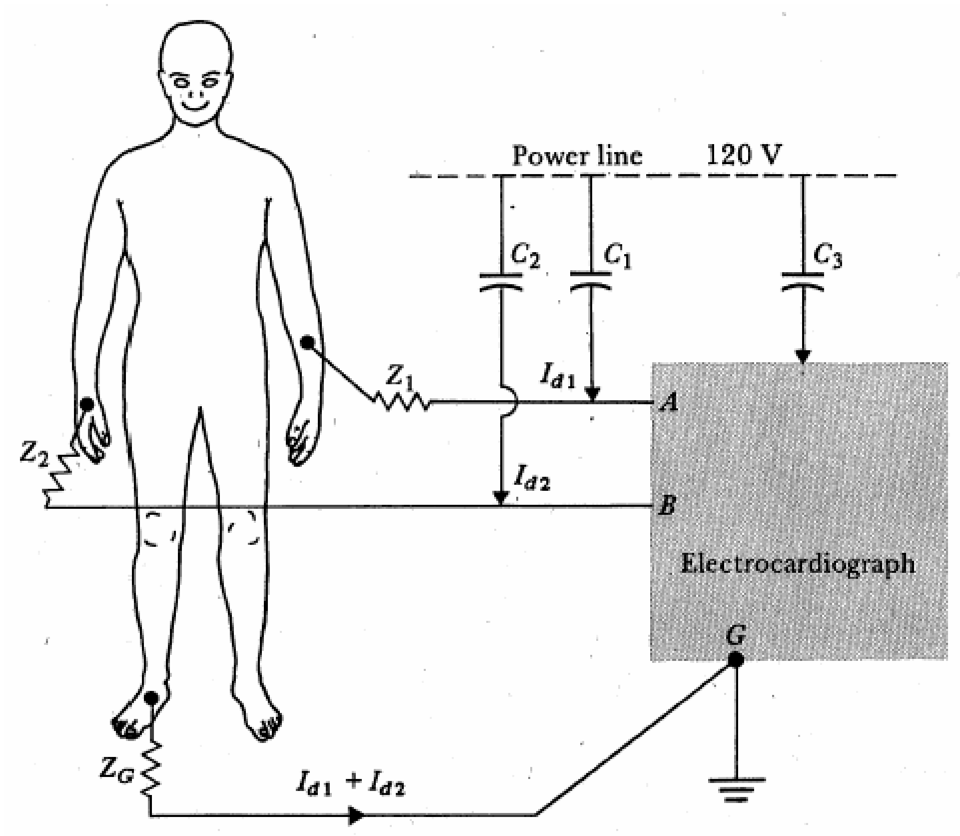
$$S/N = \frac{n}{\sqrt{n}} = \sqrt{n}$$

dn/dt is the count rate
Δt is the time interval for the measurement

Flicker (1/f) noise: power spectrum is ~1/f; somewhat mysterious; found related to resistive materials of resistors and their connections

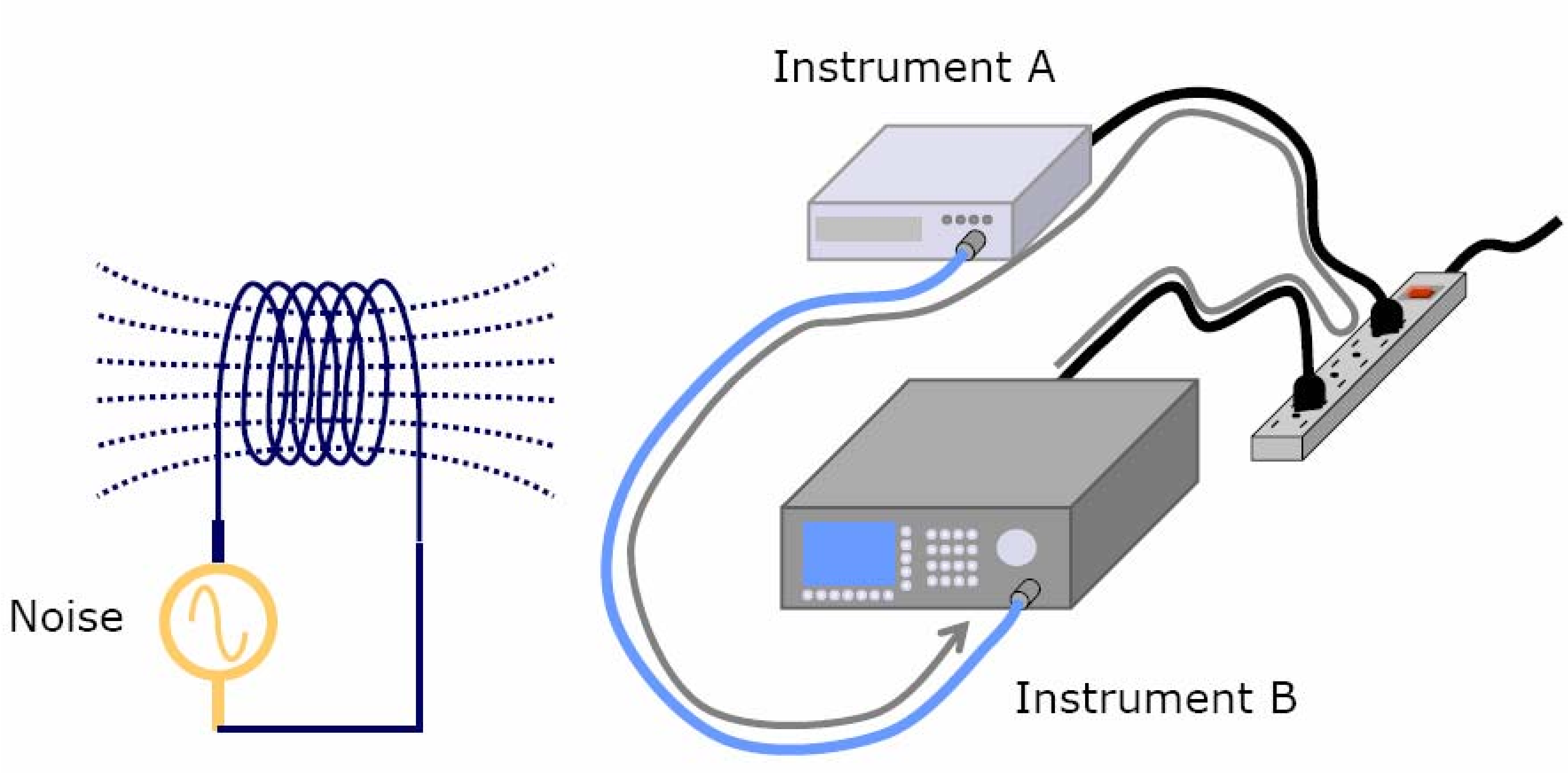
Interference

Electric fields existing in power lines can couple into instruments and human body (capacitors)



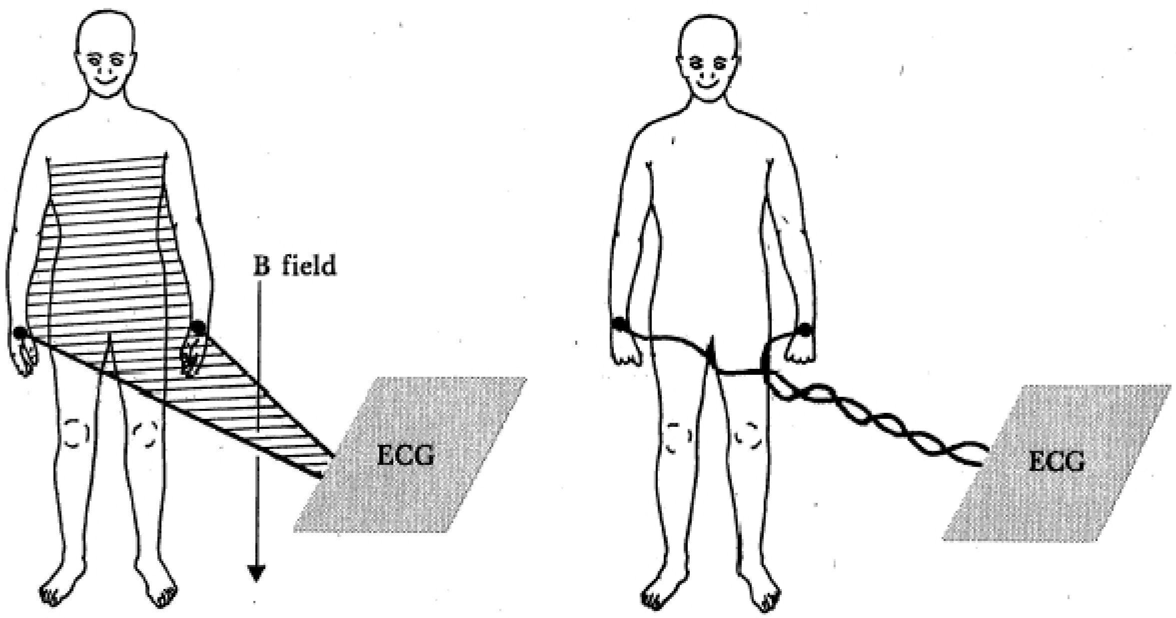
Electromagnetic interference

Magnetic fields in the environment can be picked up by a conductor and results in an induced current



Electromagnetic interference

Time-varying magnetic field induces a current in a closed loop

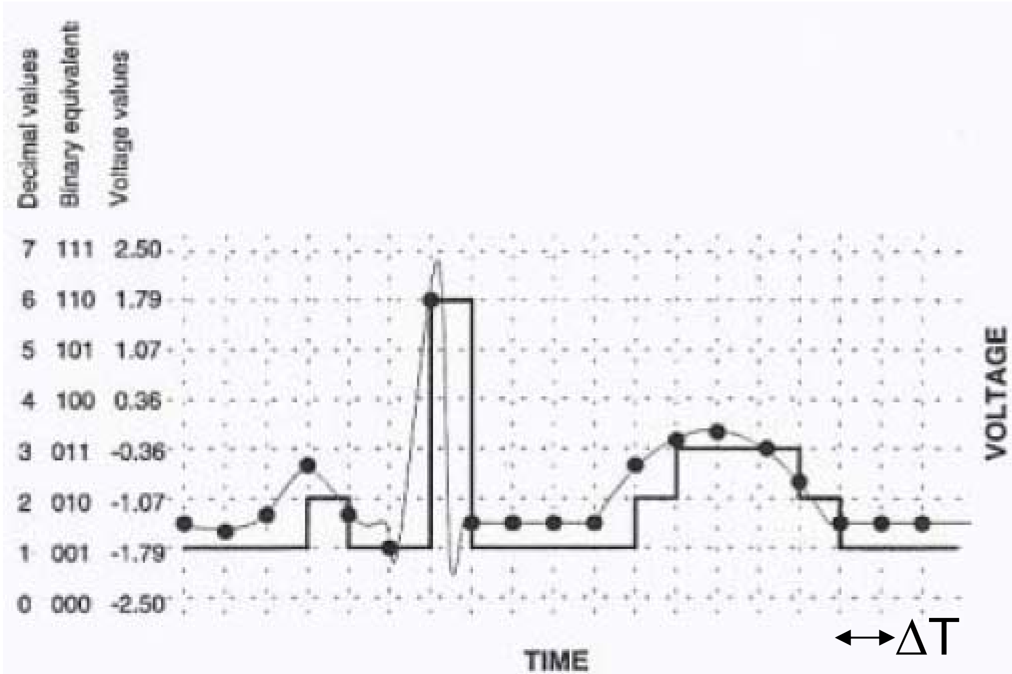


Reduce induced current by minimizing the area formed by the closed loop (twisting the lead wires and locating close to the body)

A/D conversion

Conversion of Analog signal to Digital (integer) numbers

Discrete (digital) numbers



Continuous (analog) values

Continuous time → discrete time interval ΔT

∴ A/D conversion is a process to

1. "Sample" a real world signal at finite time intervals
2. Represent the sampled signal with finite number of values

Sampling rate (frequency)

How fast do we need to sample? First define the sampling frequency:

$$f_{sampling} = \frac{1}{\Delta T} \qquad \text{(sample/s)}$$

Intuitively, we must sample fast enough to avoid distortion of the signal or loss of information $\Rightarrow$ easier to explain in the frequency domain

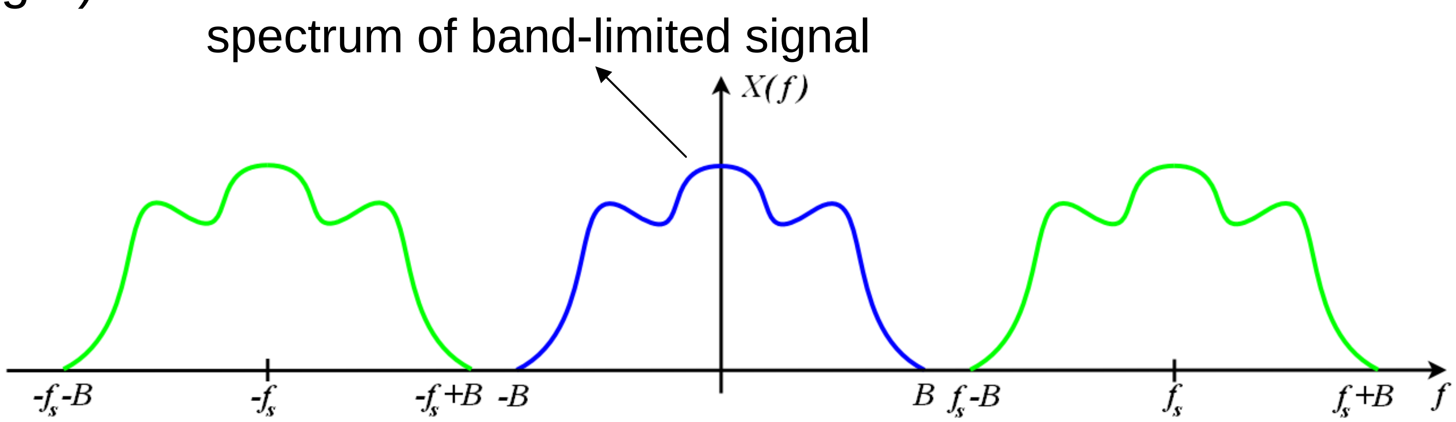
$$f_{sampling} > 2 f_{\max} \qquad \text{(sampling theorem)}$$

where f_{max} is the highest frequency present in the analog signal

What happens if the above criterion is not met?
- Loss of high frequency information in the signal
- Even worse, the data after sampling may contain false information about the original signal $\Rightarrow$ frequency aliasing

Sampling

In the frequency domain, sampling of the signal at f_{sampling} results in duplicates of the spectrum that are shifted by $m \cdot f_{\text{sampling}}$ (m is an integer)

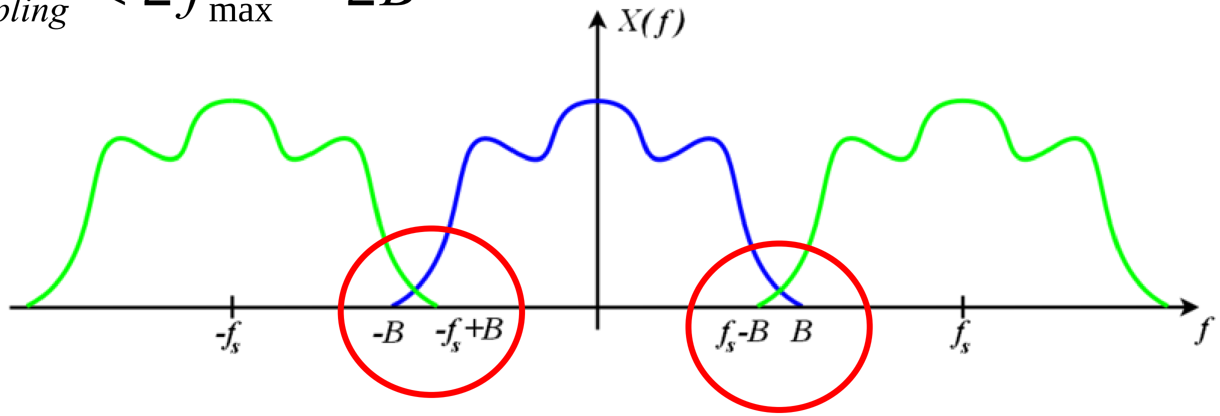


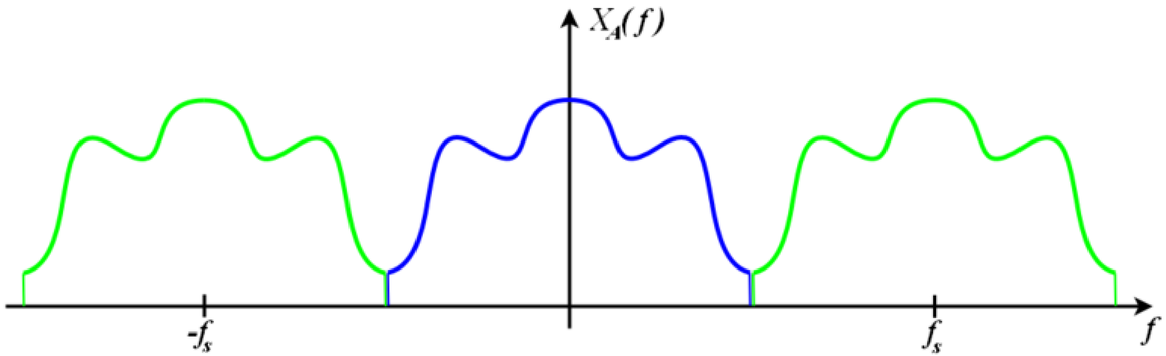
The sampling theorem essentially requires the spectrum of signal not overlapping with its duplicates

Frequency aliasing

When the sampling theorem condition is not satisfied

$$f_{sampling} < 2f_{\max} = 2B$$

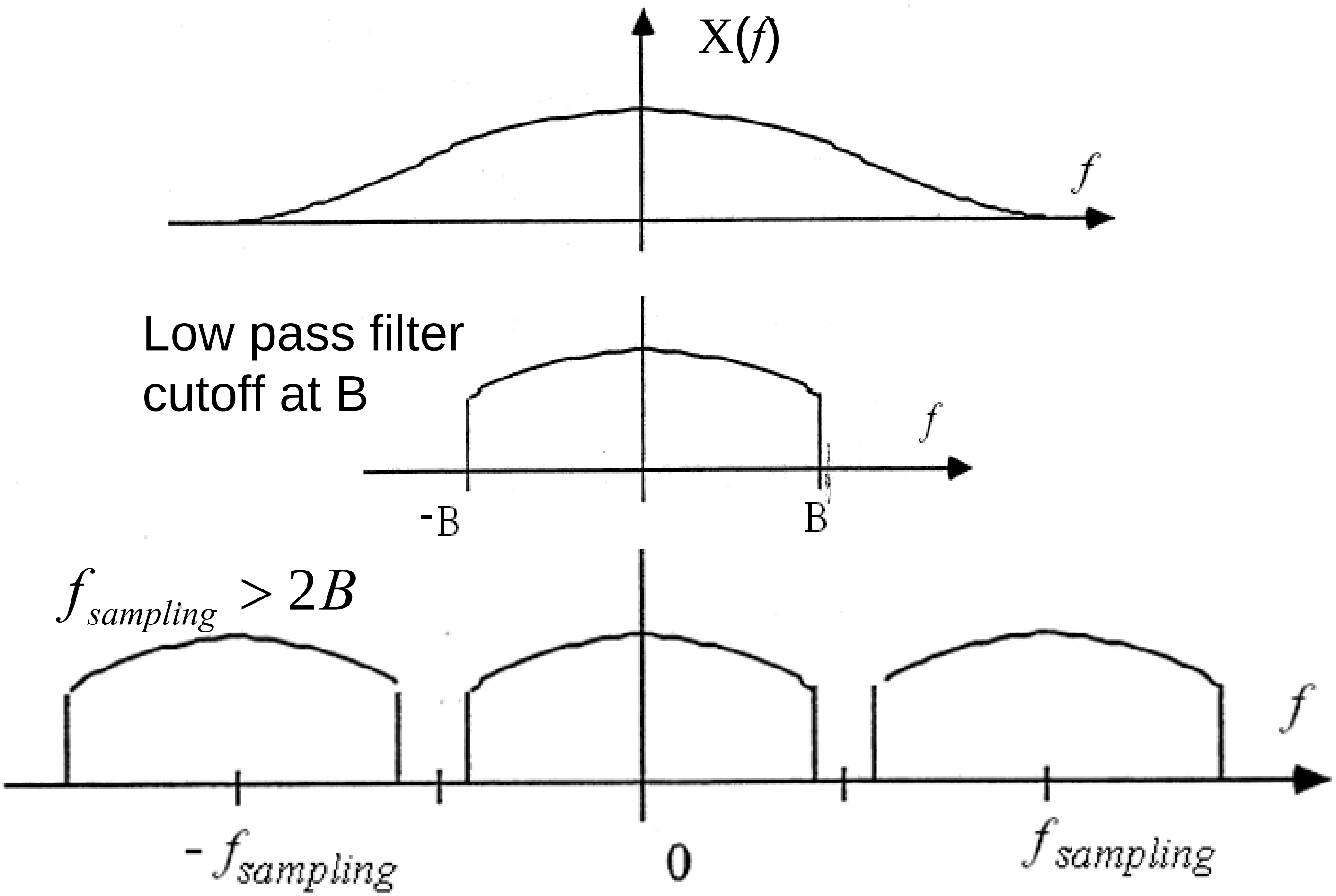




The high-frequency region overlaps and shape of spectrum is changed (summed). The process is <u>not reversible</u> $\Rightarrow$ information is lost

Anti-aliasing

- In the real world, no signal is strictly band-limited. But an effective bandwidth can be defined and used to find the sampling frequency
- To avoid frequency aliasing, a low-pass filter is applied to the signal prior to sampling



Data acquisition hardware

Lots of commercial products to choose from. National Instruments, for example, has families of products with a variety of features

Examples from National Instruments

Product	Bus	Analog Inputs[1]	Input Resolution (bits)	Aggregate Sampling Rate (kS/s)[2]	Input Range (V)
PCI-6014	PCI	16 SE/8 DI	16	200	±0.05 to ±10
PCI-6013	PCI	16 SE/8 DI	16	200	±0.05 to ±10
PCI-6010	PCI	16 SE/8 DI	16	200	±0.2 to ±5

[1]SE–Single-ended, DI–differential [2]All channels share one analog-to-digital converter.

Input resolution: for 16 bits $\Rightarrow 2^{16}$ digital levels

If the input range is ±5V , the minimum detectable signal level is

$$\frac{10V}{2^{16}} = \frac{10V}{65535} = 0.15mV$$

In practice, it is desirable to match the range of analog signal to the input range of the data acquisition hardware to increase the overall resolution of amplitude sampling

References

- The Art of Electronics (2nd ed.), by Paul Horowitz and Winfield Hill
  - Ch5: Active filters
  - Ch7
- Medical Instrumentation: application and design, 3rd ed., edited by John G. Webster
  - Ch3: Amplifiers and Signal Processing